THE SILVER-BURDETT ARITHMETICS

BOOK THREE

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THE SILVER-BURDETT ARITHMETICS

BOOK THREE

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SILVER, BURDETT AND COMPANY

BOSTON NEW YORK CHICAGO

SILVER–BURDETT
ARITHMETICS

Two Book Series

BOOK ONE
COMPLETE BOOK

Three Book Series

BOOK ONE
BOOK TWO
BOOK THREE

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PREFACE

THIS book is written to meet the needs of pupils of grades seven and eight and those of more advanced grades who wish to review arithmetic.

Like the first two books of the series, it aims to make pupils proficient in arithmetical processes, and to make them thinkers as well as doers.

To accomplish these ends it provides a great amount of drill work, both oral and written, and a wealth of well-graded problems of a varied character, yet practically all closely associated with the affairs of everyday life. In other words, it may be said that the aim of the book is to give the pupil such thorough grounding in arithmetic as will qualify him to take advantage of any opportunity that may be open to him, rather than to fit him for some special vocation in which he may not engage.

Attention is invited to the following features of the book:

1. *The thorough review and extension* of Fundamental Processes, Fractions, Decimals, and Denominate Amounts, designed to perfect the work done in the lower grades.

2. *The abundance and up-to-date character of the problems*, which are closely allied with a variety of activities and which are based on facts derived first hand from the most reliable sources.

3. The treatment of such subjects as *Parcel Post*, *Postal Savings Banks*, etc.

4. *The explanations of illustrative examples*, which set forth in correct form the logical steps leading to results, and which are designed to induce correct habits of thought and orderly arrangement of work.

5. *The treatment of Negotiable Paper and Banking*, in which are found photographic forms of business papers in everyday use and explanations of them in language free from technicalities.

6. *The extensive treatment of Mensuration* and the method employed of deriving most of the important principles of the subject.

7. *The topical arrangement* and the numerous reviews.

8. *The placing of the less important subjects* in a supplement.

9. *The omission* of subjects and problems which are *obsolete, overcomplex, and impracticable.*

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THIRD BOOK

WHOLE NUMBERS

NOTATION AND NUMERATION

Definitions

1. **Quantity** is anything that can be measured; as, *weight, length, value, time.*

2. A **unit** is any standard of reference employed in counting a collection or in measuring a quantity; as, *a pound, a yard, a dollar, a day.*

3. A **number** is the direct answer to the question: *How many?*

4. One is the simplest number and is called the **unit of number**, or **unity.**

5. The first nine numbers, sometimes called the **simple numbers**, are: *one, two, three, four, five, six, seven, eight, nine.*

6. An **amount** is an expression denoting the number and kind of units in a quantity; as, *two pounds, five yards, six dollars, five head.*

7. **Similar amounts** are such amounts as are composed of the same kind of units; as, *three bushels* and *five bushels.*

8. **Numeration** is the method of expressing numbers in words.

9. **Notation** is the method of expressing numbers by means of characters called **numerals**.

10. The **Arabic notation**, the system in general use, is so named because it was introduced into Europe by the Arabs.

11. The **Roman system of notation**, which is employed only for special purposes, is so named because it was the system in use among the ancient Romans.

Arabic System

12. The **Arabic numerals**, called **figures**, used to express numbers, are: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9; named, *naught, one, two, three, four, five, six, seven, eight, nine.*

13. The figure 0 is also called **cipher** or **zero**; the figures 1, 2, 3, 4, 5, 6, 7, 8, 9 are called **digits**.

14. Nine and one make *ten.* When ten is thought of as a single group, the group is called *a ten.* Two tens make *twenty;* three tens, *thirty;* four tens, *forty;* and so on to *ninety.* The tens are expressed in figures by writing 0 after each of the nine digits in turn; as, 10, 20, etc.

15. If each of the first nine numbers is united in turn with ten, the numbers *eleven, twelve, thirteen, fourteen* . . . to *nineteen* are formed; if united with twenty, the numbers *twenty-one, twenty-two* . . . to *twenty-nine* are formed; and so on until *ninety-one, ninety-two* . . . to *ninety-nine* are formed.

16. A number consisting of tens and units is expressed by two figures, the right-hand figure expressing the number of units, and the left-hand figure, the number of tens.

17. Ten tens make *one hundred.* One hundred thought of as a single group is called *a hundred.* Hundreds, like units and tens, are numbered from one to nine inclusive, forming the numbers *one hundred*, *two hundred*, *three hundred*, and so on to *nine hundred.* The hundreds are expressed in figures by writing 00 after each of the nine digits in turn; as, 100, 200, etc.

18. If each number less than one hundred is united in turn with one hundred, the numbers *one hundred one*, *one hundred two* . . . to *one hundred ninety-nine* are formed; if united with two hundred, the numbers *two hundred one*, *two hundred two* . . . to *two hundred ninety-nine* are formed; and so on, until the numbers *nine hundred one*, *nine hundred two* . . . to *nine hundred ninety-nine* are formed.

19. A number consisting of hundreds, tens, and units is expressed by three figures: the first, counting from the right, expresses the number of units; the second, the number of tens; and the third, the number of hundreds. When there are no tens, a cipher is written in tens' place; when no units, a cipher is written in units' place.

Thus, 268 expresses *two hundred sixty-eight;* 307, *three hundred seven;* 240, *two hundred forty.*

20. Units, tens, and hundreds are called, respectively, units of the *first*, *second*, and *third order.*

Exercise 1

1. Name the first nine numbers.

2. Nine and one combined form what number?

3. Name the tens.

4. State how the tens are expressed by figures.

5. How many numbers are there between ten and twenty?

6. The numbers between ten and twenty are formed by uniting with ten what numbers in turn?

7. With what number are the numbers from one to nine inclusive united to form the numbers between twenty and thirty?

8. State how the numbers between thirty and forty are formed; between eighty and ninety.

9. A number consisting of tens and units is expressed by how many figures?

10. In a number of two figures the left-hand figure expresses the number of what?

11. Ten tens combined form what number?

12. Name the hundreds.

13. How are the hundreds expressed by figures?

14. How many numbers are there between one hundred and two hundred?

15. A number consisting of units, tens, and hundreds is expressed by how many figures?

16. In a number of three figures, the first, counting from the right, expresses the number of what? the second? the third?

17. In 276 for what number does the 2 stand? the 7? the 6?

18. In a number of three figures what figure is written in tens' place when the number contains no tens? What figure is written in units' place when the number contains no units?

19. Write the number consisting of three units of the first order and two of the second.

20. Write the number consisting of five units of the first order, two of the second, and three of the third.

21. Write the number consisting of five units of the first order and one of the second.

22. Write the number consisting of eight units of the third order and four of the second.

23. Name the number made up of eight hundreds, five tens, and six units.

24. Name the number made up of four hundreds and seven tens.

25. Name the number made up of nine hundreds and three units.

21. Ten hundreds make *a thousand.* Thousands are numbered from one to nine hundred ninety-nine inclusive. Ten thousands make *a ten-thousand;* ten ten-thousands make *a hundred-thousand.*

22. Thousands, ten-thousands, and hundred-thousands are called, respectively, units of the *fourth*, *fifth*, and *sixth orders* and are written in the fourth, fifth, and sixth places on the *line of notation;* unit-thousands in the fourth place, ten-thousands in the fifth place, and hundred-thousands in the sixth place. Thousands are followed by three ciphers, occupying the hundreds', tens', and units' places.

23. If each number less than one thousand is united in turn with one thousand, the numbers from *one thousand* to *one thousand nine hundred ninety-nine* are formed; if united with two thousand, the numbers from *two thousand* to *two thousand nine hundred ninety-nine* are formed; and so on until the numbers from *nine hundred ninety-nine thousand* to *nine hundred ninety-nine thousand nine hundred ninety-nine* are formed.

24. A number consisting of thousands, hundreds, tens, and units is expressed by writing after the number of thousands, the number of hundreds, the number of tens, and the number of units. If units of any order are wanting, a cipher must be written in that order's place.

Thus, 375,845 expresses *three hundred seventy-five thousand eight hundred forty-five*, and 40,701, *forty thousand seven hundred one.*

25. The system of grouping and representing numbers which is in general use and which is based on the number *ten*, is called the **decimal system.** *In the decimal system of notation, ten units of any order make one unit of the next higher order.*

26. One thousand thousands make *a million*, and one thousand millions make *a billion*.

27. Millions and billions, like thousands, are numbered from one to nine hundred ninety-nine inclusive. Millions are written in the seventh, eighth, and ninth places on the line of notation and are followed by six ciphers; billions are written in the tenth, eleventh, and twelfth places, and are followed by nine ciphers.

Thus, 212,000,000 represents *two hundred twelve million;* 452,000,000,000 represents *four hundred fifty-two billion.*

28. If each number less than one million is united in turn with one million, the numbers between *one million* and *two million* are formed; if united with two million, the numbers between *two million* and *three million* are formed; and so on until the numbers between *nine hundred ninety-nine million* and *one billion* are formed. In a similar manner, if each number less than one billion is united with each of the billions, the numbers between the billions up to *a trillion* are formed.

29. A number may be expressed in figures by separating it into units, thousands, millions, billions, etc., and writing these in succession from the highest to the lowest, allowing three places for each after the highest.

Thus, five billion two hundred sixty-seven million thirty-eight thousand four hundred twenty-nine is written 5,267,038,429.

NOTE. When a number consists of more than four figures, it is customary to separate them by commas or spaces into groups of three figures each, beginning at the right.

Table of Notation

Trillions	Hundred-billions	Ten-billions	Billions	Hundred-millions	Ten-millions	Millions	Hundred-thousands	Ten-thousands	Thousands	Hundreds	Tens	Units
1	4	2	8	0	1	6	3	5	0	3	0	7
Trillions 5th period			Billions 4th period			Millions 3d period			Thousands 2d period			Units 1st period

Exercise 2

1. Beginning at billions, name the periods to units.

2. Name the orders in units' period.

3. Name the orders in millions' period.

4. Name the orders in thousands' period.

5. How many ciphers are required to write one billion?

6. Counting from the right, what is the third period?

7. Name the number made up of 267 units of the second period.

8. Name the number made up of seven units of the third order.

9. Name the number made up of 46 units of the third period.

10. Name the number made up of 5 units of the sixth order.

11. How many thousands make a million?

12. How many millions make a billion?

13. In the number 276,853 tell what number each figure represents.

14. In the number 7,840,009,060 what does the 7 represent? the 840? the 009? the 060?

15. What is the name of 1000 billions?

Read:

16. We use in a single year 90,000,000 cords of firewood and 44,000,000,000 board feet of lumber.

17. In a recent year 4,002,000 cords of wood were used in the manufacture of paper.

18. In a recent year the United States produced 3,125,713,000 bushels of Indian corn; 695,443,000 bushels of wheat; 1,126,765,000 bushels of oats; 162,227,000 bushels of barley; 33,039,000 bushels of rye; and 17,239,000 bushels of buckwheat.

Exercise 3

Copy the following, writing the numbers in figures:

1. The English language contains about six hundred thousand words.

2. Yellowstone National Park contains two million one hundred forty-two thousand seven hundred twenty acres, and Yosemite National Park contains seven hundred nineteen thousand six hundred twenty-two acres.

3. The United States produced, in a recent year, three hundred twenty-eight million one hundred ten thousand seven hundred forty-nine pounds of wool, and Argentina three hundred twenty-eight million seven hundred thirty-

one thousand pounds. The total production of wool in North America in that year was three hundred forty million three hundred twenty thousand seven hundred forty-nine pounds; the total in Europe was eight hundred four million nine hundred five thousand seventy-four pounds; in South America four hundred thirty-six million seven hundred sixteen thousand pounds.

4. The English language is spoken by more than one hundred fifty million people; the German, by more than one hundred twenty million people; the Russian, by more than ninety million; the French, by more than sixty million; the Spanish, by more than fifty million; the Italian, by more than forty million; and the Portuguese, by more than forty million.

5. The quantity of salt produced in this country in 1908, according to statistics compiled by the United States Geological Survey, amounted to twenty-eight million eight hundred twenty-two thousand sixty-two barrels of two hundred eighty pounds each, valued at seven million five hundred fifty-three thousand six hundred thirty-two dollars. The states producing the greatest quantity were as follows: Michigan, ten million one hundred ninety-four thousand two hundred seventy-nine barrels; New York, nine million seventy-six thousand seven hundred forty-three barrels; Ohio, thirteen million four hundred twenty-seven thousand four hundred seventy-eight barrels; and Kansas, two million five hundred eighty-eight thousand eight hundred fourteen barrels.

Reading Numbers Approximately

30. The distance from New York to Prescott, Ariz., is 2861 miles. **Approximate** distances from New York to Prescott are 3000 miles, 2900 miles, and 2860 miles. 3000 miles expresses the distance *correct to the nearest 1000 miles*, as 3000 miles is nearer 2861 miles than is 2000 miles. 2900 miles expresses the distance *correct to the nearest 100 miles*, as 2900 miles is nearer 2861 miles than is 2800 miles. 2860 miles expresses the distance *correct to the nearest 10 miles*, as 2860 miles is nearer 2861 miles than is 2870 miles.

DISTANCES FROM NEW YORK TO CITIES IN UNITED STATES

Cities	Miles	Cities	Miles	Cities	Miles
Birmingham, Ala.	1,043	New Orleans, La.	1,365	Seattle, Wash.	3,184
Carson City, Nev.	3,016	Jackson, Miss.	1,501	Vicksburg, Miss.	1,282
Cheyenne, Wyo.	1,966	St. Louis, Mo.	1,060	Portland, Ore.	3,248
San Francisco, Cal.	3,183	Savannah, Ga.	844	Galveston, Tex.	2,182

Exercise 4

(Refer to Table, Sec. 30, in answering these questions.)

1. Read the distance from New York to each of the cities named in the table.

2. Read the distance from New York to each of the cities named in the table, correct to the nearest 10 miles.

3. Read the distance from New York to each of the cities named in the table, correct to the nearest 100 miles.

Pictures of Numbers

31. Numbers are sometimes represented by pictures called **graphs.**

1. In Figure 1 each line opposite the name of a state stands for the amount of arid land in that state, each one eighth inch of the line representing 1 million acres. In Arizona there are two million acres of arid land. Read the amount of arid land in each state.

Fig. 1

2. In Figure 2 each unit of a heavy line represents 1 million 500-pound bales of cotton exported from the United States during the year named opposite the line.

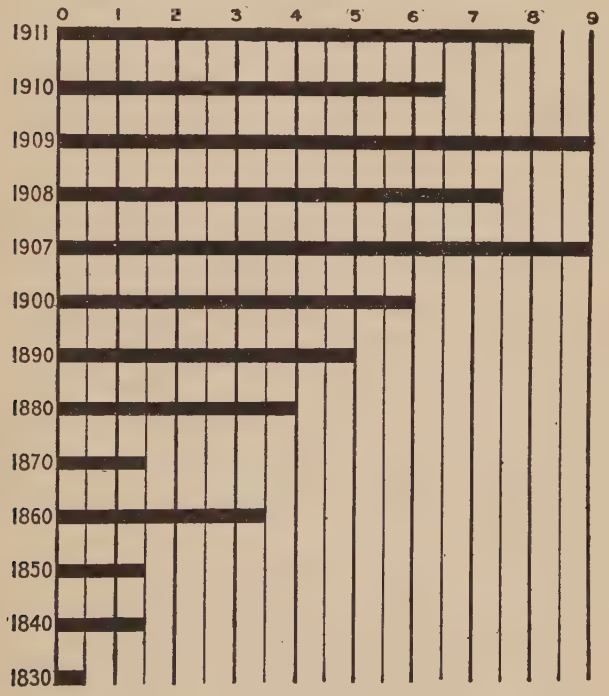


Fig. 2

(Notice that the line opposite 1911 consists of 8 units, each divided into halves. In that year there were exported 8 million 500-pound bales.)

Read the number of bales exported each year.

The Roman System

32. The Romans represented numbers by seven letters, and combinations of them. These letters are I, representing *one;* V, *five;* X, *ten;* L, *fifty;* C, *one hundred;* D, *five hundred;* and M, *one thousand.*

33. The combinations used to represent the nine simple numbers, the tens, the hundreds, and the thousands, are formed according to the following principles:

1. *Every time a letter is repeated, its value is repeated.*

2. *When a letter or a combination of letters of less value is written after a letter or a combination of letters of greater value, the sum of the values is expressed.*

3. *When a letter of less value is written before one of greater value, the difference of their values is expressed.*

4. *A dash placed over a letter or a combination of letters makes the letter or combination represent thousands.*

Table

Units	Tens	Hundreds	Thousands
I = 1	X = 10	C = 100	M = 1000
II = 2	XX = 20	CC = 200	MM = 2000
III = 3	XXX = 30	CCC = 300	MMM = 3000
IV = 4	XL = 40	CD = 400	$\overline{\text{IV}}$ = 4000
V = 5	L = 50	D = 500	$\overline{\text{V}}$ = 5000
VI = 6	LX = 60	DC = 600	$\overline{\text{VI}}$ = 6000
VII = 7	LXX = 70	DCC = 700	$\overline{\text{VII}}$ = 7000
VIII = 8	LXXX = 80	DCCC = 800	$\overline{\text{VIII}}$ = 8000
IX = 9	XC = 90	CM = 900	$\overline{\text{IX}}$ = 9000

Note. IIII = 4; XXXX = 40; CCCC = 400, are also in use.

34. In writing numbers between those in the table, Sec. 33, hundreds are written after thousands, tens after hundreds, and units after tens.

Thus, 1567 is written MDLXVII; 2053, MMLIII; 4508, $\overline{\text{IV}}$DVIII; 3600, MMMDC.

NOTE. The use of the Roman system of notation is confined chiefly to the numbering of chapters and sections of books, pages of prefaces and introductions, medical prescriptions, public documents, divisions on clock dials, etc.

Exercise 5

Write by the Roman system the following:

1. The nine simple numbers.
2. The nine tens.
3. The nine hundreds.
4. The numbers between 20 and 30.
5. The numbers between 60 and 70.
6. The numbers between 90 and 100.

7. 490	8. 940	9. 1492	10. 1664
11. 550	12. 1876	13. 1861	14. 1812
15. 1914	16. 1919	17. 1926	18. 1899

Write in figures the following:

19. CIX	20. MIV	21. LXI
22. XIX	23. XLVIII	24. MCMIX
25. XXV	26. XCIV	27. MDCLXVI
28. LXVII	29. CXIV	30. CDLIII
31. XLIV	32. CCXIX	33. MCCCXXXV
34. XCIX	35. CDXLIV	36. MCMXIX
37. MCMXX	38. MDCCLXXVI	39. MDCCCIX

FUNDAMENTAL PROCESSES

To the Teacher. Pupils have already learned how to *add*, *subtract*, *multiply*, and *divide*, but they should practice these operations until they can perform them rapidly and with accuracy.

Addition

35. Addition is the process of uniting two or more numbers into one number called their **sum**.

Note. Similar amounts, and only such, may be united (added); the result (sum) is similar to the amounts added.

Exercise 6

(Accuracy and Speed Test. Practice until you can add the exercises on this page in less than 5 minutes. Check the result by seeing if the sum found when the columns are added upward agrees with the sum found when the columns are added downward.)

1.	2.	3.	4.
1436780	2725012	3456	332
2007651	9901783	4201617	164504
3978641	1617181	302143	2
6521096	8006438	2552340	1660
7896452	3456721	120	916
4658792	7174562	164033	5152430
9786534	2406543	22578	112989
6005321	4511792	1670	13
5227001	5506713	2543789	2345
1978675	6124750	325	415679
7432186	5837592	4786	4867
5321475	8763248	285	3241
1876432	1837651	19	1876

5.

THE COTTON CROP OF THE UNITED STATES BY STATES FOR FOUR YEARS

States	1st Year	2d Year	3d Year	4th Year	Total Crop
	Bales	Bales	Bales	Bales	
North Carolina . .	675,000	725,000	660,000	777,000	
South Carolina . .	1,205,000	1,290,000	1,188,000	1,244,000	
Georgia	1,920,000	2,015,000	1,932,000	1,881,000	
Florida	62,000	75,000	64,000	68,000	
Alabama	1,202,000	1,385,000	1,088,000	1,230,000	
Mississippi	1,495,000	1,745,000	1,137,000	1,271,000	
Louisiana	700,000	528,000	286,000	274,000	
Texas	2,309,000	3,908,000	2,653,000	3,135,000	
Arkansas	775,000	1,020,000	728,000	838,000	
Tennessee	290,000	350,000	253,000	340,000	
All others	949,000	788,000	662,000	1,074,000	
Total crop . .					

Copy the above report, fill in the totals, and see that the sum of the totals of the crop by years equals the sum of the totals of the crop by states.

Subtraction

36. Subtraction is the process of finding by how many units one number exceeds another or how many are left after taking part of a number from it. The greater number is called the **minuend**, the smaller the **subtrahend**, and the result the **difference** or the **remainder**.

Note. Similar amounts, and only such, may also be subtracted; the result is similar to the amounts subtracted.

Exercise 7

Find the difference and check by seeing that the sum of the subtrahend and the difference equals the minuend:

1.	2.	3.	4.
7532	45962	38746	$ 3289.25
3401	25416	11250	$ 1096.15
5.	**6.**	**7.**	**8.**
4505	76095	87584	$ 7850.75
2354	59432	65002	$ 5611.25

Add upward; subtract across; check results:

9.
$$\begin{array}{rrr} 4165 & - & 1030 = \bullet \\ 7813 & - & 168 = \bullet \\ 5450 & - & 430 = \bullet \\ 2452 & - & 25 = \bullet \\ 1599 & - & 740 = \bullet \\ 6634 & - & 4537 = \bullet \\ \bullet & - & \bullet = \bullet \end{array}$$

10.
$$\begin{array}{rrr} 56742 & - & 16053 = \bullet \\ 78100 & - & 124 = \bullet \\ 91633 & - & 2456 = \bullet \\ 43012 & - & 13 = \bullet \\ 24357 & - & 4901 = \bullet \\ 91005 & - & 1675 = \bullet \\ \bullet & - & \bullet = \bullet \end{array}$$

Multiplication

37. **Multiplication** is the process of taking one number or amount, called the **multiplicand**, additively as many times as there are units in another number called the **multiplier**. The result of multiplication is called the **product**.

38. The sign of multiplication ($\times$) is read **times** when the multiplier is written before it and **multiplied by** when the multiplier is written after it.

Thus, 4×7 da. is read 4 *times* 7 da.; 7 days $\times$ 4 is read 7 days *multiplied by* 4.

39. The multiplier is always a *number*, and if the multiplicand is an amount, the product must be a similar amount.

REMARK. It is evident from Sec. 39 that such expressions as "2 ft. × 3 ft. = 6 sq. ft.," "2 ft. × 5 sq. ft. = 10 cu. ft.," and "15 × 2 hr. = 30°," cannot be justified and should not be employed.

Exercise 8

Read the first nine examples; name the multiplicand, the multiplier, and give at sight the product:

1. 3×7 bu.	2. $15° \times 2$	3. 4 sq. yd. × 5
4. 7 lb. × 3	5. 5 doz. × 9	6. $\$10 \times 6$
7. $9 \times \$10$	8. 4×9 lb.	9. 6 lb. × 8

10. State a short way of multiplying a whole number by 10.

11. Name at sight the product:

10×13;	10×25;	10×37;
10×99;	10×101;	10×990.

12. State a short way of multiplying a whole number by 100.

13. State at sight the product:

100×7;	100×14;	100×29;
100×90;	100×89;	100×100.

14. State a short way of multiplying a whole number by 1000.

15. State at sight the product:

1000×7; 1000×17; 1000×25.

Illustrative Examples

1. Multiply 5768 by 473.

5768	The multiplier is $400 + 70 + 3$.
473	What is the first partial product?
17304	$70 \times 5768 =$ what?
40376	What is omitted from the third partial product?
23072	Read the third partial product.
2728264	What is the product?

2. Multiply 5768 by 3007:

5768	The multiplier is $3000 + 7$.
3007	How is 40376 obtained?
40376	What is omitted from the second partial prod-
17304	uct?
17344376	What is the product?

Exercise 9

Multiply across; add upward; check results:

1. $756 \times 3406 = \bullet$
   $756 \times 4879 = \bullet$
   $756 \times 9543 = \bullet$
   $756 \times 2058 = \bullet$
   $756 \times 8878 = \bullet$
   $756 \times \bullet = \bullet$

2. $8009 \times 3457 = \bullet$
   $8009 \times 8705 = \bullet$
   $8009 \times 7056 = \bullet$
   $8009 \times 5760 = \bullet$
   $8009 \times 8956 = \bullet$
   $8009 \times \bullet = \bullet$

3. Multiply 11826 by 11826.
4. Multiply 12363 by 12363.
5. Multiply 12543 by 12543.
6. Multiply 14676 by 14676.
7. Multiply 15681 by 15681.
8. Multiply 15963 by 15963.

Notice that in Examples 3–6 each product contains the nine digits.

Division

40. **Division** is the process of finding one of the equal parts of a number or an amount, or how many times one number or amount is contained in another. The **dividend** is the number or amount divided. The **divisor** is the number or amount by which we divide. The **quotient** is the result of the division.

41. If in division there is an undivided part of the dividend less than the divisor, this undivided part is called the **remainder**.

42. Finding one of the equal parts of a number is sometimes called **partitive division**, or **partition**. In this kind of division the divisor is always a number, and if the dividend is an amount, the quotient and the remainder must be amounts similar to the dividend.

43. Finding how many times one number or amount is contained in another is sometimes called **comparative division**. In this kind of division the quotient is always a number, and if the dividend is an amount, the divisor and the remainder must be amounts similar to the dividend.

REMARK. It is evident from Secs. 42 and 43 that such expressions as "$45° \div 15 = 3$ hr.," "6 sq. ft $\div$ by 2 ft. $= 3$ ft.," and "12 cu. ft. $\div$ 4 sq. ft. $= 3$ ft." cannot be justified and should not be employed.

Exercise 10

Read the first ten examples and name at sight the quotient and the remainder in each:

1. $\$50 \div 7$
2. 52 bu. $\div 5$
3. 61 qt. $\div 8$ qt.
4. 55 ft. $\div 6$
5. 60 yd. $\div 8$ yd.
6. 53 da. $\div 7$ da.

7. 68 men ÷ 8

8. $46 ÷ $8

9. 59 in. ÷ 6 in.

10. 87¢ ÷ 9

11. If the divisor is 6 and the quotient is 8, what is the dividend, if there is no remainder?

12. If the divisor is 7, the quotient 6, and the remainder 5, what is the dividend?

13. State a short way of dividing a number ending in a cipher by 10.

14. Divide at sight:

150 by 10; 270 by 10; 1200 by 10.

15. State a short way of dividing a whole number ending in two ciphers by 100.

16. Divide at sight:

600 by 100; 1200 by 100; 5000 by 100.

17. State a short way of dividing a whole number ending in three ciphers by 1000.

18. Divide at sight:

5000 by 1000; 17,000 by 1000; 50,000 by 1000.

19. How many tens are there in 7000? how many hundreds? how many thousands?

20. How many tens are there in 9,000,000? how many hundreds? how many thousands? how many ten-thousands? how many hundred-thousands?

Illustrative Examples

1. Divide 134,568 by 452.

```
        297
452)134568
     904
     4416
     4068
      3488
      3164
       324
```

How is the figure 2 in the quotient obtained, and for what does it stand?

How was the 904 obtained?

How was the 4416 obtained?

What was divided to obtain the figure 9 in the quotient, and for what does it stand?

How was the 4068 obtained?

How was the 3488 obtained?

What was divided to obtain the 7 in the quotient?

How was the 3164 obtained?

What is the quotient, and what the remainder?

Check: $452 \times 297 + 324 = 134{,}568$.

NOTE. — The quotient may be placed to the right of the dividend thus:

452)134568(297

2. Divide 716,239 by 346.

```
        2070
346)716239
     692
     2423
     2422
       19
```

How may the first quotient figure, 2, be estimated and for what does it stand?

$346 \times 2000 =$ what?

What does the 692 represent?

How may the second quotient figure, 0, be estimated?

What was divided to obtain the 7 in the quotient, and for what does it stand?

$346 \times 70 =$ what?

What is the quotient, and what the remainder?

Check: $346 \times 2070 + 19 = 716{,}239$.

NOTE. The method of dividing by expressing the successive partial dividends is called **long division.** The method of dividing without expressing the partial dividends is called **short division.**

Special Forms

Divide 6783 by 10; by 100; by 1000.

$10\overline{)6783}$ $678|3$ $\qquad$ $100\overline{)6783}$ $67|83$ $\qquad$ $1000\overline{)6783}$ $6|783$

Name the quotient and remainder in each example.

Cutting off the last figure of a number divides the number by 10, and the figure cut off expresses the remainder.

Cutting off the last two figures of a number divides it by 100, and the figures cut off express the remainder; and so on.

Exercise 11

Divide across; add upward; check results:

1. $1541 \div 67 = \bullet$
   $4489 \div 67 = \bullet$
   $6767 \div 67 = \bullet$
   $14070 \div 67 = \bullet$
   $\bullet \div 67 = \bullet$

2. $2848 \div 89 = \bullet$
   $5785 \div 89 = \bullet$
   $8099 \div 89 = \bullet$
   $4183 \div 89 = \bullet$
   $\bullet \div 89 = \bullet$

3. $11400 \div 456 = \bullet$
   $42408 \div 456 = \bullet$
   $35568 \div 456 = \bullet$
   $29184 \div 456 = \bullet$
   $\bullet \div 456 = \bullet$

4. $366768 \div 849 = \bullet$
   $722499 \div 849 = \bullet$
   $515343 \div 849 = \bullet$
   $248757 \div 849 = \bullet$
   $\bullet \div 849 = \bullet$

5. Divide, at sight, each of the following numbers by 10, stating the quotient and the remainder:

546; 679; 591; 1021; 1466; 1923.

6. Divide, at sight, each of the following numbers by 100, stating the quotient and the remainder:

711; 910; 1565; 7706; 8109; 9001.

7. Divide, at sight, each of the following numbers by 1000, stating the quotient and the remainder:

1111; 2760; 4723; 6400; 7913; 8001.

Miscellaneous Problems

Exercise 12

1. Add 2 million 65 thousand 4 hundred, 14 million 860 thousand 16, 17 million 4 thousand 220, and 175 million 24 thousand 3 hundred 6.

2. The following are recent estimates of the wealth of the nations named; find their total wealth.

United States 130 billion dollars; Great Britain and Ireland 82 billion dollars; France 65 billion dollars; Germany 60 billion 500 million dollars; Russia 40 billion dollars; Austria-Hungary 25 billion dollars; Italy 20 billion dollars; Belgium 9 billion dollars; Spain 5 billion 400 million dollars; Netherlands 5 billion dollars; Portugal 2 billion 5 hundred million dollars; Switzerland 2 billion 4 hundred million dollars.

3. If two trains are running from a certain point, one due north at the rate of 25 miles an hour, and the other due south at the rate of 40 miles an hour, how far apart are they in 3 hours?

4. Below are given the expenditures in a recent year in the United States for the articles named; find the total.

Intoxicating liquors	$2,000,000,000
Tobacco	1,200,000,000
Jewelry	800,000,000
Automobiles	500,000,000
Confectionery	200,000,000
Soft drinks	120,000,000
Tea and Coffee	100,000,000
Millinery	90,000,000
Patent medicines	80,000,000
Chewing gum	13,000,000

5. After borrowing $175 a farmer had sufficient to pay for 25 acres of land at $50 per acre. How much had he at first?

6. A dealer owns two tracts of land, the first containing 476 acres and the second 95 acres more than the first. How many acres do both tracts contain?

7. A laborer bought a house for $1850; he paid $800 cash and agreed to pay the balance in monthly payments of $15 each. How many of these monthly payments must he make?

8. A carload of coal weighed 67,500 lb.; how many tons (2000 lb.) will this make, and how many pounds over?

9. What was the average gain per head on 25 horses bought for $4375 and sold for $5000?

10. A farmer mixed an equal number of bushels of corn and of oats for horse feed and found that the mixture

weighed 1760 lb. How many bushels of feed had he, a bushel of corn weighing 56 lb. and a bushel of oats 32 lb.?

11. In a certain city the average age of citizens over 70 years old is 75 years and the sum of their ages is 900 years. How many such citizens are there?

12. A carload of 35 trolley rails, each 56 ft. long and weighing 45 lb. to the foot, weighs how much?

13. Find the sum of all the numbers that can be written each with the figures 3, 4, 5, and 6.

14. In an election in which A, B, and C were the candidates, A received 160 votes, B 120 votes, and C sufficient to elect him by a majority of 12; how many votes were cast?

15. How far apart are two towns if one is 365 miles east of a certain point and the other is 120 miles east of the same point?

16. The farm land in the United States, exclusive of buildings, was valued at $13,058,008,000 in 1900 and $28,386,770,000 in 1910. What was the average increase in value for each of the 10 years?

17. Find the weight of a load consisting of 10 bbl. of flour (196 lb.), 3 bbl. of salt (280 lb.), and 5 bbl. of beef (200 lb.).

18. Recently it was estimated that 40,744,200 gal. of water was furnished each 24 hr. to 373,800 persons in Milwaukee. How much was furnished each person?

19. A farmer's corn crib held 1026 bu. of corn; his corn crop filled the crib and he had 126 bu. over. What was his corn crop?

20. Recently it was estimated that each person in Philadelphia used 210 gal. of water every 24 hr., but that each person should not use more than 150 gal. in that time, the additional amount used being wasted. Estimating the population of Philadelphia at 1,600,000, how much water did they waste every 24 hr.?

21. A man borrowed $5000. He paid back $2750 at one time, $300 at another, and the remainder in three equal payments. How much was each of these three payments?

22. Minnesota contains 58,915 sq. mi., and Delaware 2050 sq. mi. Minnesota would make how many states the size of Delaware and how many square miles over?

23. How many eggs are there in 24 crates each containing 36 doz.?

24. The following are the new possessions of the United States and the areas thereof. Find the total of the areas.

Philippines	115,026 sq. mi.
Porto Rico	3,435 sq. mi.
Hawaii	6,449 sq. mi.
Tutuila and islets	77 sq. mi.
Guam	210 sq. mi.
Panama Canal Zone	474 sq. mi.

25. A clock that strikes the hours strikes how many times in a week?

26. A farmer was paid for a horse 9 ten-dollar bills, 6 five-dollar bills, 8 two-dollar bills, and 14 dollar bills. How much was he paid for the horse?

27. A merchant was given a 20-dollar bill in payment of a debt of $2. What bills did he give in change if he used the least number possible?

28. The minuend is 274 and the subtrahend is 196. What is the difference?

29. How many pounds of coal remain on a car after 25,250 lb. have been unloaded, if at first it contained 58,600 lb.?

30. The multiplicand is 1728 and the multiplier is 144. What is the product?

31. The divisor is 46, the quotient 19, and the remainder 25. What is the dividend?

32. The freight rate on electrical goods shipped first-class from Chicago to Portland, Ore., is $3 per 100 pounds. Find the freight on a shipment of 2600 pounds of these goods.

33. A farm that cost $50 per acre was sold for $65 per acre. If the gain was $1125, how many acres did the farm contain?

34. How long will 4 men, each husking an average of 40 shocks of corn a day, require to husk 4800 shocks?

35. The multiplicand is 275 and the product is 218,350. What is the multiplier?

Arithmetical Expressions

44. An **arithmetical expression**, or simply an **expression**, is any number or combination of numbers expressed in arithmetical symbols.

Thus, 7, $3+2$, $6+2-5$, 3×6, and $8\div 4\times 2$ are expressions.

45. Parts of an expression connected by a plus or a minus sign are called **terms** of the expression.

Thus, in the expression $6+12-2\times 4+6\div 2$ there are 4 terms; namely, 6, 12, 2×4, and $6\div 2$.

46. The operations of multiplication and division indicated in any term of an expression should be performed before the terms are combined; and if two or more such operations are indicated in the same term, they should be performed in the order of their occurrence from left to right.

Thus, in the expression $6\times 8\div 4+12\div 6\times 2-24\div 4\div 2$,
the first term, $6\times 8\div 4$ equals $48\div 4$, or 12,
the second term, $12\div 6\times 2$ equals 2×2, or 4,
the third term, $24\div 4\div 2$ equals $6\div 2$, or 3,
and the whole expression, equals $12+4-3$, or 13.

47. The value of an expression is the result obtained by performing the operations indicated.

48. When an expression is enclosed in **parentheses** ( ), in **braces** { }, in **brackets** [ ], or placed under the **vinculum** $\overline{\quad}$, the value of the expression is to be found and used instead of the expression as a whole.

Exercise 13

Find the value of:

1. $5+4+7-2-5\times 2$
2. $6\times 5+3\times 2-4\times 3$
3. $12\div 6+6\div 2-4\div 4$
4. $64\div 8+6\times 4\div 2$
5. $8\div 4\times 2+8\times 4\div 2-8\div 4\div 2$
6. $4\times\sqrt{125+250}-6\times(250-125)+\sqrt{500-250}\div 25$
7. $375\times 40\div 50+1200\div 60\times 40-(1000-600)$
8. $1760\div 40+3\times 1200\div 60+4\times 1500\div 30$

FACTORING

49. The **factors** of a number are such numbers as multiplied together will produce the number.

50. A number that has no other factor than itself and 1 is a **prime number.** A number composed of other factors than itself and 1 is a **composite number.**

51. When all the factors of a number are prime numbers, they are called the **prime factors** of the number.

52. An **even number** is a number that is divisible by 2; an **odd number,** one that is not divisible by 2.

Exercise 14

Name at sight two factors that will produce:

1. 54; 63; 80; 96.
2. 120; 132; 110.
3. 56; 48; 39; 72.
4. 108; 100; 150.

Tell which of the following numbers are prime:

5. 11; 12; 13; 14; 15; 16; 17; 18; 19; 20.

6. 41; 43; 47; 49; 55; 59; 63; 31; 39; 81.

7. Name all the composite numbers from 30 to 50 inclusive.

8. Name the prime factors of 12; 18; 21; 24; 25; 27; 28; 30; 33.

9. Name the odd numbers to 20.

Divisors

53. A number that will divide a given number without a remainder is called an **exact divisor** or a **divisor** of the given number.

54. A divisor of each of two or more numbers is called a **common divisor** of them.

55. The greatest number that will divide each of two or more numbers is called their **greatest common divisor** (g. c. d.).

56. Some easy tests of divisibility:

A number is divisible:

by 2, if it ends in 0, 2, 4, 6, or 8.
by 3, if the sum of its digits is divisible by 3.
by 4, if the number expressed by the two right-hand digits is divisible by 4.
by 5, if it ends in 0 or 5.
by 6, if it is divisible by 2 and by 3.
by 9, if the sum of its digits is divisible by 9.
by 12, if it is divisible by 3 and by 4.
by 15, if it is divisible by 3 and by 5.
by 25, if it ends in 00, 25, 50, or 75.

57. If a number is divided by one of its prime divisors, the quotient by one of its prime divisors, the next quotient by one of its prime divisors, and so on until the quotient becomes 1, these successive divisors are called the **continued prime divisors** of the number.

$$\begin{array}{r|l} 2 & 210 \\ 3 & 105 \\ 5 & 35 \\ 7 & 7 \\ \hline & 1 \end{array}$$

Thus, 2, 3, 5, and 7 are the continued prime divisors of 210.

58. The continued prime divisors of a number are the prime factors of that number.

$$\begin{array}{r|l} 2 & 210 \\ 3 & 105 \\ 5 & 35 \\ 7 & 7 \\ \hline & 1 \end{array}$$

Thus, 2, 3, 5, and 7, the continued prime divisors of 210, are the prime factors of 210; for, $210 = 2 \times 3 \times 5 \times 7$.

59. The continued common prime divisors of two or more numbers are their common prime factors.

$$\begin{array}{r|rrr} 2 & 84 & 126 & 210 \\ 3 & 42 & 63 & 105 \\ 7 & 14 & 21 & 35 \\ \hline & 2 & 3 & 5 \end{array}$$

Thus, 2, 3, and 7, the continued common prime divisors of 84, 126, and 210, are the common prime factors of 84, 126, and 210.

60. Any of the common prime factors of two or more numbers, or the product of any two or more of such factors, is a common divisor of the numbers.

Thus, the common prime factors of 84, 126, and 210 are 2, 3, and 7; hence, 84, 126, and 210 are each divisible by 2, 3, 7, 2×3, 2×7, 3×7, and $2 \times 3 \times 7$.

61. The product of all the common prime factors of two or more numbers is their greatest common divisor.

Thus, the greatest common divisor of 84, 126, and 210 is $2 \times 3 \times 7$

Exercise 15

1. Name at sight two exact divisors of 54; 63; 90; 72; 56; 39; 66.

2. Name a common divisor of 12 and 18; 54 and 63; 72 and 96; 110 and 120.

3. Name a common divisor of 12, 18, and 24; 42, 49, and 56; 54, 63, and 72.

Tell at sight which of the numbers 2, 3, 4, 5, 6, 9, 12, 15, and 25, will divide:

4. 84 5. 99 6. 105 7. 75 8. 78 9. 146

10. 230 11. 324 12. 425 13. 423 14. 552 15. 627

Illustrative Example

Find the greatest common divisor of 90, 120, and 240.

Explanation:

2	90	120	240
3	45	60	120
5	15	20	40
	3	4	8

2, 3, and 5 are the common prime factors of 90, 120, and 240.

The product of all the common prime factors of two or more numbers is their greatest common divisor.

$\therefore 2 \times 3 \times 5$, or 30, is the g. c. d. of 90, 120, and 240.

Exercise 16

Find the greatest common divisor of:

1. 42, 49, and 56
2. 40, 56, and 72
3. 70, 80, and 140
4. 63, 81, and 90
5. 144, 180, and 200
6. 245, 280, and 322
7. 138, 276, and 414
8. 288, 384, and 528
9. 330, 495, and 660
10. 432, 540, and 756
11. 224, 336, and 560
12. 234, 312, and 390
13. 1875, 3125, and 4375
14. 392, 504, and 560

Find all the common divisors of:

15. 150, 180, and 200
16. 180, 300, and 210
17. 300, 750, and 1050
18. 420, 630, and 1050

Multiples

62. A number that can be exactly divided by a given number is called a **multiple** of the given number.

63. A number that can be exactly divided by each of two or more numbers is called a **common multiple** of them.

64. The least number that can be exactly divided by each of two or more numbers is called their **least common multiple** (l.c.m.).

65. The least common multiple of two or more numbers is the product of all their different prime factors, each taken the greatest number of times that it occurs in any one of them.

Illustrative Examples

1. Find the least common multiple of 18, 24, and 36.

$18 = 2 \times 3 \times 3$
$24 = 2 \times 3 \times 2 \times 2$
$36 = 2 \times 3 \times 3 \times 2$

Explanation:

The different prime factors of 18, 24, and 36 are 2 and 3. The greatest number of times that 2 occurs as a factor in any of the numbers is *three;* the greatest number of times that 3 occurs as a factor in any of the numbers is *two.*

$\therefore$ $2 \times 2 \times 2 \times 3 \times 3$, or 72, is the l. c. m. of 18, 24, and 36.

2. Find the l. c. m. of 120, 168, and 180.

2	120	168	180
3	60	84	90
2	20	28	30
5	10	14	15
2	2	14	3
	1	7	3

Explanation:

2, 3, and 2 are common prime factors of the numbers. 5 is a common prime factor of 120 and 180; 2 again a common prime factor of 120 and 168; 3 again a prime factor of 180; and 7 a prime factor of 168.

The different prime factors are 2, 3, 5, and 7. The greatest number of times that 2 occurs as a factor is *three* in 120; that 3 occurs is *two* in 180; that 5 occurs is *once* in either 120 or 180; that 7 occurs is *once* in 168.

$\therefore$ $2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 7$, or 2520, = l. c. m. of 120, 168, and 180.

NOTE. When the numbers are easily factored, the first form is preferable.

Exercise 17

Find the least common multiple of:

1. 12, 30, and 42
2. 28, 36, and 44
3. 48, 60, and 72
4. 50, 75, and 125
5. 144, 132, and 120
6. 100, 140, and 180
7. 176, 125, and 275
8. 144, 180, and 200
9. 125, 500, and 750
10. 144, 216, and 600

Cancellation

66. Cancellation is the process of crossing out a number and writing the result obtained by multiplying or dividing the number.

Important Principles

67. *Multiplying one number and dividing another by the same number does not change their product.*

Thus, $6 \times 10 = 60$, and $(6 \times 2) \times (10 \div 2)$, or $12 \times 5 = 60$.

68. *Multiplying both dividend and divisor by the same number does not change the quotient.*

Thus, $12 \div 3 = 4$, and $(5 \times 12) \div (5 \times 3)$, or $60 \div 15 = 4$.

69. *Dividing both dividend and divisor by the same number does not alter the quotient.*

Thus, $18 \div 6 = 3$, and $(18 \div 3) \div (6 \div 3)$, or $6 \div 2 = 3$.

Illustrative Examples

1. Find the product of 25 and 48.

Thus, $\overset{100}{\cancel{25}} \times \overset{12}{\cancel{48}} = 1200$ — 25 has been multiplied and 48 divided by what number?

2. Divide 415 by 5.

Thus, $\overset{830}{\cancel{415}} \div \overset{10}{\cancel{5}} = 83$ — 415 and 5 have each been multiplied by what number?

3. Divide 144 by 24.

Thus, $\overset{36}{\cancel{144}} \div \overset{6}{\cancel{24}} = 6$ — 144 and 24 have each been divided by what number?

4. Find the value of $\frac{24 \times 36 \times 45}{9 \times 16 \times 5}$, read *24 times 36 times 45, divided by 9 times 16 times 5.*

$$\frac{\overset{3}{\cancel{24}} \times \overset{18}{\cancel{36}} \times \overset{9}{\cancel{45}}}{\underset{2}{\cancel{9}} \times \cancel{16} \times \cancel{5}} = 54$$

24 and 16 have each been divided by what number? How has the 9 in the dividend been obtained? By what has the 9 in the dividend been divided? How has the 18 in the dividend been obtained? What factors remain uncancelled?

5. A laborer who saves \$ 20 a month must save for how many years to have sufficient to pay for 40 acres of land at \$ 60 an acre?

Explanation:

$12 \times \$\,20 =$ the amount the laborer saves each year.
$40 \times \$\,60 =$ the cost of the land.

$\therefore \frac{\overset{2}{\cancel{40}} \times \overset{5}{\cancel{60}}}{\underset{1}{\cancel{12}} \times \cancel{20}}$, or $10 =$ the number of years required.

Exercise 18

Solve by cancellation:

1. 60×25
2. 35×500
3. 28×50
4. 16×250
5. $132 \div 22$
6. $144 \div 36$
7. $140 \div 28$
8. $180 \div 45$
9. $\frac{36 \times 27 \times 56}{18 \times 9 \times 28}$
10. $\frac{24 \times 35 \times 72}{32 \times 54}$

11. $\frac{96 \times 90 \times 150}{72 \times 180}$

12. $\frac{12 \times 39 \times 42 \times 36}{4 \times 13 \times 14 \times 9}$

13. How many groups of children of 16 each can be formed from 24 groups of 18 each?

14. How many pounds of sugar at 6 ¢ a pound should be given for 20 doz. eggs at 30 ¢ a dozen?

15. Find the quotient of 48×27 divided by 12×9.

16. How many strips of paper 8 ft. long can be cut from 9 rolls each 48 feet long?

17. How many 2-bushel bags can be filled from 4 bins of grain, each containing 180 bu.?

18. 48 five-hundred-pound bales of cotton weigh how many tons (2000 lb.)?

19. How many years must a farmer save at the rate of $ 25 a month to have sufficient to pay for 36 acres of land at $ 75 an acre?

20. The yield of corn from a 20-acre field is estimated at 50 bu. per acre. A bushel of corn weighs 56 lb.; how many tons of 2000 lb. are there in the yield?

21. 6 loads of flour, each containing 25 barrels and each barrel weighing 196 pounds, will make how many 50-pound sacks of flour?

22. Divide the product of 15, 21, and 35, by the product of 9, 25, and 49.

FRACTIONS

GENERAL DEFINITIONS

70. Any one of the equal parts of a unit is called a **fractional unit**.

Thus, *one half*, *one fifth*, and *one tenth* are fractional units.

71. Any number of equal fractional units is a **fraction**.

Thus, *one third*, *four fifths*, *five fifths*, and *eight fifths* are fractions.

72. The number of equal parts into which a unit is divided to form fractional units is the **denominator**, and the number of equal fractional units taken to make a fraction is the **numerator**, of the fraction.

73. The numerator and denominator are together called the **terms** of the fraction.

74. A fraction is usually written in figures by writing the denominator below, and the numerator above, a line.

Thus, *four fifths* is written $\frac{4}{5}$.

75. A fraction whose numerator is less than its denominator; that is, a fraction whose value is less than unity, is called a **proper fraction**; as, $\frac{2}{3}$.

76. A fraction whose numerator is equal to or greater than its denominator; that is, a fraction whose value is equal to or greater than unity, is called an **improper fraction**; as, $\frac{3}{3}$, $\frac{3}{2}$.

77. A number made up of a whole number and a fraction combined is called a **mixed number**.

Thus, $2\frac{1}{2}$, which equals $2+\frac{1}{2}$, is a mixed number.

78. A whole number is called an **integral number** or an **integer**.

79. A fraction may be considered to denote that the numerator is to be divided by the denominator.

For, $5 \div 2 = \frac{1}{2}$ of 5

$= \frac{1}{2}$ of $(1+1+1+1+1)$

$= \frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}$, or $\frac{5}{2}$;

therefore, $\frac{5}{2} = 5 \div 2$.

80. The **inverse** of a given fraction is the fraction obtained by interchanging the terms of the given fraction; as, $\frac{5}{4}$ is the inverse of $\frac{4}{5}$.

81. The **reciprocal** of a number is *one* divided by the number.

Thus, the reciprocal of 5 is $\frac{1}{5}$.

GENERAL PRINCIPLES

82. Multiplying the numerator of a fraction multiplies the value of the fraction.

Thus, $3 \times \frac{2}{5} = \frac{3 \times 2}{5}$, or $\frac{6}{5}$, because if the numerator is multiplied by 3 and the denominator remains unchanged, there will be 3 times as many fractional units, each of the same value as before.

83. Multiplying the denominator of a fraction divides the value of the fraction.

Thus, $\frac{2}{5} \div 3 = \frac{2}{5 \times 3}$, or $\frac{2}{15}$, because if the denominator is multiplied by 3 and the numerator remains unchanged, there will be the

same number of fractional units as before, but each will be only one third as great.

84. Dividing the numerator of a fraction divides the value of the fraction.

Thus, $\frac{9}{10} \div 3 = \frac{9 \div 3}{10}$, or $\frac{3}{10}$, because if the numerator is divided by 3 and the denominator remains unchanged, there will be only one third as many fractional units, each of the same value as before.

85. Dividing the denominator of a fraction multiplies the value of the fraction.

Thus, $3 \times \frac{7}{12} = \frac{7}{12 \div 3}$, or $\frac{7}{4}$, because if the denominator is divided by 3 and the numerator remains unchanged, there will be the same number of fractional units as before, but each will be three times as great.

86. Multiplying both terms of a fraction by the same number does not alter the value of the fraction.

Thus, $\frac{4}{5} = \frac{3 \times 4}{3 \times 5}$, or $\frac{12}{15}$, because multiplying the numerator by 3 multiplies the value of the fraction by 3, and multiplying the denominator by 3 divides the value of the fraction by 3, and doing both, multiplies and divides the value of the fraction by 3, which does not alter its value.

87. Dividing both terms of a fraction by the same number does not alter the value of the fraction.

Thus, $\frac{12}{16} = \frac{12 \div 4}{16 \div 4}$, or $\frac{3}{4}$, because dividing the numerator by 4 divides the value of the fraction by 4, and dividing the denominator by 4 multiplies the value of the fraction by 4, and doing both, divides and multiplies the value of the fraction by 4, which does not alter its value.

Exercise 19

1. Name the fraction whose numerator is 4 and denominator 5.

2. Name the fraction whose numerator is 7 and denominator 4.

3. Name the fraction whose numerator is 5 and denominator 5.

4. Name the number made up of 3 and $\frac{1}{2}$ combined.

5. Name three proper fractions each having the denominator 4.

6. Name three improper fractions each having the numerator 4.

7. What is the inverse of $\frac{4}{5}$?

8. What is the inverse of $\frac{3}{2}$?

9. Express as a fraction, $3 \div 2$; $4 \div 5$; $3 \div 3$; $10 \div 9$.

Name at sight the answer:

10. $3 \times \frac{2}{7}$	11. $4 \times \frac{5}{8}$	12. $3 \times \frac{2}{9}$
13. $3 \times \frac{2}{3}$	14. $3 \times \frac{7}{24}$	15. $3 \times \frac{7}{12}$
16. $3 \times \frac{5}{18}$	17. $\frac{6}{7} \div 2$	18. $\frac{3}{7} \div 2$
19. $\frac{1}{2} \div 2$	20. $\frac{7}{10} \div 2$	21. $\frac{1}{4} \div 3$
22. $\frac{5}{9} \div 2$	23. $4 \times \frac{7}{4}$	24. $\frac{9}{10} \div 9$
25. $\frac{3}{5} \div 3$	26. $\frac{1}{2} \div 4$	27. $\frac{3}{8} \div 2$
28. $\frac{2}{3} = \frac{}{6}$	29. $\frac{4}{5} = \frac{8}{}$	30. $\frac{8}{12} = \frac{}{3}$
31. $\frac{9}{12} = \frac{3}{}$	32. $\frac{1}{2} = \frac{}{10}$	33. $\frac{12}{15} = \frac{4}{}$

REDUCTION

88. Reduction is the process of changing the form of an expression without changing its value.

89. A fraction is said to be in its **lowest terms** when its numerator and denominator are prime to each other.

Thus, $\frac{10}{15}$ when reduced to its lowest terms equals $\frac{2}{3}$.

90. A fraction may be reduced to its lowest terms by dividing both of its terms by their greatest common divisor.

91. Fractions are said to be **similar** when they have a common denominator.

Thus, $\frac{3}{5}$ and $\frac{4}{5}$ are similar fractions.

92. Fractions having different denominators may be reduced to fractions having a common denominator equal to the least common multiple of their denominators.

Thus, $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{4}{5}$ may each be reduced to 60ths.

NOTE 1. Any common multiple of the denominators of two or more fractions may be taken as a common denominator of them. Thus, 30, 60, 90, etc., may each be taken as a common denominator of $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{4}{5}$.

NOTE 2. If fractions having different denominators are in their lowest terms, the least common multiple of their denominators is their **least common denominator** (l. c. d.).

93. A mixed number is equal to the whole number multiplied by the denominator of the fraction, plus the numerator of the fraction, all divided by the denominator of the fraction.

Thus, $18\frac{3}{5} = \frac{5 \times 18 + 3}{5}$, or $\frac{93}{5}$.

Exercise 20

Supply at sight the missing numerators:

1. $1\frac{1}{2} = \frac{\quad}{2}$ 2. $3\frac{1}{3} = \frac{\quad}{3}$ 3. $2\frac{3}{4} = \frac{\quad}{4}$ 4. $7\frac{1}{2} = \frac{\quad}{2}$
5. $1\frac{1}{3} = \frac{\quad}{3}$ 6. $6\frac{2}{3} = \frac{\quad}{3}$ 7. $8\frac{1}{3} = \frac{\quad}{3}$ 8. $12\frac{1}{2} = \frac{\quad}{2}$
9. $1\frac{1}{4} = \frac{\quad}{4}$ 10. $3\frac{1}{5} = \frac{\quad}{5}$ 11. $13\frac{1}{3} = \frac{\quad}{3}$ 12. $16\frac{2}{3} = \frac{\quad}{3}$
13. $2\frac{1}{2} = \frac{\quad}{2}$ 14. $2\frac{1}{4} = \frac{\quad}{4}$ 15. $4\frac{4}{5} = \frac{\quad}{5}$ 16. $33\frac{1}{3} = \frac{\quad}{3}$

Reduce at sight to whole or mixed numbers:

17. $\frac{7}{2}$ 18. $\frac{10}{3}$ 19. $\frac{12}{4}$ 20. $\frac{9}{4}$
21. $\frac{20}{3}$ 22. $\frac{18}{6}$ 23. $\frac{15}{4}$ 24. $\frac{25}{3}$
25. $\frac{25}{6}$ 26. $\frac{50}{6}$ 27. $\frac{25}{8}$ 28. $\frac{42}{6}$
29. $\frac{50}{3}$ 30. $\frac{45}{2}$ 31. $\frac{75}{2}$ 32. $\frac{100}{3}$

Reduce at sight to lowest terms:

33. $\frac{2}{4}$ 34. $\frac{4}{6}$ 35. $\frac{8}{10}$ 36. $\frac{6}{12}$
37. $\frac{6}{8}$ 38. $\frac{10}{15}$ 39. $\frac{8}{12}$ 40. $\frac{9}{12}$
41. $\frac{12}{16}$ 42. $\frac{12}{18}$ 43. $\frac{24}{30}$ 44. $\frac{20}{30}$
45. $\frac{16}{24}$ 46. $\frac{32}{48}$ 47. $\frac{25}{45}$ 48. $\frac{28}{32}$

Name at sight the least common denominator of:

49. $\frac{1}{2}$ and $\frac{2}{3}$ 50. $\frac{3}{4}$ and $\frac{2}{3}$ 51. $\frac{3}{4}$ and $\frac{4}{5}$
52. $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{3}{4}$ 53. $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{4}{5}$ 54. $\frac{2}{3}$, $\frac{5}{6}$, and $\frac{7}{8}$
55. $\frac{2}{3}$, $\frac{1}{5}$, and $\frac{1}{10}$ 56. $\frac{1}{2}$, $\frac{1}{8}$, and $\frac{5}{12}$ 57. $\frac{2}{9}$, $\frac{5}{12}$, and $\frac{1}{24}$
58. $\frac{3}{4}$, $\frac{2}{5}$, and $\frac{3}{8}$ 59. $\frac{1}{2}$, $\frac{7}{8}$, and $\frac{3}{5}$ 60. $\frac{4}{5}$, $\frac{3}{4}$, and $\frac{7}{12}$

Illustrative Examples

1. Reduce $32\frac{7}{8}$ to an improper fraction.

Thus, $32\frac{7}{8} = \frac{8 \times 32 + 7}{8}$, or $\frac{263}{8}$. [See § 93]

2. Reduce $\frac{315}{12}$ to a mixed number.

Thus, $\frac{315}{12} = 12\overline{)315}$ $26\frac{1}{4}$. [See § 79]

3. Reduce $\frac{144}{156}$ to its lowest terms.

Thus, the g. c. d. of 144 and 156 = 12.

$\frac{144}{156} = \frac{144 \div 12}{156 \div 12}$, or $\frac{12}{13}$. [See § 90]

4. Reduce $\frac{3}{8}$, $\frac{7}{12}$, and $\frac{5}{16}$ to fractions having the least common denominator.

Thus, the l. c. m. of 8, 12, and 16 is 48.

$\frac{3}{8} = \frac{18}{48}$; $\frac{7}{12} = \frac{28}{48}$; $\frac{5}{16} = \frac{15}{48}$. [See § 92]

Exercise 21

Reduce to improper fractions:

1. $12\frac{3}{8}$	2. $37\frac{1}{2}$	3. $14\frac{2}{7}$	4. $66\frac{2}{3}$
5. $15\frac{7}{8}$	6. $39\frac{5}{6}$	7. $87\frac{1}{4}$	8. $123\frac{3}{8}$
9. $156\frac{8}{9}$	10. $312\frac{2}{5}$	11. $481\frac{3}{15}$	12. $98\frac{5}{16}$
13. $145\frac{7}{25}$	14. $256\frac{1}{50}$	15. $506\frac{3}{20}$	16. $300\frac{2}{25}$

Reduce to whole or mixed numbers:

17. $\frac{300}{3}$ 18. $\frac{200}{6}$ 19. $\frac{100}{7}$ 20. $\frac{350}{16}$

21. $\frac{92}{8}$ 22. $\frac{112}{4}$ 23. $\frac{760}{12}$ 24. $\frac{386}{24}$

25. $\frac{198}{15}$ 26. $\frac{640}{8}$ 27. $\frac{720}{5}$ 28. $\frac{876}{10}$

29. $\frac{1200}{24}$ 30. $\frac{1200}{9}$ 31. $\frac{1500}{6}$ 32. $\frac{1000}{8}$

Reduce to lowest terms:

33. $\frac{45}{75}$ 34. $\frac{72}{90}$ 35. $\frac{108}{144}$ 36. $\frac{132}{168}$

37. $\frac{624}{800}$ 38. $\frac{515}{900}$ 39. $\frac{500}{625}$ 40. $\frac{270}{288}$

41. $\frac{162}{168}$ 42. $\frac{279}{288}$ 43. $\frac{232}{248}$ 44. $\frac{369}{423}$

45. $\frac{405}{450}$ 46. $\frac{108}{324}$ 47. $\frac{625}{1000}$ 48. $\frac{875}{1000}$

Reduce to fractions having the least common denominator:

49. $\frac{2}{3}, \frac{3}{4}, \frac{4}{5}$ 50. $\frac{2}{3}, \frac{4}{5}, \frac{5}{6}$ 51. $\frac{2}{3}, \frac{7}{12}, \frac{3}{8}$

52. $\frac{9}{15}, \frac{8}{20}, \frac{15}{25}$ 53. $\frac{4}{5}, \frac{7}{10}, \frac{2}{15}$ 54. $\frac{3}{8}, \frac{5}{24}, \frac{3}{16}$

55. $\frac{7}{12}, \frac{3}{20}, \frac{1}{30}$ 56. $\frac{1}{12}, \frac{1}{18}, \frac{5}{9}$ 57. $\frac{5}{12}, \frac{7}{16}, \frac{1}{24}$

58. $\frac{8}{15}, \frac{2}{25}, \frac{3}{40}$ 59. $\frac{3}{25}, \frac{7}{30}, \frac{9}{40}$ 60. $\frac{5}{16}, \frac{7}{24}, \frac{11}{18}$

61. $\frac{7}{25}, \frac{3}{40}, \frac{7}{45}$ 62. $\frac{9}{10}, \frac{11}{25}, \frac{3}{40}$ 63. $\frac{5}{14}, \frac{5}{21}, \frac{9}{28}$

64. $\frac{5}{6}, \frac{7}{36}, \frac{5}{72}$ 65. $\frac{7}{15}, \frac{3}{20}, \frac{8}{30}$ 66. $\frac{7}{12}, \frac{11}{15}, \frac{5}{18}$

67. $\frac{1}{2}, \frac{2}{9}, \frac{3}{10}$ 68. $\frac{7}{24}, \frac{11}{20}, \frac{17}{30}$ 69. $\frac{2}{25}, \frac{3}{50}, \frac{7}{100}$

ADDITION AND SUBTRACTION

94. Fractions that do not have a common denominator must be reduced to similar fractions before they can be added or subtracted.

95. 1. **The sum of two or more similar fractions equals the sum of their numerators divided by their common denominator.**

2. **The difference of two similar fractions equals the difference of their numerators divided by their common denominator.**

Illustrative Examples

1. Find the sum of $\frac{5}{12}$, $\frac{7}{30}$, and $\frac{11}{75}$.

Explanation:

l. c. d. = 300.

$300 = 25 \times 12$, 10×30, or 4×75.

$$\frac{5}{12} = \frac{25 \times 5}{25 \times 12} = \frac{125}{300}$$

$$\frac{7}{30} = \frac{10 \times 7}{10 \times 30} = \frac{70}{300}$$

$$\frac{11}{75} = \frac{4 \times 11}{4 \times 75} = \frac{44}{300}$$

$$\therefore \text{Sum} = \frac{239}{300}.$$

2. Find the difference between $\frac{17}{20}$ and $\frac{5}{12}$.

Explanation:

l. c. d. = 60.

$60 = 3 \times 20$, or 5×12.

$$\frac{17}{20} = \frac{3 \times 17}{3 \times 20} = \frac{51}{60}$$

$$\frac{5}{12} = \frac{5 \times 5}{5 \times 12} = \frac{25}{60}$$

Difference $= \frac{26}{60} = \frac{13}{30}$.

3. Find the sum of $212\frac{3}{4}$, $118\frac{3}{16}$, and $237\frac{5}{9}$.

Explanation:

$212\frac{3}{4} = 212\frac{108}{144}$

$118\frac{3}{16} = 118\frac{27}{144}$

$237\frac{5}{9} = 237\frac{80}{144}$

$\therefore$ Sum $= 567\frac{215}{144}$, or $568\frac{71}{144}$.

4. Find the difference between $211\frac{1}{4}$ and $37\frac{7}{10}$.

Explanation:

$211\frac{1}{4} = 210\frac{25}{20}$

$37\frac{7}{10} = 37\frac{14}{20}$

Difference $= 173\frac{11}{20}$.

Exercise 22

Add or subtract the following as indicated:

1. $\frac{7}{15} + \frac{3}{10}$
2. $\frac{5}{12} + \frac{7}{18}$
3. $\frac{3}{25} + \frac{9}{50}$
4. $\frac{11}{64} - \frac{1}{16}$
5. $\frac{7}{48} - \frac{1}{16}$
6. $\frac{25}{72} - \frac{1}{12}$
7. $\frac{2}{3} + \frac{1}{4} + \frac{2}{5}$
8. $\frac{3}{4} + \frac{3}{8} + \frac{7}{12}$
9. $\frac{5}{6} + \frac{5}{12} + \frac{7}{18}$
10. $\frac{3}{50} + \frac{3}{25} + \frac{7}{75}$
11. $\frac{3}{8} + \frac{3}{16} + \frac{3}{32}$
12. $\frac{7}{15} + \frac{9}{10} + \frac{5}{12}$

13. $\frac{31}{75} - \frac{3}{25}$
14. $\frac{7}{100} - \frac{1}{20}$
15. $\frac{51}{100} - \frac{3}{10}$
16. $\frac{7}{10} + \frac{3}{50} + \frac{1}{20}$
17. $\frac{5}{16} + \frac{1}{64}$
18. $\frac{7}{10} + \frac{3}{15} - \frac{1}{75}$
19. $\frac{7}{8} + \frac{9}{10} + \frac{7}{15}$
20. $\frac{3}{7} + \frac{4}{5} + \frac{8}{35}$
21. $\frac{9}{10} + \frac{1}{2} - \frac{1}{3}$
22. $\frac{7}{15} - \frac{1}{5} + \frac{1}{3}$
23. $\frac{9}{10} - \frac{3}{100} + \frac{1}{20}$
24. $\frac{7}{15} + \frac{1}{18} + \frac{1}{24}$
25. $2\frac{1}{2} + \frac{2}{3} + \frac{4}{5}$
26. $\frac{2}{3} + 4\frac{1}{5} + \frac{7}{8}$
27. $\frac{3}{4} + \frac{9}{10} + 12\frac{1}{2}$
28. $1 - \frac{1}{5} + \frac{1}{6}$
29. $2 + \frac{3}{8} - \frac{1}{3}$
30. $\frac{15}{16} - \frac{3}{8} + \frac{1}{24}$
31. $1 - \frac{3}{26} + \frac{5}{13}$
32. $12\frac{3}{4} + 5\frac{1}{8} + 7\frac{1}{32}$
33. $16\frac{1}{10} - 14\frac{3}{5}$
34. $22\frac{2}{7} - 20\frac{5}{7}$
35. $28\frac{4}{5} - 7\frac{9}{10}$
36. $12\frac{1}{2} + 13\frac{2}{3} + 31\frac{1}{4}$
37. $\frac{2}{3} + \frac{3}{4} + \frac{4}{5} + \frac{7}{8}$
38. $\frac{9}{10} + \frac{3}{5} + \frac{1}{2} - \frac{1}{4}$
39. $\frac{3}{5} + \frac{9}{10} + \frac{7}{15} + \frac{9}{25}$
40. $\frac{11}{12} + \frac{7}{8} - \frac{3}{4} + \frac{1}{16}$
41. $\frac{3}{8} + \frac{9}{16} + \frac{5}{12} + \frac{1}{18}$
42. $\frac{3}{25} + \frac{1}{75} + 1\frac{1}{2} - \frac{7}{12}$
43. $42\frac{2}{7} + 35\frac{5}{14} + 40\frac{2}{21} + 12\frac{1}{7}$
44. $15\frac{2}{3} + 25\frac{1}{6} + 30\frac{3}{4} - 5\frac{7}{12}$
45. $12\frac{1}{2} + 13\frac{1}{3} + 28\frac{3}{4} + 9\frac{1}{12}$
46. $100 - 87\frac{1}{2} + 6\frac{1}{4} - 3\frac{1}{8}$

MULTIPLICATION

96. To multiply by a fraction is to take such a part of the multiplicand as is denoted by the denominator of the fraction as many times as is indicated by its numerator.

Thus, $\frac{3}{4}$ of 5, or $\frac{3}{4} \times 5$, means $3 \times \frac{1}{4}$ of 5;
also, $\frac{2}{3}$ of $\frac{4}{5}$, or $\frac{2}{3} \times \frac{4}{5}$, means $2 \times \frac{1}{3}$ of $\frac{4}{5}$.

97. To find the product of a whole number multiplied by a fraction, use the whole number as the multiplier.

For, $\frac{3}{4}$ of $5 = 3 \times \frac{1}{4}$ of 5 [§ 96]
$= 3 \times \frac{5}{4}$ [§ 79]
$= \frac{15}{4}$ [§ 82]
$= 5 \times \frac{3}{4}$.

98. **The product of two fractions equals the product of their numerators divided by the product of their denominators.**

For, $\frac{2}{3} \times \frac{4}{5} = 2 \times \frac{1}{3}$ of $\frac{4}{5}$ [§ 96]

$$= 2 \times \frac{4}{3 \times 5} \quad [\S\ 83]$$

$$= \frac{2 \times 4}{3 \times 5}. \quad [\S\ 82]$$

Exercise 23

Name at sight the product:

1. $\frac{2}{3} \times 4$	2. $\frac{3}{4} \times 3$	3. $\frac{7}{8} \times 2$
4. $\frac{9}{10} \times 5$	5. $\frac{3}{8} \times 4$	6. $\frac{7}{12} \times 3$
7. $\frac{2}{9} \times 3$	8. $\frac{7}{8} \times 4$	9. $\frac{5}{18} \times 6$
10. $\frac{11}{12} \times 4$	11. $\frac{7}{8} \times 3$	12. $\frac{9}{10} \times 5$
13. $\frac{4}{7} \times 8$	14. $\frac{7}{15} \times 5$	15. $\frac{11}{12} \times 8$
16. $\frac{1}{2} \times \frac{1}{4}$	17. $\frac{1}{2} \times \frac{1}{3}$	18. $\frac{1}{4} \times \frac{1}{5}$
19. $\frac{1}{2} \times \frac{3}{4}$	20. $\frac{1}{3} \times \frac{2}{3}$	21. $\frac{1}{2} \times \frac{3}{8}$
22. $\frac{3}{4} \times \frac{3}{5}$	23. $\frac{4}{5} \times \frac{6}{5}$	24. $\frac{1}{5} \times \frac{3}{4}$
25. $\frac{5}{9} \times \frac{6}{7}$	26. $\frac{3}{5} \times \frac{4}{9}$	27. $\frac{3}{8} \times \frac{5}{9}$

99. **Cancelling a factor common to the numerator of one fraction and the denominator of another does not change the value of their product.**

Thus, $\frac{\overset{2}{\cancel{6}}}{5} \times \frac{4}{\underset{3}{\cancel{9}}} = \frac{8}{15}$

For, dividing the numerator of $\frac{6}{5}$ by 3 and the denominator of $\frac{4}{9}$ by 3, divides $\frac{6}{5}$ by 3 [§ 84] and multiplies $\frac{4}{9}$ by 3 [§ 85], and, therefore, does not change the value of their product [§ 67].

Illustrative Examples

1. Multiply $\frac{3}{25}$ by 4.

Thus, $4 \times \frac{3}{25} = \frac{12}{25}$. [§ 82]

2. Multiply $\frac{3}{25}$ by 5.

Thus, $5 \times \frac{3}{25} = \frac{3}{5}$. [§ 85]

3. Multiply 5 by $\frac{5}{7}$.

Thus, $\frac{5}{7} \times 5 = \frac{25}{7}$. [§ 97]

4. Multiply $\frac{11}{12}$ by $\frac{7}{8}$.

Thus, $\frac{7}{8} \times \frac{11}{12} = \frac{77}{96}$. [§ 98]

5. Multiply $\frac{15}{38}$ by $\frac{19}{25}$.

Thus, $\frac{\cancel{19}}{\cancel{25}_{5}} \times \frac{\overset{3}{\cancel{15}}}{\cancel{38}_{2}} = \frac{3}{10}$. [§ 99]

6. Multiply $3\frac{1}{2}$ by $2\frac{1}{4}$.

Thus, $2\frac{1}{4} \times 3\frac{1}{2} = \frac{9}{4} \times \frac{7}{2} = \frac{63}{8}$, or $7\frac{7}{8}$.

Multiply:

7. $275\frac{3}{4}$ by 42. 8. 756 by $18\frac{3}{5}$. 9. $276\frac{1}{3}$ by $32\frac{2}{5}$.

In practice the work may be done thus:

$275\frac{3}{4}$	756	$276\frac{1}{3}$
42	$18\frac{3}{5}$	$32\frac{2}{5}$
550	6048	552
1100	756	828
$31\frac{1}{2}$	$453\frac{3}{5}$	$110\frac{2}{5}$
$11581\frac{1}{2}$	$14061\frac{3}{5}$	$10\frac{2}{3}$
		$\frac{2}{15}$
		$8953\frac{1}{5}$

Exercise 24

Multiply:

1. $9 \times \frac{2}{15}$
2. $8 \times \frac{7}{32}$
3. $9 \times \frac{25}{12}$
4. $12 \times \frac{3}{25}$
5. $15 \times \frac{7}{75}$
6. $20 \times \frac{7}{50}$
7. $\frac{3}{16} \times 9$
8. $\frac{7}{24} \times 24$
9. $\frac{9}{50} \times 30$
10. $\frac{7}{144} \times 48$
11. $\frac{3}{200} \times 75$
12. $\frac{5}{288} \times 72$
13. $\frac{7}{12} \times \frac{9}{10}$
14. $\frac{7}{12} \times \frac{15}{16}$
15. $\frac{24}{25} \times \frac{15}{16}$
16. $\frac{32}{35} \times \frac{75}{64}$
17. $\frac{19}{32} \times \frac{3}{38}$
18. $\frac{42}{75} \times \frac{50}{63}$
19. $2\frac{2}{3} \times 12\frac{1}{2}$
20. $6\frac{2}{3} \times 3\frac{1}{3}$
21. $7\frac{1}{2} \times 3\frac{3}{4}$
22. $5\frac{1}{2} \times 7\frac{1}{2}$
23. $18\frac{3}{4} \times 1\frac{1}{2}$
24. $3\frac{1}{3} \times 7\frac{1}{2}$
25. $\frac{7}{12} \times \frac{3}{28} \times \frac{4}{9}$
26. $\frac{5}{12} \times \frac{18}{25} \times \frac{2}{9}$
27. $\frac{3}{4} \times \frac{16}{27} \times \frac{9}{8} \times \frac{4}{5}$
28. $\frac{2}{3} \times \frac{5}{4} \times \frac{4}{5} \times \frac{7}{12}$

29. $156\frac{1}{2} \times 12$
30. $315\frac{2}{3} \times 24$
31. $186\frac{2}{5} \times 72$
32. $436\frac{8}{9} \times 80$
33. $275 \times 12\frac{1}{2}$
34. $333 \times 42\frac{2}{3}$
35. $437 \times 68\frac{3}{4}$
36. $396 \times 79\frac{3}{5}$
37. $348\frac{1}{4} \times 32\frac{1}{3}$
38. $375\frac{1}{4} \times 60\frac{2}{3}$
39. $429\frac{3}{4} \times 92\frac{2}{3}$
40. $376\frac{4}{5} \times 86\frac{3}{4}$

DIVISION

100. To divide by a fraction is to find a result, called the quotient, which multiplied by the divisor will produce the dividend.

Thus,

(1) the expression $7 \div \frac{2}{3}$ means: find the number, $\frac{2}{3}$ of which is 7;

(2) the expression $\frac{2}{3} \div \frac{3}{4}$ means: find the number, $\frac{3}{4}$ of which is $\frac{2}{3}$.

101. To divide an integer by a fraction, multiply the integer by the inverse of the fraction.

Thus, $7 \div \frac{2}{3} = 7 \times \frac{3}{2}$;
for, $\frac{2}{3}$ of the quotient $= 7$, [§ 100]
$\frac{1}{3}$ of the quotient $= \frac{1}{2}$ of 7,
$\frac{3}{3}$ of the quotient $= \frac{3}{2} \times 7$, or $7 \times \frac{3}{2}$. [§ 38]

102. The quotient of two fractions equals the dividend multiplied by the inverse of the divisor.

Thus, $\frac{2}{3} \div \frac{3}{4} = \frac{2}{3} \times \frac{4}{3}$;
for, $\frac{3}{4}$ of the quotient $= \frac{2}{3}$, [§ 100]
$\frac{1}{4}$ of the quotient $= \frac{1}{3} \times \frac{2}{3}$,
$\frac{4}{4}$ of the quotient $= \frac{4}{3} \times \frac{2}{3}$, or $\frac{2}{3} \times \frac{4}{3}$. [§ 38]

Illustrative Examples

1. Divide $\frac{8}{9}$ by 4.

Thus, $\frac{8}{9} \div 4 = \frac{2}{9}$. [§ 84]

2. Divide $\frac{3}{11}$ by 4.

Thus, $\frac{3}{11} \div 4 = \frac{3}{44}$. [§ 83]

3. Divide $7\frac{1}{2}$ by 8.

Thus, $7\frac{1}{2} \div 8 = \frac{15}{2} \div 8$, or $\frac{15}{16}$.

4. Divide 5 by $\frac{7}{12}$.

Thus, $5 \div \frac{7}{12} = 5 \times \frac{12}{7}$, or $8\frac{4}{7}$. [§ 101]

5. Divide 3 by $3\frac{1}{3}$.

Thus, $3 \div 3\frac{1}{3} = 3 \div \frac{10}{3}$, or $\frac{9}{10}$.

6. Divide $\frac{8}{45}$ by $\frac{12}{25}$.

Thus, $\frac{8}{45} \div \frac{12}{25} = \frac{\overset{2}{\cancel{8}}}{\underset{9}{\cancel{45}}} \times \frac{\overset{5}{\cancel{25}}}{\underset{3}{\cancel{12}}}$, or $\frac{10}{27}$. [§§ 102, 99]

7. Divide $2\frac{2}{3}$ by $5\frac{1}{2}$.

Thus, $2\frac{2}{3} \div 5\frac{1}{2} = \frac{8}{3} \div \frac{11}{2}$
$= \frac{8}{3} \times \frac{2}{11}$, or $\frac{16}{33}$.

8. Divide $456\frac{2}{3}$ by $37\frac{1}{2}$.

Thus,

$37\frac{1}{2}$ $456\frac{2}{3}$
6 6

$$225)2740(12\tfrac{8}{45}$$
$$\underline{225}$$
$$490$$
$$\underline{450}$$
$$40$$

$\frac{40}{225} = \frac{8}{45}$

Suggestion. Multiply both dividend and divisor by the least common denominator of the fractions.

Exercise 25

Find the quotient:

1. $\frac{8}{25} \div 2$ 2. $\frac{8}{25} \div 5$ 3. $\frac{21}{50} \div 7$ 4. $\frac{1}{7} \div 7$
5. $\frac{24}{5} \div 8$ 6. $\frac{2}{15} \div 3$ 7. $\frac{4}{15} \div 4$ 8. $\frac{24}{5} \div 7$
9. $7\frac{1}{2} \div 6$ 10. $14\frac{2}{7} \div 4$ 11. $33\frac{1}{3} \div 5$ 12. $66\frac{2}{3} \div 10$
13. $37\frac{1}{2} \div 15$ 14. $22\frac{2}{9} \div 8$ 15. $87\frac{1}{2} \div 7$ 16. $112\frac{1}{2} \div 9$
17. $7 \div \frac{1}{9}$ 18. $9 \div \frac{1}{12}$ 19. $7 \div \frac{2}{3}$ 20. $12 \div \frac{3}{5}$
21. $9 \div \frac{3}{20}$ 22. $12 \div \frac{4}{25}$ 23. $15 \div \frac{5}{9}$ 24. $18 \div \frac{7}{9}$
25. $9 \div 1\frac{1}{2}$ 26. $12 \div 1\frac{1}{3}$ 27. $15 \div 2\frac{1}{2}$ 28. $24 \div 3\frac{1}{2}$
29. $\frac{3}{8} \div \frac{9}{10}$ 30. $\frac{5}{8} \div \frac{2}{3}$ 31. $\frac{6}{25} \div \frac{15}{16}$ 32. $\frac{4}{15} \div \frac{2}{25}$
33. $\frac{5}{12} \div \frac{5}{6}$ 34. $\frac{11}{12} \div \frac{1}{36}$ 35. $\frac{3}{10} \div \frac{9}{5}$ 36. $\frac{7}{20} \div \frac{21}{40}$
37. $\frac{9}{40} \div \frac{3}{50}$ 38. $\frac{5}{18} \div \frac{25}{42}$ 39. $\frac{9}{5} \div \frac{10}{27}$ 40. $\frac{8}{5} \div \frac{32}{25}$
41. $112\frac{1}{2} \div 8\frac{1}{4}$ 42. $133\frac{1}{3} \div 6\frac{1}{6}$ 43. $150\frac{1}{2} \div 16\frac{2}{3}$
44. $133\frac{1}{3} \div 18\frac{2}{5}$ 45. $126\frac{3}{4} \div 56\frac{3}{5}$ 46. $312\frac{1}{4} \div 48\frac{5}{6}$

COMPLEX FRACTIONS

Definitions

103. A **complex fraction** is an expression in the form of a fraction, one or both of whose terms are fractional or mixed; as,

$$\frac{3}{\frac{4}{5}}, \quad \frac{5\frac{1}{2}}{6}, \quad \frac{\frac{1}{4}}{\frac{2}{3}}, \quad \frac{15\frac{1}{3}}{2\frac{3}{4}}.$$

104. A complex fraction denotes that the numerator is to be divided by the denominator.

Thus, $\frac{3}{\frac{4}{5}}$ denotes $3 \div \frac{4}{5}$.

Exercise 26

Simplify:

1. $\dfrac{\frac{5}{8}}{3}$

2. $\dfrac{20}{\frac{3}{11}}$

3. $\dfrac{5\frac{3}{4}}{24}$

4. $\dfrac{25}{1\frac{2}{3}}$

5. $\dfrac{\frac{4}{5}}{\frac{5}{21}}$

6. $\dfrac{\frac{4}{15}}{3\frac{1}{12}}$

7. $\dfrac{42\frac{1}{4}}{\frac{8}{9}}$

8. $\dfrac{37\frac{1}{2}}{6\frac{1}{4}}$

9. $\dfrac{12\frac{1}{2}+2\frac{1}{8}}{4\frac{5}{6}-1\frac{7}{12}}$

10. $\dfrac{\frac{2}{3}+\frac{3}{4}}{\frac{3}{4}-\frac{2}{3}}$

11. $\dfrac{\frac{4}{5}\times\frac{3}{4}}{\frac{4}{5}\div\frac{3}{4}}$

12. $\dfrac{12\frac{1}{2}\div 87\frac{1}{2}}{112\frac{1}{2}\div 37\frac{1}{2}}$

13. $\dfrac{4}{1+\frac{5}{6}}$

14. $\dfrac{1}{8\frac{1}{3}+\dfrac{3\frac{1}{2}}{2\frac{5}{8}}}$

15. $\dfrac{2\frac{3}{8}\times 1\frac{1}{3}}{13\frac{3}{4}\div\dfrac{\frac{1}{8}}{\frac{1}{2}}}$

16. $\dfrac{\dfrac{3-\frac{1}{5}}{4}-\dfrac{3+\frac{1}{5}}{8}}{16}$

17. $\dfrac{\frac{3}{100}+\frac{9}{50}+\frac{4}{25}+\frac{3}{5}}{\frac{92}{100}-\frac{23}{50}+\frac{12}{25}-\frac{16}{25}}$

18. $\dfrac{(\frac{2}{3}+\frac{3}{4})\times(\frac{3}{4}-\frac{2}{3})}{(\frac{9}{16}-\frac{4}{9})\div(\frac{2}{3}+\frac{3}{4})}$

MISCELLANEOUS PROBLEMS

Exercise 27

1. How many quarter-acre lots can be made from $6\frac{1}{2}$ acres of land?

2. A field was formed from 13 half-acre lots. How many acres did the field contain?

3. A bar of metal $1\frac{1}{2}$ in. long was cut into pieces, each $\frac{1}{12}$ in. long. How many pieces did it make?

4. Find the sum of all proper fractions that have the denominator of each less than 5.

5. From a board 12 ft. long 2 pieces were cut, the first $2\frac{1}{2}$ ft. long, and the second $2\frac{3}{4}$ ft. long. What was the length of the piece that remained?

6. If 2 is added to both terms of the fraction $\frac{3}{4}$, is the resulting fraction greater or less than $\frac{3}{4}$, and how much?

7. If 2 is added to both terms of the fraction $\frac{4}{3}$, is the resulting fraction greater or less than $\frac{4}{3}$, and how much?

8. A woman bought $3\frac{1}{2}$ doz. eggs at 30 ¢ a dozen, and $1\frac{1}{4}$ lb. of dried beef at 40 ¢ a pound. What was her bill?

9. When plums are selling at the rate of 2 qt. for $\$\frac{1}{4}$, how many quarts can be bought for $\$\frac{3}{4}$?

10. On a certain scale of measurement $\frac{3}{4}$ in. is taken to represent 10 ft. What should represent 25 ft.?

11. The product of 3 fractions is $1\frac{1}{15}$. Two of them are $\frac{3}{2}$ and $\frac{4}{5}$. What is the third?

12. If $\frac{3}{4}$ in. on a map represents 10 mi., what is the distance between two towns if the space between them on the map is $4\frac{1}{8}$ in.?

13. During a certain year (365 da.) which began on Saturday, a rural mail carrier traveled $24\frac{1}{2}$ mi. each week day, excepting February 22 and July 4. How many miles did he travel that year in delivering his mail?

14. Ice is $\frac{10}{11}$ as heavy as water, and a cubic foot of water weighs $62\frac{1}{2}$ lb. How many cubic feet of ice will weigh a ton (2000 lb.)?

15. Supply the missing number:

$$7\tfrac{1}{2} + 18\tfrac{3}{4} + 33\tfrac{1}{3} + \ ? = 150.$$

16. If A can do a piece of work in 4 days, what part of it can he do in 1 day? What part of it can he do in 3 days?

17. If A can do a piece of work in 4 days and B can do it in 3 days, what part of the work can each do in 1 day? What part can each do in 2 days? What part can both do in 1 day? What part can both do in 2 days?

18. If A can do a piece of work in 4 days and B can do it in 3 days, how long will both, working together, require to do the work?

19. Two thousand pounds make a short ton, and 2240 lb. make a long ton. Express in lowest terms the fraction that denotes the part a short ton is of a long ton.

20. The time consumed by a man in doing $\frac{5}{6}$ of a piece of work was $12\frac{1}{2}$ da.; how long should it take him to finish it?

21. Hay well dried in the field loses about one third of its weight, if it is allowed to dry out and is then packed in the mow. Which is cheaper and how much, to buy 6 tons of hay from the field at \$15 a ton, or to buy from the mow at \$20 a ton?

22. The following is a recipe for muffins, the quantities here given being sufficient to make five. How much of each of these substances is necessary to make 5 doz. muffins?

1 cup of milk
$1\frac{1}{2}$ cups of flour
2 eggs
$\frac{1}{2}$ teaspoonful of salt
$1\frac{1}{2}$ teaspoonfuls of baking powder

23. A tenant, working a farm, receives for his labor $\frac{1}{3}$ of all the grain he raises, and the owner $\frac{2}{3}$. If the tenant's share of the crop is 104 bu. of oats, 121 bu. of wheat, 320 bu. of corn, and $30\frac{1}{2}$ bu. of rye, what is the owner's share?

24. Estimating the weight of a gallon of water to be $8\frac{1}{3}$ lb., what weight of water will a tank hold whose capacity is 12 hogsheads (63 gal.)?

25. A dealer bought a live turkey weighing 14 lb. at $12\frac{1}{2}$ ¢ a pound. If it loses $\frac{1}{4}$ of its weight in dressing, at what price per pound must the dressed turkey be sold that the gain may be 35 ¢?

26. Divide 160 into two such parts that $\frac{2}{3}$ of the smaller shall equal $\frac{2}{5}$ of the larger.

SUGGESTION. $\frac{2}{3}$ of the smaller $= \frac{2}{5}$ of the larger.
$\frac{1}{3}$ of the smaller $= \frac{1}{5}$ of the larger.
$\frac{3}{3}$ of the smaller $= \frac{3}{5}$ of the larger.
$\frac{8}{5}$ of the larger = the smaller and the larger, or 160.

27. The following are the ingredients necessary to make a filling for a small lemon pie for 4 persons; how much of each of these ingredients must be used for a dozen such pies?

$1\frac{1}{2}$ tablespoonfuls of cornstarch
$\frac{1}{2}$ cup of sugar
$\frac{1}{2}$ cup of boiling water
1 teaspoonful of butter
$\frac{1}{2}$ lemon
1 egg
$\frac{1}{2}$ teaspoonful of pulverized sugar

28. A dealer paid \$45 for potatoes at \$$\frac{3}{4}$ a bushel; what must he sell them for per bushel to gain \$30?

29. If one machine can skim 75 gallons of milk in $1\frac{3}{4}$ hours and another 60 gallons per hour, how long must both run to skim 300 gallons of milk?

30. The sum of two numbers is 98, and $\frac{4}{5}$ of the larger is $\frac{5}{6}$ of the smaller. What are the numbers?

31. 56 lb. of sweet corn were put up in $\frac{1}{2}$-pound, $\frac{1}{4}$-pound, and $\frac{1}{8}$-pound packages, an equal number of each; how many packages of each kind were made?

32. How much must a contractor pay 18 men for working 6 days, if each works 10 hr. a day at \$$\frac{2}{5}$ an hour?

33. A wage rate of men on farms in Montana is \$49 per month without board and \$35 per month with board. What fractional part of the \$49 is deducted for board?

34. What part of June has passed by midnight, June 20?

35. The following are the ingredients and quantities necessary to make enough apple pudding for 3 persons. How much of each of these ingredients should be used to make enough apple pudding for 12 persons?

1 cup of flour
$\frac{1}{4}$ teaspoonful of salt
$1\frac{1}{2}$ teaspoonfuls of baking powder
$1\frac{1}{2}$ tablespoonfuls of sugar
1 egg
$\frac{3}{4}$ cup of milk
2 tablespoonfuls of butter
2 apples
$\frac{1}{4}$ teaspoonful of cinnamon

36. Find the total length of the edges of a box $3\frac{1}{2}$ ft. long, $2\frac{1}{2}$ ft. wide, and $1\frac{3}{4}$ ft. high.

37. What should $2\frac{1}{2}$ lb. of tea cost if $\frac{1}{2}$ lb. cost $\$\frac{1}{4}$?

38. When $\$233\frac{1}{3}$ was paid for 8 months' rent, what rent was charged per year?

39. Find the cost of $8\frac{1}{2}$ doz. plants at $3\not{c}$ each.

40. If one stove consumes $\frac{3}{4}$ gal. of oil in a day and another $\frac{5}{8}$ gal., what part of a day will a gallon last both stoves?

41. A and B together purchased 150 acres of land, of which A took $30\frac{1}{2}$ acres more than B. How many acres did each take?

SUGGESTION. $2 \times$ B's $+ 30\frac{1}{2}$ acres $= 150$ acres.

$2 \times$ B's $= 150$ acres $- 30\frac{1}{2}$ acres.

42. A man and his son earned $\$25\frac{1}{2}$ in a week. The man earned $\$5\frac{1}{2}$ more than the son. How much did each earn?

43. If $2\frac{1}{2}$ in. on a certain drawing represent 10 mi., what should $\frac{3}{4}$ of an inch represent on the same drawing?

44. Two loads of corn contained together $75\frac{1}{2}$ bushels. They were sold at $\$\frac{3}{4}$ a bushel. $\$4\frac{1}{2}$ more was received for one load than for the other. How many bushels did each contain?

45. A dealer weighed 3 drafts of lard in a can which weighed $2\frac{3}{4}$ lb. The weight of the first draft and can was $21\frac{1}{2}$ lb., the weight of the second draft and can was $18\frac{1}{2}$ lb., the weight of the third draft and can was $20\frac{3}{4}$ lb. What was the weight of the lard in the three drafts?

DECIMALS

105. One tenth of a unit is *a tenth;* one tenth of a tenth is *a hundredth;* one tenth of a hundredth is *a thousandth;* one tenth of a thousandth is *a ten-thousandth;* and so on.

106. A tenth, a hundredth, a thousandth, a ten-thousandth, and so on, are called **decimal units.** A tenth is a decimal unit of the *first order;* a hundredth is a decimal unit of the *second order;* a thousandth is a decimal unit of the *third order;* and so on.

REMARK. In practice a decimal unit less than one ten-thousandth is seldom used.

107. Any number of decimal units of the same order is called a **decimal fraction,** or simply a **decimal.**

Thus, *7 tenths, 21 hundredths,* and *325 thousandths* are decimals.

108. If the line of notation in the decimal system is extended to the right from units' place, and a point (.), called the **decimal point,** is placed after the units' order, a figure standing in the *first* place to the right of this point, represents a number of tenths; in the *second* place, a number of hundredths; in the *third* place, a number of thousandths; in the *fourth* place, a number of ten-thousandths; and so on.

Thus, .375 represents 3 tenths, 7 hundredths, and 5 thousandths.

Table

Hundreds	Tens	Units	.	Tenths	Hundredths	Thousandths	Ten-thousandths	Hundred-thousandths	Millionths	Ten-millionths	Hundred-millionths	Billionths
3	1	2	.	8	1	4	9	6	3	1	9	5

NUMERATION AND NOTATION

109. Since $.375 = \frac{3}{10} + \frac{7}{100} + \frac{5}{1000}$, or $\frac{375}{1000}$, which is read *three hundred seventy-five thousandths*, .375 may be read *three hundred seventy-five thousandths.*

110. To read a decimal, read the number expressed by the figures of the decimal, and pronounce the name of the place occupied by the right-hand figure.

111. To write a decimal in figures, write the number of decimal units, and so place the decimal point that the right-hand figure will stand in the place that has the same denomination as the decimal.

Illustrative Examples

1. Read .426.

Thus, .426 is read *four hundred twenty-six thousandths.*

2. Read 3.45.

Thus, 3.45 is read *three and forty-five hundredths.*

3. Read .16$\frac{2}{3}$.

Thus, .16$\frac{2}{3}$ is read *sixteen and two thirds hundredths.*

4. Read .0$\frac{1}{3}$.

Thus, .0$\frac{1}{3}$ is read *one third of a tenth.*

5. Read 6.257 correct to the nearest hundredth.

Thus, 6.257, read correct to the nearest hundredth, is *six and twenty-six hundredths.*

6. Read 2.783 correct to the nearest hundredth.

Thus, 2.783, read correct to the nearest hundredth, is *two and seventy-eight hundredths.*

7. Write in figures 8 thousandths.

Thus, write 8, and so place the decimal point that 8 stands in the thousandths' place; as, .008.

8. Write in figures 75 tenths.

Thus, write 75, and so place the decimal point that 5 stands in the tenths' place; as, 7.5.

Exercise 28

Read:

1. .7	2. .35	3. .609	4. .0017
5. 12.5	6. 35.04	7. .008	8. .0004
9. 100.01	10. 300.03	11. 300.003	12. .303
13. .0$\frac{2}{3}$	14. .0$\frac{4}{5}$	15. 1.0$\frac{8}{9}$	16. .12$\frac{1}{2}$
17. .00$\frac{2}{3}$	18. .02$\frac{7}{8}$	19. .03$\frac{1}{3}$	20. 4.66$\frac{2}{3}$

21. Read 3.68 correct to the nearest tenth.

22. Read 3.52 correct to the nearest tenth.

23. Read 7.651 correct to the nearest hundredth.

24. Read 8.757 correct to the nearest hundredth.

25. Read 4.7321 correct to the nearest thousandth.

26. Read 4.7348 correct to the nearest thousandth.

27. Read \$7.8746 correct to the nearest mill; correct to the nearest cent; correct to the nearest dime; correct to the nearest dollar.

Write in figures:

28. 7 tenths
29. 45 hundredths
30. 3 hundredths
31. 9 thousandths
32. 25 thousandths
33. 2 and 7 tenths
34. 135 thousandths
35. 18 tenths
36. 375 hundredths
37. 8 ten-thousandths
38. $37\frac{1}{2}$ hundredths
39. $87\frac{1}{2}$ hundredths
40. 707 thousandths
41. 700 and 7 thousandths
42. 303 hundredths
43. 4325 ten-thousandths
44. $62\frac{1}{2}$ ten-thousandths
45. 300 and 3 hundredths

REDUCTION

Illustrative Examples

1. Reduce .375 to a common fraction.

Thus, $.375 = \frac{375}{1000}$, or $\frac{3}{8}$.

2. Reduce $.33\frac{1}{3}$ to a common fraction.

Thus, $.33\frac{1}{3} = \frac{33\frac{1}{3}}{100} = \frac{3 \times 33\frac{1}{3}}{3 \times 100} = \frac{100}{300}$, or $\frac{1}{3}$.

3. Reduce $\frac{4}{25}$ to a decimal.

Thus, $\frac{4}{25} = \frac{16}{100}$, or .16.

REMARK. Notice that the denominator of $\frac{4}{25}$ exactly divides 100.

4. Reduce $\frac{7}{8}$ to a decimal.

Thus, $8\overline{)7.000}$
$.875$

SUGGESTION. Annex ciphers to the numerator and divide by the denominator. Compare the number of decimal places in the quotient with the number of ciphers annexed.

5. Reduce $\frac{2}{9}$ to hundredths.

Thus, $9\overline{)2.00}$
$.22\frac{2}{9}$

6. Reduce $.6\frac{2}{3}$ to thousandths.

Thus, $.6\frac{2}{3} = .6 + .0\frac{2}{3}$
$= .6 + \frac{1}{3}$ of .2
$= .6 + \frac{1}{3}$ of .200
$= .6 + .066\frac{2}{3}$, or $.666\frac{2}{3}$.

7. Reduce $.466\frac{2}{3}$ to tenths.

Thus, $.466\frac{2}{3} = .4\frac{66\frac{2}{3}}{100}$
$= .4\frac{2}{3}\frac{0}{0}\frac{0}{0}$, or $.4\frac{2}{3}$.

Exercise 29

Reduce to fractions or mixed numbers:

1. .08	2. .025	3. .375	4. .625
5. 4.25	6. 5.05	7. 3.75	8. .450
9. .004	10. .0375	11. 6.005	12. .875
13. $.8\frac{1}{3}$	14. $.4\frac{1}{6}$	15. $.6\frac{1}{4}$	16. $.62\frac{1}{2}$
17. $.3\frac{1}{8}$	18. $.37\frac{1}{2}$	19. $.66\frac{2}{3}$	20. $.87\frac{1}{2}$

21. $.83\frac{1}{3}$ 22. $.22\frac{2}{9}$ 23. $.18\frac{3}{4}$ 24. $.14\frac{2}{7}$

25. $.44\frac{4}{9}$ 26. $.28\frac{4}{7}$ 27. $1.12\frac{1}{2}$ 28. $1.37\frac{1}{2}$

Reduce to decimals:

29. $\frac{3}{4}$ 30. $\frac{3}{8}$ 31. $\frac{7}{8}$ 32. $\frac{3}{16}$

33. $\frac{3}{30}$ 34. $\frac{9}{40}$ 35. $\frac{7}{32}$ 36. $\frac{9}{64}$

Reduce to tenths:

37. $\frac{2}{3}$ 38. $\frac{5}{9}$ 39. $\frac{1}{12}$ 40. $\frac{3}{7}$

Reduce to hundredths:

41. $\frac{5}{6}$ 42. $\frac{8}{9}$ 43. $\frac{5}{12}$ 44. $\frac{7}{15}$

Reduce to thousandths:

45. $\frac{3}{7}$ 46. $\frac{4}{9}$ 47. $\frac{7}{12}$ 48. $\frac{11}{15}$

Reduce:

49. $.4\frac{1}{6}$ to hundredths 50. $.8\frac{1}{3}$ to hundredths

51. $.16\frac{2}{3}$ to thousandths 52. $.14\frac{2}{7}$ to thousandths

53. $.5\frac{5}{6}$ to thousandths 54. $.28\frac{4}{7}$ to ten-thousandths

55. .125 to hundredths 56. .6375 to hundredths

57. .5625 to tenths 58. $.333\frac{1}{3}$ to hundredths

ADDITION AND SUBTRACTION

112. To add or subtract decimals, write the numbers so that units of the same order stand in the same column; add or subtract as in addition or subtraction of whole numbers, and place the decimal point between units and tenths in the result.

Illustrative Examples

Add:

1.	2.
7.568	$.33\frac{1}{3} = .333\frac{1}{3}$
3.8	$.888\frac{8}{9} = .888\frac{8}{9}$
7.84	$.6\frac{2}{3} = .666\frac{2}{3}$
19.208	Sum $= 1.888\frac{8}{9}$

Subtract:

3.	4.
15.3	$15.5\frac{1}{3} = 15.53\frac{1}{3}$
7.375	$7.66\frac{2}{3} = 7.66\frac{2}{3}$
7.925	Difference $= 7.86\frac{2}{3}$

Exercise 30

Add:

1.	2.	3.
2.3	3.125	7.602
4.07	14.57	15.9
6.875	123.3	8.013
25.046	.0462	103.0035

4.	5.	6.
$.33\frac{1}{3}$	$2.97\frac{1}{2}$	$.16\frac{2}{3}$
$2.562\frac{1}{6}$	$.014\frac{2}{3}$	$.87\frac{1}{2}$
$14.6\frac{2}{3}$	$351.7\frac{1}{3}$	$.333\frac{1}{3}$
$9.074\frac{1}{3}$	$.01\frac{1}{9}$	$.6\frac{2}{3}$

Subtract:

7.	8.	9.
2.3	.01	10.101
1.998	.009808	9.0191

10.	11.	12.
$6.6\frac{2}{3}$	$3.3\frac{1}{3}$	$18.7\frac{1}{2}$
$3.3331\frac{1}{3}$	$.7666\frac{2}{3}$	$6.88\frac{8}{9}$

Find the value of:

13. $3.125 + 14.36 + 125.3 + .0462$.

14. $4.325 + 6.0035 - 2.05 - 3.0573$.

MULTIPLICATION

Multiplying by 10, 100, 1000, etc.

1. $10 \times .375 = 10 \times \frac{375}{1000} = \frac{375}{100}$, or 3.75.

2. $100 \times .375 = 100 \times \frac{375}{1000} = \frac{375}{10}$, or 37.5.

3. $1000 \times .375 = 1000 \times \frac{375}{1000}$, or 375.

113. Moving the decimal point one place to the right multiplies by 10; moving it two places to the right multiplies by 100; moving it three places to the right multiplies by 1000, etc.

Exercise 31

Read, supplying at sight the missing products:

1. $10 \times .5$	2. 10×1.3
3. $10 \times .06$	4. 10×2.01
5. $10 \times .35$	6. 10×4.13
7. $100 \times .312$	8. 100×5.10
9. $100 \times .58$	10. 100×3.75
11. $100 \times .43$	12. 100×7.3

13. $1000 \times .5$
14. 1000×3.06
15. $1000 \times .66$
16. 1000×21.716
17. $1000 \times .250$
18. 1000×5.005
19. $10000 \times .17$
20. 10000×4.45

Multiplying by Integers and by Decimals

Multiply: *In practice find products thus:*

1. 6.25 by 45.

$45 \times 6.25 = 45 \times \frac{625}{100}$
$= \frac{28125}{100}$, or 281.25.

```
 6.25
   45
 3125
2500
281.25
```

2. 27 by 4.5.

$4.5 \times 27 = \frac{45}{10} \times 27$
$= \frac{1215}{10}$, or 121.5.

```
  27
 4.5
 135
108
121.5
```

3. 3.56 by 6.4.

$6.4 \times 3.56 = \frac{64}{10} \times \frac{356}{100}$
$= \frac{22784}{1000}$, or 22.784.

```
 3.56
  6.4
 1424
2136
22.784
```

114. To find the product of two numbers, one or both of which are decimals, multiply as in whole numbers and point off as many decimal places in the product as there are in the multiplicand and the multiplier together.

Exercise 32

Multiply:

1. 2.37 by 26
2. 3.85 by 60
3. 7.24 by .08
4. 3.16 by 37
5. 4.06 by 50
6. 37.4 by .007

7. 8.75 by .25 8. 632 by .008 9. 3.42 by .024

10. 6.35 by 7.56 11. .607 by 96 12. .389 by 4.8

13. 4.56 by .32 14. .008 by .048 15. .756 by .095

Exercise 33

1. Find the cost of all:

150 lb. sugar at $5\frac{1}{2}¢$
14 doz. eggs at $40¢$
12 lb. butter at $48¢$

2. What wages must a contractor pay 6 carpenters for working 28 da. at $\$2.75$, and 3 plasterers for working 8 da. at $\$3.25$?

3. Find the cost of a ton (2000 lb.) of nails at $\$3\frac{1}{2}$ per 100 lb.

4. Find the cost of 6000 rails at $\$12$ per 100.

5. Find the cost of shipping 600 lb. of goods at $\$0.48$ per 100 lb.

6. Below is given the acreage in various grains in Oregon during a recent year and the average yield per acre. Fill in the missing totals and find the total value of all these grains at the prices now prevailing.

Grain	Acreage	Average Acre Yield	Total Yield
Wheat	770,000	20.8 bu.	
Oats	300,000	35 bu.	
Barley	83,000	30 bu.	
Rye	11,000	18 bu.	

7. A merchant had on hand at the beginning of a day $287.50. During the day he received $86.75, and paid out $47.75. How much had he on hand at the close of the day?

8. How many decimal places has the product when tenths are multiplied by tenths? hundredths by tenths? thousandths by thousandths? hundredths by thousandths?

9. Find, correct to the nearest cent, the value of 240.6 bushels of wheat at $0.98 a bushel.

10. Express each in dollars and find the sum: 18 cents, $37\frac{1}{2}$ cents, $2\frac{3}{4}$ dollars, 18 mills, 6 cents 7 mills.

DIVISION

Dividing by 10, 100, 1000, etc.

1. $625 \div 10 = \frac{625}{10}$, or 62.5.
2. $62.5 \div 10 = \frac{625}{10} \div 10 = \frac{625}{100}$, or 6.25.
3. $62.5 \div 100 = \frac{625}{10} \div 100 = \frac{625}{1000}$, or .625.
4. $62.5 \div 1000 = \frac{625}{10} \div 1000 = \frac{625}{10000}$, or .0625.

115. Moving the decimal point one place to the left divides by 10; moving it two places to the left divides by 100; moving it three places to the left divides by 1000, etc.

Read, supplying at sight the missing quotients:

1. $25 \div 10$	2. $2.5 \div 10$
3. $.07 \div 10$	4. $.75 \div 10$
5. $250 \div 100$	6. $37.5 \div 100$
7. $.5 \div 1000$	8. $62.5 \div 1000$

Dividing by Integers and by Decimals

1. $.015 \div 3 = \frac{15}{1000} \div 3.$
$= \frac{5}{1000}$, or $.005.$

That is, $.015 \div 3 = .005.$ (A)

2. $.008 \div .04 = \frac{8}{1000} \div \frac{4}{100}.$
$= \frac{100}{4} \times \frac{8}{1000}$
$= \frac{1}{4} \times \frac{8}{10}$, or $.8 \div 4.$

That is, $.008 \div .04 = .8 \div 4.$ (B)

3. $.04 \div 20 = \frac{4}{100} \div 20.$
$= \frac{1}{20} \times \frac{4}{100}$
$= \frac{1}{2} \times \frac{4}{1000}$ [§ 67]
$= .004 \div 2.$

That is, $.04 \div 20 = .004 \div 2.$ (C)

Observe in (A) that the divisor is a whole number and that the quotient contains as many decimal places as there are in the dividend.

Observe from (B) and (C) that the decimal point may be moved to the right or to the left in both dividend and divisor the same number of places without changing the quotient.

From (A), (B), and (C) it follows that, in division of decimals we may:

116. Move the decimal point to the right or to the left in both dividend and divisor such a number of places as will make the divisor a whole number not ending in ciphers; then divide and point off in the quotient as many places as there are in the dividend.

Illustrative Examples

Divide:

1. .625 by .25.

$$.625 \div .25 = 25\overline{)62.5}\quad \begin{array}{r} 2.5 \\ \hline 50 \\ \hline 125 \\ \underline{125} \end{array}$$

2. 6.25 by .025.

$$6.25 \div .025 = 25\overline{)6250}\quad \begin{array}{r} 250 \\ \hline 50 \\ \hline 125 \\ \underline{125} \\ \hline 0 \end{array}$$

3. 6.25 by 250.

$$6.25 \div 250 = 25\overline{).625}\quad \begin{array}{r} .025 \\ \hline 50 \\ \hline 125 \\ \underline{125} \end{array}$$

Check: (1) $.25 \times 2.5 = .625$. (2) $.025 \times 250 = 6.25$.
(3) $250 \times .025 = 6.25$.

Exercise 34

Find the quotient of:

1. $.05 \div .5$
2. $.325 \div 25$
3. $3.075 \div 75$
4. $.00252 \div 42$
5. $461.5 \div 142$
6. $98.1 \div 3000$
7. $191.52 \div 4200$
8. $180 \div 50$
9. $.0045 \div 1500$
10. $3.6 \div 1500$
11. $5 \div .005$
12. $.005 \div .5$
13. $8.64 \div 2.4$
14. $.6048 \div .48$
15. $.1728 \div .0144$
16. $4840 \div .16$
17. $3.75 \div 1.5$
18. $3.84 \div .16$
19. $48.3 \div .021$
20. $.308 \div 2.2$

Exercise 35

Divide:

1. $446.2 \div 4600$	2. $50.73 \div 890$
3. $100.5 \div .67$	4. $10.71 \div 5.1$
5. $.4012 \div 5.9$	6. $98.01 \div .99$
7. $75.69 \div 870$	8. $15.08 \div .58$
9. $29.68 \div 2.8$	10. $345.6 \div 3.2$
11. $16.686 \div .54$	12. $27.202 \div 6700$
13. $35.956 \div 89$	14. $6.6552 \div 9.4$
15. $792.82 \div .98$	16. $7092.8 \div .88$
17. $182.40 \div .024$	18. $.47795 \div .79$
19. $590.73 \div .97$	20. $899.91 \div .99$

Finding the Quotient Approximately

117. When the division does not terminate, in practice it is seldom carried beyond the fifth decimal place in the quotient, which is usually expressed to the nearest *ten-thousandth* (.0001).

Thus, the quotient of $7.4 \div 35$ expressed correct to the nearest ten-thousandth is found as follows:

$.21142^{+} = .2114$ correct to the nearest .0001.

$$
\begin{array}{r}
35\overline{)7.40000} \\
\underline{70} \\
40 \\
\underline{35} \\
50 \\
\underline{35} \\
\text{etc.}
\end{array}
$$

NOTE. If the quotient were .21146, expressed to the nearest .0001 it would be .2115.

Exercise 36

Express to the nearest ten-thousandth the value of:

1. $7 \div 15$	2. $9 \div 28$	3. $7.5 \div 22$
4. $6.75 \div 34$	5. $.05 \div 32$	6. $.075 \div 48$
7. $2.04 \div 65$	8. $.384 \div 81$	9. $.147 \div 300$
10. $15 \div 38$	11. $8 \div 75$	12. $1 \div 300$
13. $4 \div 270$	14. $2.1 \div 275$	15. $48 \div 780$
16. $304 \div 41$	17. $3 \div 8.50$	18. $7 \div 7.75$
19. $9.1 \div 22.51$	20. $5 \div 87.5$	21. $4 \div 61.25$
22. $8 \div 97.50$	23. $76 \div 76.67$	24. $600 \div 900$
25. $81 \div 243$	26. $100 \div 333$	27. $700 \div 1050$

28. $48 \div 780$	29. $304 \div 41$
30. $3 \div 8.50$	31. $7 \div 7.75$
32. $9.1 \div 22.51$	33. $5 \div 87.5$
34. $4 \div 61.25$	35. $8 \div 97.50$
36. $86 \div 76.67$	37. $700 \div 900$
38. $91 \div 243$	39. $200 \div 333$
40. $800 \div 1050$	41. $187 \div 623$
42. $85.67 \div 666$	43. $794 \div 1111$

BILLS AND ACCOUNTS

118. A **bill** is a written statement of the names and prices of goods sold or services rendered, together with the names of the parties and the date of the transaction.

119. An **account** is a record of business transactions between two parties at a given date.

120. A bill is **filled** by writing in the cost of each item; it is **footed** by adding the cost of the different items to find the total cost.

(The terms used in Bills and Accounts were explained in Book Two; if the pupil is unable to use them understandingly, they should be reviewed.)

Exercise 37

1. Complete and verify the footing of the following:

Bill for Goods purchased on the Same Date

Engravings do not become the property of the party ordering unless a special agreement is made. All claims must be made within 3 days from receipt of goods.

E. A. WRIGHT BANK NOTE COMPANY

BANK NOTE & GENERAL ENGRAVERS
PRINTERS, LITHOGRAPHERS & MANUFACTURING STATIONERS

Mr. James Warring,
Union & Planters Bank Bldg.,
Memphis, Tenn.

1108 Chestnut Street
Philadelphia 10/26/12.

TERMS: Cash at first of month following date of shipment
" Subject to sight draft "
Not responsible for money paid to Agents

ORDER No. 42710 SHIPPED VIA Adams Exp. F. O. B. PHILA.

5000	Steel Engraved	Letter Heads	@ $12	40 per M	$62 00	
2000	" "	Bill Heads	@ 18	00 " "		
1500	" "	Bank Checks	@ 72	00 " "		
1500	" "	Opening Announcements	@ 34	00 " "		
500	" "	Business Cards	@ 29	00 " "		$271 50

RECEIVED PAYMENT
Nov 23 1912
E.A.WRIGHT BANK NOTE CO.
Per M.H.B

The person who receives money for a firm in payment of a bill usually receipts the bill and signs the firm name; also his own name or initials, thus: "*Per M. H. B.*"

Find out what "F. O. B. Phila." as printed on this bill means.

Remark. Sometimes columns are ruled in which to place the extension of the items and the total.

2. Verify the footing of the columns of Daily Totals and of Credits, and see that the balance is correct, in the following:

Bill for the Entire Purchases of One Month

MARSHALL FIELD & COMPANY

Importers, Retailers, Manufacturers

State, Washington, Randolph & Wabash

Chicago, Dec. 2, 1912

Sold to Mr. Frank P. Price,
124 Main St.,
Galesburg, Ill.

31/2365

Notice. — Merchandise to be credited on current bill must be returned before the last day of the month.

Nov.				Daily Totals	Credits
2	2 yd. lace	75	1 50		
	1 pr. blankets		4 75		
	2 doz. napkins	4 00	8 00	14 25	
16	1 pr. blankets				4 75
19	1 dining table			24 00	
23	2 yd. cloth	3 85	7 70		
	1 pr. shoes		4 00		
	4 yd. ribbon	18	72		
	1 hand bag		2 00	14 42	
25	1 pr. shoes				4 00
				52 67	8 75
				8 75	
				43 92	

3. Extend the items in the following account and find the balance:

DAVENPORT, IOWA, April 1, 1913.

Mr. J. B. Chamberlin,

In account with SIMPSON & CRAWFORD

Dr.			
4 Hand Saws	*$ 1.80*		
1 Hardwood Tool Chest		*12*	*50*
2 Iron Block Planes	*0.75*		
3 Auger Handles	*2.50*		
2 Brace and Bit Outfits	*10.50*		
Cr.			
8 days' carpentering	*$ 3.60*		
4 days' painting	*3.50*		
2 days' roofing	*3.50*		
5 days' labor	*1.50*		

Rule bill forms and on them make out bills for Examples 4 and 5; use your own name for that of the buyer, and the name of some firm that deals in the articles purchased for that of the seller. Fill, foot, and receipt the bills, dating them appropriately.

4.

1 Mission Bookcase	@	$ 5.75
1 Library Table	@	9.95
1 Cheval Mirror	@	8.75
2 Parlor Roman Chairs	@	3.50
6 Dining Chairs	@	2.40
2 Rockers	@	4.95
4 Porch Rockers	@	2.80

5.	9 yd. Velvet Ribbon	@	$0.21
	$9\frac{1}{2}$ yd. Corduroy	@	1.80
	7 yd. Serge	@	1.75
	8 yd. Sheeting	@	0.35
	2 pr. Lace Curtains	@	3.50
	4 pr. Sash Curtains	@	2.75
	1 pr. Shoes	@	3.50

6. F. P. Duffy presents the following bill to you, Oct. 1, 1913: 28 days' carpenter work, $2.25 per day; 12 days' plastering, $2.75 per day; 8 days' painting, $3 per day; 6 days' hauling, $4.50 per day; 12 days' common labor, $1.50 per day. Fill, foot, and receipt the bill for these items.

Render the following bill for the month named in the current year; receipt the bill:

7.	Nov. 5	1 pr. Shoes	@	$ 5.00
		1 suit Clothes	@	18.50
		4 yd. Silk	@	1.50
	Nov. 12	1 Reading Lamp	@	8.75
	Nov. 17	1 pr. Shoes returned	@	5.00
	Nov. 25	4 doz. Cards	@	0.10
		2 pr. Blankets	@	6.00
		1 doz. Handkerchiefs	@	3.00
	Nov. 30	1 pr. Blankets returned	@	6.00
		2 pr. Gloves	@	0.75
		1 Table Cover	@	3.50
		2 yd. Lace	@	.45

8. Rule account form and on it make out the following account and find the balance:

Sept. 1, 1913, B. F. Heistand & Sons, bought of J. W. Williams, Dayton, Ohio, 250 bu. wheat @ $0.92 per bushel; 160 bu. oats @ $0.48 per bushel; 6 T. of hay @ $21 per ton; 375 bu. corn @ $0.78 per bushel. At the same time J. W. Williams bought from B. F. Heistand & Sons 2000 ft. of boards @ $40 per 1000 ft.; 600 posts @ $28 per 100; 800 rails @ $12 per 100.

PARCEL POST

121. The parcel post law provides:

"That fourth-class mail matter shall embrace all other matter, including farm and factory products and books, not now embraced by law in either the first, second, or third class, not exceeding fifty pounds in weight (twenty pounds if intended for delivery in zones beyond the second), nor greater in size than seventy-two inches in length and girth combined, nor in form or kind likely to injure the person of any postal employee or damage the mail equipment or other mail matter and not of a character perishable within a period reasonably required for transportation and delivery."

For the purpose of carrying the law into effect the United States is divided into zones with different rates of postage applicable to each, except in the case of parcels weighing four ounces or less and books weighing eight ounces or less.

Parcels weighing four ounces or less are mailable at the rate of *one cent for each ounce or fraction thereof*, regardless of distance, and books weighing eight ounces or less are mailable at the rate of *one cent for each two ounces or fraction thereof*, regardless of distance. Parcels weighing more than four ounces, and books weighing more than eight ounces, are mailable at the pound rate, as shown in the following table:

Weight	1st zone		2d zone, 50 to 150 miles	3d zone, 150 to 300 miles	4th zone, 300 to 600 miles	5th zone, 600 to 1000 miles	6th zone, 1000 to 1400 miles	7th zone, 1400 to 1800 miles	8th zone, all over 1800 miles
	Local rate	Zone rate, 50 miles	*Rate*	*Rate*	*Rate*	*Rate*	*Rate*	*Rate*	*Rate*
1 pound	$0.05	$0.05	$0.05	$0.06	$0.07	$0.08	$0.09	$0.11	$0.12
2 pounds	.06	.06	.06	.08	.11	.14	.17	.21	.24
3 pounds	.06	.07	.07	.10	.15	.20	.25	.31	.36
4 pounds	.07	.08	.08	.12	.19	.26	.33	.41	.48
5 pounds	.07	.09	.09	.14	.23	.32	.41	.51	.60
	and 1 cent for each additional 2 pounds or fraction thereof	and 1 cent for each additional pound or fraction thereof	and 1 cent for each additional pound or fraction thereof	and 2 cents for each additional pound or fraction thereof	and 4 cents for each additional pound or fraction thereof	and 6 cents for each additional pound or fraction thereof	and 8 cents for each additional pound or fraction thereof	and 10 cents for each additional pound or fraction thereof	and 12 cents for each additional pound or fraction thereof
	Weight limit 50 pounds			Weight limit 20 pounds					

The local rate is applicable to parcels intended for delivery at the office of mailing or on a rural route starting therefrom.

NOTE. Except as provided for in Section 121, a fraction of a pound is considered a full pound.

Exercise 38

From the table, state at sight the postage on the following:

	Weight of Parcel	Distance carried from Office of Mailing
1.	2 pounds	60 miles
2.	4 pounds	30 miles
3.	8 pounds	350 miles
4.	10 pounds	750 miles
5.	2 pounds	2000 miles
6.	2 oz.	Any distance
7.	3 pounds	Within third zone
8.	$7\frac{1}{2}$ pounds	400 miles

9. What is the postage on a parcel weighing 4 pounds, mailed for delivery at the local office?

10. What is the postage on a parcel weighing 11 pounds, mailed for delivery at the local office?

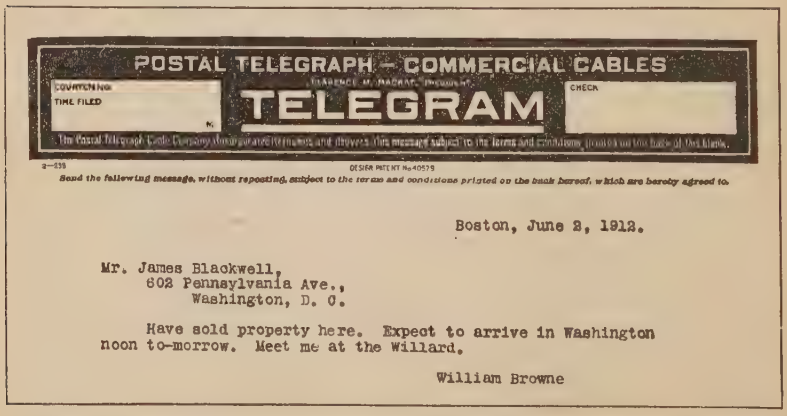
11. What is the postage on a parcel weighing 6 pounds, mailed for delivery on a rural route, starting from the office of mailing?

12. What is the postage on a parcel weighing 10 pounds, mailed for delivery on a rural route, starting from the office of mailing?

MESSAGES

Telegram

122. Find the cost of sending the following telegram, the day rate from Boston to Washington being 40¢ for ten words or less, and 3¢ for each word over ten (address and one signature free):



POSTAL TELEGRAM – COMMERCIAL CABLES

CLARENCE H. MACKAY, PRESIDENT

CHECK

TIME FILED

TELEGRAM

The Postal Telegraph-Cable Company does not guarantee transmission and delivery of this message beyond the terms and conditions printed on the back hereof.

2—539 DESIGN PATENT No. 40519

Send the following message, without repeating, subject to the terms and conditions printed on the back hereof, which are hereby agreed to.

Boston, June 2, 1913.

Mr. James Blackwell,
603 Pennsylvania Ave.,
Washington, D. C.

Have sold property here. Expect to arrive in Washington noon to-morrow. Meet me at the Willard.

William Browne

NOTE. The night rate for telegrams is usually lower than the day rate.

Cablegram

123. Find the cost of sending the following cablegram, the rate from New York to London being 25¢ per word, all words except those of the date to be counted and charged for. In the address the name of "*place to*" and the name of the territorial division of the country in which the "*place to*" is situated are each counted as one word without regard to number of letters they contain.

The following are applicable to cable messages:

1. In messages in plain language the maximum length of a word is fixed at fifteen letters.

2. Punctuation marks, hyphens, apostrophes, and pound sterling marks are counted as one word each.

Form 1.

WESTERN UNION *Cable Message* WESTERN UNION

THE WESTERN UNION TELEGRAPH COMPANY

INCORPORATED

THEO. N. VAIL, PRESIDENT BELVIDERE BROOKS, GENERAL MANAGER

THE LARGEST TELEGRAPH AND CABLE SYSTEM IN EXISTENCE. CABLE SERVICE TO ALL THE WORLD

25,000 OFFICES AND 35,000 ADDITIONAL TELEGRAPH AND TELEPHONE CONNECTIONS IN NORTH AMERICA

DIRECT AMERICAN CABLES NEW YORK TO GREAT BRITAIN
CONNECTS ALSO WITH ANGLO-AMERICAN AND DIRECT U. S. ATLANTIC CABLES
DIRECT COMMUNICATION WITH GERMANY AND FRANCE, CUBA, WEST INDIES, MEXICO AND CENTRAL AND SOUTH AMERICA
WITH PACIFIC CABLES TO ALASKA, HONOLULU, AUSTRALIA, GUAM, THE PHILIPPINES, JAPAN, ETC.
MONEY TRANSFERRED BY TELEGRAPH AND CABLE TO ALL THE WORLD

BRANCH OFFICES IN PRINCIPAL CITIES OF GREAT BRITAIN AND THE EUROPEAN CONTINENT. ALL FOREIGN TELEGRAPH STATIONS ACCEPT MESSAGES TO BE SENT "Via WESTERN UNION"

New York, June 5, 1914

SEND THE FOLLOWING MESSAGE, SUBJECT TO TERMS ON BACK HEREOF, WHICH ARE HEREBY AGREED TO

To A. T. Smith.

820 Oxford St. London.

We sail Tuesday morning

George Hallett

Night Letter

124. Telegraph companies will receive not later than midnight **night letters** for delivery only on the morning of the next ensuing business day, at rates lower than their standard night message rates, and usually stated as follows:

"The standard day rate for ten words shall be charged for the transmission of fifty words or less, and one fifth of the standard day rate for ten words shall be charged for each additional ten words or less."

Find the cost of sending the following night letter, the *day rate* from Albany, N.Y., to Elizabeth, N.J., being 30¢ for ten words or less and 2¢ for each word over ten (address and one signature free).

Form 2260 J

NIGHT LETTER

THE WESTERN UNION TELEGRAPH COMPANY

INCORPORATED

25,000 OFFICES IN AMERICA CABLE SERVICE TO ALL THE WORLD

THEO. N. VAIL, PRESIDENT BELVIDERE BROOKS, GENERAL MANAGER

RECEIVER'S No.	TIME FILED	CHECK

SEND the following NIGHT LETTER subject to the terms on back hereof, which are hereby agreed to } Albany, N.Y., June 5, 1913

To Hitchings & Company,

Elizabeth, N.J.

We have just sold for you four greenhouses to be shipped to Quarryville, Pa., at the earliest date possible. We must guarantee that they will be up and ready for use in thirty days. Ship in our name and send workmen on at once to put the houses together. Send us fifty more booklets: we expect more large sales to-morrow.

J. M. Thompson & Co.

Day Letter

125. Telegraph companies will receive **day letters,** to be transmitted at rates lower than their standard day message rates, and usually stated as follows:

"One and one half times the standard night letter rate shall be charged for the transmission of fifty (50) words or less, and one fifth of the initial rate for such fifty words shall be charged for each additional ten (10) words or less."

Find the cost of sending the following day letter, the rate from Minneapolis, Minn., to Lexington, Ky., being 50¢ for ten words or less, and 3¢ for each word over ten (address and one signature free).

Form ...

DAY LETTER

THE WESTERN UNION TELEGRAPH COMPANY
INCORPORATED

25,000 OFFICES IN AMERICA — CABLE SERVICE TO ALL THE WORLD

THEO. N. VAIL, PRESIDENT — BELVIDERE BROOKS, GENERAL MANAGER

RECEIVER'S No.	TIME FILED	CHECK

SEND the following DAY LETTER subject to the terms on back hereof which are hereby agreed to

Minneapolis, Minn., June 5, 1912.

Mr. G. L. Brown,

Lexington, Kentucky.

Notify me as soon as you have shipped horses. I understand that you are shipping two carloads. There is ready sale here for good horses. Prices are advancing.

F. K. Lewis.

Exercise 39

In the columns headed respectively "Day" and "Night," in the following, the figures before the hyphen indicate rates for ten words or less (address and signature free), and the figures after the hyphen indicate the rates for each word over ten, for messages sent between Philadelphia and the cities named; find the missing costs.

Place	Number of Words	Rate		Cost of Day Telegram	Cost of Night Telegram	Cost of Night Letter	Cost of Day Letter
		Day	Night				
1. Mobile, Ala.	17	60–4	50–3				
2. Phœnix, Ariz.	22	100–7	100–7				
3. Los Angeles, Cal.	25	100–7	100–7				
4. Denver, Colo.	18	75–5	60–4				
5. Hartford, Conn.	60	30–2	25–1				
6. Wilmington, Del.	21	25–2	25–1				
7. Jacksonville, Fla.	40	60–4	50–3				
8. Atlanta, Ga.	15	60–4	50–3				
9. Boise, Id.	24	100–7	100–7				
10. Chicago, Ill.	35	50–3	40–3				
11. Columbus, O.	19	50–3	40–3				
12. Des Moines, Ia.	8	60–4	50–3				
13. Kansas City, Mo.	40	60–4	50–3				
14. Louisville, Ky.	26	50–3	40–3				
15. Bar Harbor, Me.	50	50–3	40–3				
16. Baltimore, Md.	16	25–2	25–1				
17. Boston, Mass.	32	35–2	25–1				
18. Albany, N.Y.	45	30–2	25–1				
19. Harrisburg, Pa.	27	25–2	25–1				

Refer to the above table and state at sight the cost of sending a telegram of ten words from Philadelphia to each of the cities named.

REVIEW OF DECIMALS

Exercise 40

1. Milch cows should be fed each day 2.25 lb. of hay or its equivalent for each 100 lb. of their live weight. How much should a cow weighing 800 lb. be fed each day?

2. One cubic foot of water at the rate prescribed by the Philadelphia council costs .03¢. The average family should use not more than about 2400 cu. ft. per month. What must it pay for water, at the rate prescribed, for one year?

3. The domestic postage rate on letters and sealed matter is 2¢ for each ounce or fraction; what is the postage on a letter weighing $2\frac{1}{2}$ oz.? 3 oz.? 4 oz.? 5 oz.?

4. The domestic postage rate on newspapers or periodicals is 1¢ for each 4 oz. or fraction; what is the postage on a package of newspapers weighing 9 oz.? 5 oz.? 13 oz.? 15 oz.?

5. The domestic postage rate on all miscellaneous printed matter, which includes books weighing 8 ounces or less, is 1¢ for each 2 oz. or fraction thereof; what is the postage on a book weighing 6 oz.? 7 oz.? $7\frac{1}{2}$ oz.? 5 oz.?

6. The domestic postage rate on merchandise not exceeding 4 oz. is 1¢ for each ounce or fraction thereof; what is the postage on a package of merchandise weighing 3 oz.? $3\frac{1}{2}$ oz.? 2 oz.? $1\frac{1}{2}$ oz.?

To the Teacher. Pupils should be required to gain full information through practical experience, if possible, concerning the sending of post-office money orders, registered letters, and special delivery letters.

7. Find the cost of sending a registered letter weighing $3\frac{1}{2}$ oz.

8. Find the cost of sending a registered parcel containing merchandise and weighing $3\frac{1}{2}$ oz.

9. What is the postage on a letter weighing 2 oz., if the envelope bears also a special delivery stamp?

10. What should be charged for 2240 lb. of coal at \$ 0.32 per 100 lb.?

11. Anna has \$ 2.75 and Harriet has \$ 0.15 more than 3 times the amount that Anna has. How much money have both?

12. If a dealer gains \$ 5.50 on \$ 75 invested, find, correct to the nearest cent, his gain on each dollar invested.

13. One barrel of good lime is used in laying 1000 bricks; how many barrels must be used in laying 7450 bricks?

14. Find the cost of 275 rails at \$ 14 per 100.

15. Two bushels of hair are used for each coat in a hundred square yards of plastering. How many bushels of hair are needed to give 5760 sq. yd. of surface two coats of plaster?

16. Of what denomination is the product when tenths are multiplied by tenths? hundredths by hundredths? tenths by hundredths? thousandths by hundredths?

17. Of what denomination is the quotient when tenths are divided by tenths? hundredths by tenths? thousandths by hundredths?

18. Water weighs 62.5 lb. per cubic foot, and lead is 11.35 times as heavy as water. Find, correct to the nearest tenth of a pound, the weight of a cubic foot of lead.

19. A man that earns $2.50 a day earns how much during the working days of October, if the first day of October is Tuesday?

20. If a man is paid $2.75 for working a 10-hour day, what should he be paid for working during a certain day from 7 A.M. to noon, and from 2 P.M. to 4 P.M.?

21. There are 1728 cubic inches in a cubic foot and 231 cubic inches in a gallon. Estimate, correct to the nearest hundredth, the number of gallons in a cubic foot.

22. A cubic foot of water weighs 62.5 lb.; milk is 1.03 times as heavy as water. What is the weight of 10 cu. ft. of milk?

23. At $24.50 per 1000 feet, what is the cost of 1275 feet of boards?

24. Estimating that a cubic foot of water equals 7.48 gallons, and that milk is 1.03 times as heavy as water, find, correct to the nearest hundredth of a pound, the weight of a gallon of milk, water weighing 62.5 lb. per cubic foot.

25. Cast iron is 7.2 times as heavy as water; how many cubic feet are there in a ton of cast iron?

26. When 2.5 bushels of seed yield a crop of 40 bushels of grain, what is the yield for each bushel sowed?

27. If a mechanic who receives $3.60 for a 10-hour day is paid $1\frac{1}{2}$ times his regular wages for each hour he works

over 10 hours, how much should be paid him for working during a certain day from 7 A.M. to noon, and from 1 P.M. to 8 P.M.?

28. A gallon contains 231 cubic inches and a bushel 2150.42 cubic inches; find, correct to the nearest hundredth, the number of gallons in a bushel.

29. Simplify: $\frac{.125 \div .25}{.625 \div .375}$.

30. When 9¢ is paid for the loan of $2, the sum paid is what decimal part of the loan?

31. A mechanic is receiving $3 a day (10 hours) and 50¢ an hour for overtime. During a certain week he works each working day from 7 A.M. to noon, from 1 P.M. to 6 P.M., and from 7 P.M. to 10 P.M.; what are his wages for that week?

32. The average age of the 24 girls in a certain school is 13.55 years; what will be the average age, correct to the nearest hundredth, of the girls in this school, if 6 more enter whose ages average 14.62 years?

33. Simplify: $.006 \div .02 \times .02 \div .006 + .002 \times 60 \div .12$.

34. How many pounds of butter fat does a herd of 25 cows produce in a week, if they average 24 lb. of milk daily, and the milk tests .04 butter fat?

35. A dealer bought 10,000 cigars at $4.50 per 100 and retailed them at 3 for 25¢; find his profit.

36. The weight of a common brick is about 4.5 lb. A man who wishes to haul as nearly as possible 2000 lb. should load how many bricks?

37. A dairyman in Wisconsin recently reported an average of 420 pounds of butter fat per year from a herd of 29 cows. The average price of butter fat at the local creamery was $0.32 a pound. What was the average profit made from each cow of this herd? What was the profit made from the whole herd? The cost of keeping this herd was estimated at $50 per cow. What was the average net profit from each cow, and the net profit from the whole herd?

38. Fill in the missing averages:

DAILY AVERAGE OF HOURS OF LABOR OF REGULAR FARM WORKERS, BY MONTHS, FOR ONE YEAR

1910	1.—New England Dairy Farm		2.—Iowa Grain and Stock Farm	
	Week days	Sundays	Week days	Sundays
	Hours	*Hours*	*Hours*	*Hours*
January	11.4	7.2	10.8	5.9
February	11.7	7.3	11.3	4.9
March	11.4	5.1	12.3	4.8
April	12.2	5.3	12.2	4.7
May	11.9	4.4	12.3	3.6
June	11.9	3.7	12.7	3.3
July	11.8	3.6	12.6	3.1
August	11.3	3.3	12.6	3.2
September	11.2	4.0	11.8	2.7
October	11.5	3.5	12.3	2.9
November	10.3	4.9	12.2	3.3
December	9.7	4.3	12.4	8.2
Yearly average				

DENOMINATE AMOUNTS

126. Denominate units are units established by custom or law to measure such quantities as *weight, capacity, length, surface,* etc.

Thus, *a pound, a bushel, a yard, an acre,* etc., are denominate units.

REMARK. For convenience there are usually several units of the same kind of quantity adopted, having fixed relations to one another.

Thus, the *yard* is the standard unit of length. The yard is divided into three equal parts called *feet,* and the foot is divided into twelve equal parts called *inches.*

127. A **simple denominate amount** is any number of like denominate units.

Thus, 3 *bushels,* $5\frac{1}{2}$ *yards,* and $\frac{1}{2}$ *of a gallon* are simple denominate amounts.

REDUCTION

[Review thoroughly tables studied in the lower grades. See page 360.]

Illustrative Examples

1. Reduce .6 rd. to inches.

Explanation:

.6 rd. = .6 × 5.5 yd., or 3.3 yd.
3.3 yd. = 3.3 × 3 ft., or 9.9 ft.
9.9 ft. = 9.9 × 12 in., or 118.8 in.
∴ .6 rd. = 118.8 in.

The answer may be found thus:

$$\begin{array}{r} 5.5 \\ .6 \\ \hline 3.30 \\ 3 \\ \hline 9.9 \\ 12 \\ \hline 118.8 \end{array}$$

Another explanation:

1 rd. $= 5\frac{1}{2}$ yd.
$= 16\frac{1}{2}$ ft.
$= 198$ in.
$\therefore .6$ rd. $= .6 \times 198$ in., or 118.8 in.

The answer may be found thus:

$$\begin{array}{r} 5\frac{1}{2} \\ 3 \\ \hline 16\frac{1}{2} \\ 12 \\ \hline 198 \\ .6 \\ \hline 118.8 \end{array}$$

2. Reduce 495 in. to rods.

Explanation:

495 = the no. of in.
The no. of in. $\div 12$ = the no. of ft.
$\therefore 41\frac{1}{4}$ = the no. of ft.
The no. of ft. $\div 3$ = the no. of yd.
$\therefore 13\frac{3}{4}$ = the no. of yd.
The no. of yd. $\div 5\frac{1}{2}$ = the no. of rd.
$\therefore 2\frac{1}{2}$ = the no. of rd.
$\therefore$ 495 in. $= 2\frac{1}{2}$ rd.

The answer may be found thus:

$12)\underline{495}$, the no. of in.
$3)\underline{41\frac{1}{4}}$, the no. of ft.
$5\frac{1}{2})\underline{13\frac{3}{4}}$, the no. of yd.
$\frac{4}{22})\underline{\frac{4}{55}}$
$2\frac{1}{2}$, the no. of rd.

Another explanation:

1 rd. $= 5\frac{1}{2}$ yd.
$= 16\frac{1}{2}$ ft.
$= 198$ in.
$\therefore 495 \div 198$, or $2\frac{1}{2}$ = the no. of rd.

The answer may be found thus:

$$\begin{array}{r} 5\frac{1}{2} \\ 3 \\ \hline 16\frac{1}{2} \\ 12 \\ \hline 198)495(2\frac{1}{2} \\ 396 \\ \hline 99 \\ \hline 198 \end{array} = \frac{1}{2}$$

Exercise 41

Express:

1. 3 bu. as quarts.
2. $4\frac{1}{2}$ gal. as pints.
3. 3.5 yd. as inches.
4. 2 long tons as pounds.
5. 2.75 rd. as feet.
6. $2\frac{1}{2}$ wk. as hours.

7. 4 da. as minutes.
8. 3.25 sq. yd. as square inches.
9. 1.6 in. as feet.
10. $4\frac{1}{2}$ hhd. as gallons.
11. 3 reams as sheets.
12. 100 qt. as bushels.
13. 75 pt. as gallons.
14. 288 in. as yards.
15. 15,730 sq. yd. as acres.
16. 825 in. as rods.

MISCELLANEOUS PROBLEMS

Exercise 42

1. How many sacks, holding 25 lb. each, can be filled from a ton of sugar?

2. A long ton is how many times a short ton?

3. A ship sank in 30 fathoms of water; how many feet deep is the water?

4. A horse whose height is $15\frac{1}{2}$ hands is how many feet high?

5. If a horse eats 1 pk. of oats (standard weight) at a meal, in how many days, eating 3 meals a day, will he eat a ton of oats?

6. A farmer stepped the length of his field and found it to be 160 paces (3.3 ft.); find the length of the field in rods.

7. Show that a mile equals 1760 yd., or 5280 ft.

8. Show that a bushel equals 32 qt.

9. How many square inches are there in a square yard?

10. A peck of chestnuts was retailed at 5¢ a pint; how much was received for them?

11. A school that opens at 8.30 A.M. and closes at 2 P.M. is in session how many hours, if an intermission of 30 minutes is allowed at noon?

12. At 7¢ an ounce, what should be charged for $\frac{1}{4}$ lb. of cloves?

13. Find a boy's gain on $\frac{1}{4}$ of a gross of boxes of matches which he bought at $1 a gross, if he retailed them at 4 boxes for 5¢.

14. What should be charged for 846 lb. of feed at $22 per ton?

SUGGESTION. 846 lb. = .846 of a thousand pounds,
= .423 of a ton.

15. Find the cost of 1860 lb. of hay at $18 per ton.

16. Find the cost of 2575 lb. of bran at $23 per ton.

17. Find the cost of 2600 lb. of straw at $8 per ton.

18. Find the cost of 2000 lb. of shelled corn at $0.75 per bushel (56 lb.).

19. Find, correct to the nearest hundredth, the number of cubic inches in a dry quart.

20. A dealer wishes to sell apples that cost $2 a bushel, so as to gain $1.20 a bushel; what should he charge for $\frac{1}{2}$ pk.?

21. What should be charged for 16 lemons at 18¢ per dozen?

22. How many pint boxes are required to hold $\frac{1}{2}$ bu. of berries?

23. How many quarts of ice cream must be bought to serve 50 persons at a hotel, if one pint is served to 2 persons?

24. Name instantly the tenth month; the fifth; the eighth; the eleventh; the fourth; the first; the ninth; the third; the sixth; the second; the seventh; the twelfth.

25. Find the cost of 50 eggs at 30¢ a dozen.

26. What should be paid for 50 bu. of oats weighing 28 lb. to the bushel, when the market price is 50¢ per bushel (32 lb.)?

27. What date is 30 days after May 31st?

28. What date is 90 days after May 31st?

29. What date is 2 months after February 28th?

30. What part of a bushel is $2\frac{1}{2}$ pk.?

31. What part of a yard is 9 in.?

32. A man who was born 14 B.C. and died 44 A.D. lived to what age?

SUGGESTION. "14 B.C." means 14 years before Christ, and "44 A.D." means 44 years after Christ.

33. What time elapsed between two events, one of which happened 150 B.C. and the other 200 A.D.?

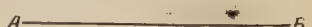
34. What date is 3 months after September 30th?

PRACTICAL MEASUREMENTS

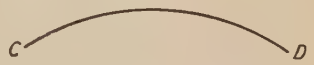
Lines, Angles, and Surfaces

[For a fuller treatment of Surfaces, see page 279.]

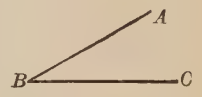
128. A **straight line** is a line that has the same direction throughout; as, AB.



129. A **curved line** is a line no part of which is straight; as, CD.

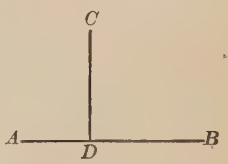


130. The opening between two straight lines drawn from the same point is called an **angle** ($\angle$).



Thus, the figure ABC is an angle.

131. If one straight line meets another straight line so as to form two equal angles, the angles are called **right angles**.



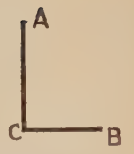
Thus, if the line CD meets the line AB so as to make $\angle ADC = \angle BDC$, each angle is a right angle.

132. An **acute angle** is an angle that is less than a right angle. An **obtuse angle** is an angle that is greater than a right angle.

Acute Angle

Right Angle

Obtuse Angle



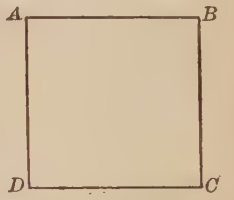
133. When one straight line meets another straight line so as to form with it a right angle, the lines are said to be **perpendicular** to each other.

Thus, if $\angle ACB$ is a right angle, AC and BC are perpendicular to each other.

134. Anything that has length and breadth without thickness is called **surface.**

135. A surface on which any number of straight lines may be drawn in different directions is a **plane surface.**

136. The *number* of square units in a surface is its **area.**



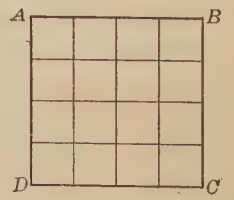
137. A **square** is a portion of a plane bounded by four equal straight lines and having four right angles; as, $ABCD$.

138. A **square unit** is a square a unit long; that is, a square of any desired length.

Thus, a *square inch* is a square an inch long.

NOTE. The result obtained by using a number twice as a factor is called the square of the number.

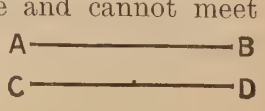
Thus, 16 is the square of 4. The square of 4 may be written 4^2, read *4 squared.*



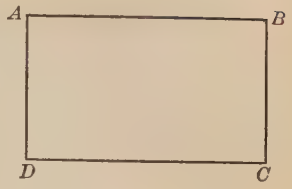
139. The area of a square is the square of the number of units in its length.

Thus, 4×4, or 16, is the number of square units in $ABCD$.

140. Lines that lie in the same plane and cannot meet however far produced are called **parallel lines**; as, AB and CD.



141. A **rectangle** is a portion of a plane bounded by four straight lines and having four right angles; as, *ABCD*.



142. *The opposite sides of a rectangle are equal.*

143. The area of a rectangle is the number of units in its length multiplied by the number of units in its width.

Thus, 3×5, or 15, is the number of square units in *ABCD*.

Exercise 43

1. Show why a square foot contains 144 sq. in.

2. Show why a square yard contains 9 sq. ft.

3. How many square inches are there in a 6-in. square?

4. What part of a square rod does a square contain that is $\frac{1}{2}$ rd. long?

5. Show why $30\frac{1}{4}$ sq. yd. equal a square rod.

6. How many square feet of canvas will cover a cubical box 4 ft. long?

7. How many square feet are there in the walls and ceiling of a room 12 ft. long, 12 ft. wide, and 12 ft. high?

8. A piece of land in the form of a square is $2\frac{1}{2}$ rd. long; how many square rods does it contain? How many feet of fence will inclose it?

9. Find the area of a rectangle 6 ft. long and 2 ft. wide.

10. How many square feet are there in a rectangle 8 ft. long and 6 ft. wide?

11. How many square feet are there in a rectangle 3 yd. long and 2 yd. wide?

12. How many square rods are there in a rectangle 40 rd. long and 4 rd. wide?

Land Measure

Surveyors' Linear Measure

144. For measuring distances, surveyors generally use a **chain** (4 rd.) made of 100 links, or a **tape measure**, generally made of steel or cloth, and usually 50 ft. or 100 ft. long, graduated to feet, tenths, and sometimes hundredths, of a foot.

145. Table

7.92 inches	= 1	link (li.)
100 links	= 1	chain (ch.)
80 chains	= 1	mile (mi.)

146. The unit of land measure is the **acre**, which may be of any shape; it is equivalent to a rectangular tract 20 rd. long and 8 rd. wide. In cities land is sometimes sold by the square foot.

147. Building lots are usually sold by the **front foot** (per foot); thus, if a lot extends 60 ft. along a certain street, it is said to have a frontage of 60 ft. on that street, and if it is sold at $10 per front foot (per foot), the selling price is 60 × $10, or $600.

NOTE. By the depth of a lot is meant the distance it extends back from the street along which its frontage is measured.

148. Table of Land Measure

10,000 square links (sq. li.) 16 square rods (sq. rd.)	= 1 square chain (sq. ch.)
10 square chains 160 square rods	= 1 acre (A.)
640 acres	= 1 square mile (sq. mi.)

NOTE. A square rod is sometimes called a **perch.**

Exercise 44

1. How many acres are there in a lot 40 rd. long and 30 rd. wide?

2. How many acres are there in a lot 12 ch. long and 5 ch. wide?

3. At \$ 0.90 per rod, find the cost of inclosing a field with a wire fence, if the sides of the field measure 6 ch., 8 ch. 50 li., 7 ch. 25 li., 10 ch. 40 li., and 7 ch. 20 li.

4. How wide must a strip of land be that is 121 yd. long, so that it may contain 2 A.?

5. How many feet frontage on Walnut Street has the shaded lot? How many feet deep is this lot? What is it worth at \$ 10 per front foot?

6. How many rods of fencing are necessary to inclose a tract of land 1 mile square, and divide it into 4 equal squares?

7. A lot with a frontage of 60 ft. and a depth of 121 ft. sold at $20 per front foot. How much was that an acre?

Carpeting

149. Carpets are usually 1 yard or $\frac{3}{4}$ of a yard wide. The cheaper carpets are 1 yard wide, and the higher grades, such as velvet, Wilton, and Axminster, are $\frac{3}{4}$ of a yard wide. Linoleum and oilcloth are usually sold by the square yard.

150. In determining the number of yards of carpet required for a floor, the number of strips needed should be found; a fractional part of a strip should be regarded as a full strip.

REMARK. In practice it is seldom possible, because of the great variety of patterns with which carpets are woven, to estimate the exact amount necessary for a room.

151. Sometimes the carpet of a room is surrounded by a border; the length of the border required is then the entire distance around the room. The standard widths of borders are $\frac{3}{8}$ yd., $\frac{5}{8}$ yd., and $\frac{3}{4}$ yd.

Illustrative Example

Find the amount of carpet and of border that must be bought to carpet a room 12 ft. long and 10 ft. 6 in. wide with carpet $\frac{3}{4}$ of a yard wide, the strips running lengthwise and the carpet surrounded by a border $\frac{5}{8}$ of a yard wide.

Explanation:

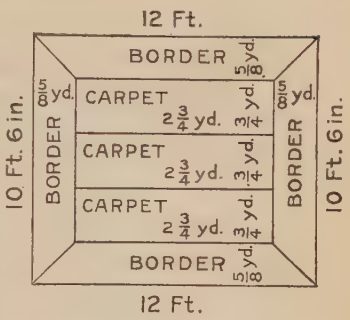
12 ft. = 4 yd.; $10\frac{1}{2}$ ft. = $3\frac{1}{2}$ yd.

4 yd. − ($2 \times \frac{5}{8}$ yd.), or $2\frac{3}{4}$ yd. = the length of each strip.

$3\frac{1}{2}$ yd. − ($2 \times \frac{5}{8}$ yd.), or $2\frac{1}{4}$ yd. = the total width of carpet.

$2\frac{1}{4} \div \frac{3}{4}$, or 3 = the number of strips required.

$3 \times 2\frac{3}{4}$ yd., or $8\frac{1}{4}$ yd. = the amount of carpet required.



$2 \times (4 \text{ yd.} + 3\frac{1}{2} \text{ yd.})$, or 15 yd. = the amount of border required.

Exercise 45

1. How many strips of carpet $\frac{3}{4}$ yd. wide must be bought to carpet a room 13 ft. 6 in. wide, the strips to be laid lengthwise?

2. How many strips of carpet $\frac{3}{4}$ yd. wide must be bought to carpet a room 12 ft. 9 in. wide, the strips to be laid lengthwise?

3. How many yards of carpet $\frac{3}{4}$ yd. wide must be bought to carpet a room 24 ft. long and 18 ft. wide, the strips to be laid lengthwise, and $4\frac{1}{2}$ yd. to be allowed for waste in matching?

4. How many yards of carpet $\frac{3}{4}$ yd. wide must be bought for a floor 18 ft. long and 16 ft. wide, if the strips are laid lengthwise, 3 yards being allowed for waste in matching?

5. Find the cost of carpeting a room 16 ft. long, 14 ft. wide, with carpet $\frac{3}{4}$ yd. wide at $1.60 a yard, if the strips run lengthwise, $\frac{1}{2}$ yd. being allowed on each strip for matching the pattern.

6. How many square yards are there in a piece of linoleum 2 yd. long, if its width is $2\frac{1}{2}$ yd.?

7. Find the total weight of a piece of linoleum $3\frac{1}{2}$ yd. long and $2\frac{1}{2}$ yd. wide, which weighs $3\frac{1}{2}$ lb. to the square yard.

8. What is linoleum selling for per square yard, if it is 2 yd. wide and quoted at \$ 2 per running yard?

9. How many square yards of linoleum are required for a bathroom 12 ft. long and 9 ft. wide?

10. At \$ 0.85 per square yard, find the cost of the linoleum that must be bought to cover a kitchen floor 12 ft. long and $10\frac{1}{2}$ ft. wide.

11. Make a drawing on the blackboard (scale: 2 in. = 1 ft.) showing 4 strips of carpet $\frac{3}{4}$ yd. wide and 15 ft. long surrounded by a border $\frac{5}{8}$ yd. wide. If this carpet and border exactly cover a floor, how long is the floor? how wide?

12. Make a drawing on paper of a floor $16\frac{1}{2}$ ft. long and $13\frac{1}{2}$ ft. wide (scale: 1 in. = 2 ft.) showing strips of carpet $\frac{3}{4}$ yd. wide running lengthwise, and surrounded by a border $\frac{3}{4}$ yd. wide. How many yards of carpet and how many yards of border must be bought to carpet this floor?

Lumber Measure

152. Lumber is usually bought and sold by **board measure**. The unit of board measure is the **board foot**, which is equivalent to the contents of a board a foot square and an inch thick.

153. The number of board feet in a board 16 ft. long, 9 in. wide, and 1 in. thick ($16' \times 9'' \times 1''$) may be found thus:

16 = the number of feet in the length.

$\frac{9}{12}$ = the number of feet in the width.

$\frac{9 \times 16}{12}$ = the number of board feet in the board.

From the above explanation we have the following:

154. The number of board feet in a board one inch thick equals the number of inches in the width times the number of feet in the length, divided by 12.

REMARK. All boards less than one inch thick are bought and sold by surface measure, the thickness not being considered. The number of board feet in a board one inch or less in thickness is therefore the number of square feet of surface in one side of the board.

155. A piece of timber 14 ft. long, 10 in. wide, and 3 in. thick may be regarded as the equivalent of three boards each 14 ft. long, 10 in. wide, and 1 in. thick.

$\therefore \frac{3 \times 10 \times 14}{12}$ = the number of board feet in the piece of timber.

From the above statement we have the following:

156. The number of board feet in a piece of timber more than an inch thick equals the number of inches in the thickness, times the number of inches in the width, times the number of feet in the length, divided by 12.

157. Lumber is usually quoted by the 1000 feet, meaning board feet.

For example, 200 feet of lumber means 200 board feet.

Roofing laths are usually sold by the hundred linear (lin.) feet.

Exercise 46

Find the number of feet of inch boards in each of the following:

	Number of Boards	Size	Number of Feet
1.	10	$16' \times 9''$	
2.	20	$12' \times 12''$	
3.	28	$12' \times 9''$	

Find the number of feet in each of the following:

	Number of Pieces	Size	Number of Feet
4.	24	$3'' \times 5'' \times 16'$	
5.	2	$2'' \times 6'' \times 18'$	
6.	2	$2'' \times 6'' \times 14'$	
7.	2	$2'' \times 4'' \times 8'$	
8.	12	$4'' \times 6'' \times 12'$	
9.	12	$\frac{7}{8}'' \times 3'' \times 18'$	
10.	12	$\frac{7}{8}'' \times 9'' \times 3'\ 6''$	
11.	13	$\frac{7}{8}'' \times 8'' \times 10'\ 6''$	

12. Find the cost of 112 roofing laths averaging 12 ft. long at \$ 1.15 per 100 linear feet.

13. Find the cost of the following bill of lumber:

275 lin. ft. of roofing laths at \$ 1.15 per 100 lin. ft.
300 ft. of white pine siding at \$ 50 per M.
12 boards $1'' \times 12'' \times 16'$ at \$ 50 per M.
2200 plastering laths at \$ 0.60 per C.
24 pieces $3'' \times 5'' \times 12'$ at \$ 30 per M.

Papering

158. Wall paper is generally 18 inches wide and is sold by the roll. A single roll is 8 yards long, and a double roll is 16 yards long. **Border paper** is sold by the linear yard and is of various widths. **Molding** is sold by the foot.

NOTE. Imported papers vary in width and in length of roll.

Wall paper is generally put up in double rolls, but prices quoted are usually for the single roll.

159. No definite rule can be given for determining the exact amount of paper required for a room, because of the allowance that must be made for windows, doors, matching of patterns, and width of border. Usually, however, one of three methods is followed by paper hangers in determining the amount.

First Method. The areas of the doors and windows are deducted from the area of the entire surface of the side walls above the baseboard, and the number of square feet remaining divided by the number of square feet in a double roll, thus determining the number of double rolls required for the side walls.

Second Method. The width of the doors and windows is deducted from the perimeter of the room, and the number of feet remaining is divided by the number of feet ($1\frac{1}{2}$) in the width of a strip of paper; the result is the number of strips required for the side walls. The number of double rolls required is then ascertained by dividing the number of strips determined by the number of strips of required length that may be cut from a double roll.

NOTE. This rule is sometimes modified by counting two strips for each yard in the perimeter after the width of the doors and windows is deducted.

Third Method. The number of square feet in the side walls is divided by the number of square feet in a double roll, and then half

of a single roll is deducted for each door and window, thus determining the number of double rolls required for the side walls.

NOTE. Sometimes the side wall is measured from the ceiling to the floor instead of from the ceiling to the baseboard. In measuring, follow the custom prevailing in your community.

160. Paper for the ceiling being of a pattern different from that used on the side walls, the quantity required should be calculated separately.

NOTE 1. Usually no deduction is made for border, thereby making sufficient allowance for waste in matching.

NOTE 2. A fractional part of a roll is considered a whole roll.

Illustrative Example

Find by the first method the cost of the paper and the border required for the walls of a room 18 ft. long, 12 ft. wide, and 9 ft. high above the baseboard, if the room contains two windows 6 ft. by 3 ft. and a door $7\frac{1}{2}$ ft. by 4 ft., the paper to cost \$0.25 per roll and the border \$0.10 per yard.

Explanation:

$2 \times (18 \text{ ft.} + 12 \text{ ft.})$, or 60 ft. = the perimeter of the room.

9 ft. = the estimated height of the room.

9×60, or 540 = the number of square feet in the surface of the walls.

$2 \times (3 \times 6) + (4 \times 7\frac{1}{2})$, or 66 = the number of square feet to be deducted.

$540 - 66$, or 474 = the number of square feet to be papered.

$48 \times 1\frac{1}{2}$, or 72, is the number of square feet in a double roll.

$474 \div 72 = 6\frac{7}{12}$.

7 double rolls, or 14 single rolls, are required.

$14 \times \$0.25$, or \$3.50, is the cost of the paper.

60 ft., or 20 yd. = the length of the border required.

$20 \times \$0.10$, or \$2.00 = the cost of the border.

\$3.50 + \$2, or \$5.50 = the required cost.

Exercise 47

1. How many whole strips of paper can be cut from a double roll, if the strips are 16 ft. long? 12 ft. long? 10 ft. long?

2. How many strips of paper are required to cover a ceiling 24 ft. long and 21 ft. wide, when the strips run lengthwise? How many are required when the strips run crosswise?

3. How many strips of paper are required for a ceiling 20 ft. long and 16 ft. wide, when the strips run lengthwise? How many are required when the strips run crosswise?

Suggestion. Consider a fractional part of a strip a whole strip.

4. How many double rolls of paper must be bought for a ceiling 18 ft. long and 15 ft. wide, if the strips run lengthwise? How much waste will there be in each double roll?

5. How many double rolls of paper must be bought for a ceiling 18 ft. long and 15 ft. wide, if the strips run crosswise? How much waste will there be in each double roll?

6. Estimate the number of double rolls of paper required for a ceiling 20 ft. long and 16 ft. wide, if the strips run crosswise.

7. Estimate the number of double rolls of paper that must be bought for the four walls of a room 24 ft. long and 16 ft. wide, allowing for two doors, each 4 ft. wide, and four windows, each $3\frac{1}{2}$ ft. wide, the walls being 9 ft. high above the baseboard. (Use second method.)

8. Find the cost of the paper required for the walls of a room 18 ft. long, 14 ft. wide, and 9 ft. high above the baseboard, allowing for 1 door $7\frac{1}{2}$ ft. $\times$ 4 ft., and 4 windows, each 6 ft. $\times$ 4 ft., the paper to cost \$ 0.35 per roll and the border \$ 0.20 per yard. (Use first method.)

9. Find the cost of the paper required for the walls and ceiling of a room 20 ft. long, 16 ft. wide, and 9 ft. high above the baseboard, allowing for 2 doors and 6 windows, for each of which $\frac{1}{2}$ of a single roll is deducted; the paper to cost \$ 0.22 per roll and the border \$ 0.15 per yard.

10. Find the cost of the paper required for the walls and ceiling of a room 18 ft. long, 16 ft. wide, and 10 ft. high, allowing for 2 doors, each $7\frac{1}{2}$ ft. $\times$ 4 ft., 4 windows, each 6 ft. $\times$ 4 ft., and a baseboard 9 in. wide; the paper to cost \$ 0.18 per roll and the border \$ 0.10 per yard. (Use first method.)

Plastering, Paving, and Painting

161. In making estimates for the cost of labor, the square yard or the square foot is taken as the unit in plastering, painting, and paving.

162. No definite rule is followed in plastering and painting in making allowances for doors, windows, and other openings. In some localities half the areas of all openings is deducted; in others no deductions are made.

163. The approximate amount of ready-mixed paint that is required for the outside of a building may be found thus:

Measure in feet the height of the building at one corner, and the

perimeter; find their product and divide by 250. The result is the approximate number of gallons required for two coats.

Thus, if a building is 40 ft. long, 35 ft. wide, and 20 ft. high at one corner, the number of square feet estimated is $20 \times (2 \times 40 + 2 \times 35)$, or 3000, and $3000 \div 250$, or 12, is the number of gallons of paint required for two coats.

NOTE. The area of the gables is usually ignored in the calculations. The quantity as found by the above method is sufficient to paint the gables also, unless the building has more than two gables.

Exercise 48

1. About 1300 laths are required for every 100 sq. yd. of plastering; how many laths are required for 250 sq. yd.?

2. 9 lb. of 3-penny nails are required for the laths for 100 sq. yd. of plastering. At $3\frac{1}{2}¢$ per pound, find the cost of the lath nails that must be bought for 240 sq. yd. of plastering.

3. Estimating plastering laths at \$ 0.60 per hundred, find the cost of the plastering laths that must be bought for 350 sq. yd. of plastering, 1300 plastering laths being necessary for 100 sq. yd. of plastering.

4. 10 bu. of lime and 10 lb. of hair are required for 100 sq. yd. of plastering. Estimating the lime at \$ 0.30 a bushel and the hair at $5\frac{1}{2}¢$ a pound, find the cost of lime and hair necessary for 180 sq. yd. of plastering.

5. Estimating the cost of labor at \$12 for each 100 sq. yd. of plastering (two coats), find the cost of the labor for plastering the walls and ceiling of a room 20 ft. long, 16 ft. wide, and 10 ft. high.

6. Some contractors estimate the cost of labor and material for each square yard of plastering at $0.35. At this figure, find the cost of plastering the walls and the ceiling of a room 20 ft. long, 18 ft. wide, and 9 ft. high.

7. A plasterer made the following estimate for 100 sq. yd. of plastering (two coats):

Material	Quantity			Cost
Laths	1300	@	$0.60 per C	?
Nails	9 lb.	@	0.03	?
Hair	10 lb.	@	$0.05\frac{1}{2}$	?
Lime	10 bu.	@	0.30	?
Sand	2 T.	@	1.00	?
	Total cost of material			?
	Labor			$18.00
	Total			?

8. How many gallons of ready-mixed paint are required to give two coats to a building 50 ft. long, 36 ft. wide, and 18 ft. high at one corner? Estimate as in § 163.

9. How many gallons of ready-mixed paint are needed to give two coats of paint to a building 45 ft. long, 32 ft. wide, and 18 ft. high at one corner? Estimate as in § 163.

10. At $0.15 per square foot, find the cost of a cement pavement 60 ft. long and $4\frac{1}{2}$ ft. wide.

Roofing

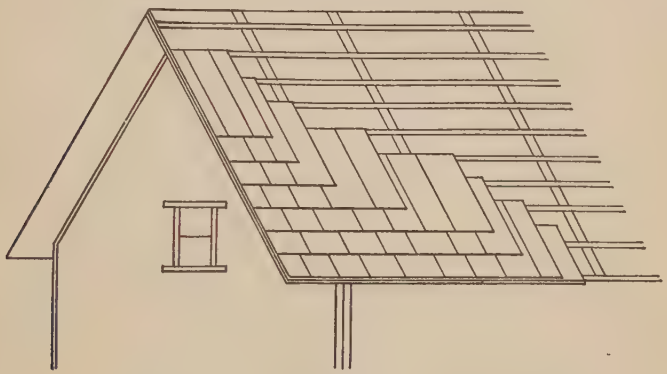
164. Various materials are used for roofing, some of the most common of which are shingles, slate, and tin.

165. Shingles are of various lengths, as 24 inches, 20 inches, 18 inches, and 16 inches; they are usually estimated to be $3\frac{1}{2}$ inches,

4 inches, $4\frac{1}{2}$ inches, 5 inches, or 6 inches wide, and are put up in bundles of 25 and 50 of uniform widths; also in bales of 200 and 167 of varying widths, 6 bundles of the last size being estimated to contain a thousand.

166. Shingles, whether in bales or loose, are quoted by the thousand.

NOTE. "*A thousand* four-inch" shingles means a quantity of shingles equivalent to 1000 uniformly 4 inches wide.



Shingles are laid in courses as shown in the accompanying figure, the first course usually being double. If shingles average 4 inches wide and are *exposed 6 inches to the weather*, then each shingle covers 24 square inches of roof surface.

167. Slate is usually sold by the **square**, which is a quantity sufficient to roof 100 square feet of surface. Slates usually range in size from 6 inches by 12 inches to 14 inches by 24 inches, and are roofed so that they are exposed to the weather $1\frac{1}{2}$ inches less than half of their length.

Thus, if a building is roofed with slate $9'' \times 18''$ the portion of the slate exposed to the weather is $7\frac{1}{2}''$ long; that is, $67\frac{1}{2}$ sq. in. of each slate is exposed.

168. Roofing tin is usually sold in boxes of 112 sheets, 20 inches by 28 inches. Making proper allowance for waste and seams, a box of tin is estimated to cover 360 square feet of roof.

Exercise 49

1. How many shingles averaging 4 in. wide are required for a course 40 ft. long?

2. Roof shingles should not exceed in exposure to the weather the length of the shingle less $1\frac{1}{2}$ in. divided by 3. At this estimate, how much of the length of a 24-in. shingle should be exposed to the weather?

3. Estimating shingles 6 in. wide and laid $7\frac{1}{2}$ in. to the weather, how many shingles are required for 100 sq. ft. of roof?

4. Estimating shingles 5 in. wide and laid 6 in. to the weather, how many shingles are required for 100 sq. ft. of roof?

5. Estimating shingles 4 in. wide and laid 6 in. to the weather, how many square feet of roof will a thousand shingles cover?

6. Find the number of shingles, averaging 4 in. wide and laid 6 in. to the weather, required for a roof 30 ft. long and extending 21 ft. from ridge to eaves, the first course on each side of the roof being double, and an allowance of 100 shingles being made for waste.

7. An architect made the following estimate of the cost of labor and the quantity and cost of the material required for a roof 40 ft. long and extending 24 ft. from ridge to

eaves, the first course of shingles on each side being double, and an ample number being allowed for waste; find the cost of each item and the total cost.

Item	Quantity	Kind	Price	Cost
Roofing Laths	3200 lin. ft.	$1'' \times 2\frac{1}{2}''$	\$1.15 per 100 lin. ft.	?
Shingles	9700	$4'' \times 24''$	\$12.00 per M.	?
Nails	64 lb.	10 d.	\$2.55 per cwt.	?
Nails	68 lb.	4 d.	\$2.85 per cwt.	?
Labor	72 hr.	Carpentry	\$0.47½ per hour	?

8. Estimate the number of slates (correct to the nearest unit) required for 100 sq. ft. of roofing, if the slates are 24 in. long and 14 in. wide, and $1\frac{1}{2}$ in. less than one half the length of each slate is exposed to the weather.

9. Find the cost, at \$ 4 per square, of the slate necessary for $3\frac{1}{2}$ squares of roofing; if the slates are 18 in. × 9 in., and $1\frac{1}{2}$ in. less than half the length of each slate is exposed to the weather, estimate the number of slates required, to the nearest slate.

10. Find the cost of the slate necessary for a roof 36 ft. long and 24 ft. from ridge to eaves, estimating that each slate will roof $67\frac{1}{2}$ sq. in., and that 154 slates are required for the double courses and breakage, the slate costing \$ 5.50 per square.

11. Estimating that a box of tin will cover 360 sq. ft. of roof and that tin is selling for \$ 20 per box, find the cost of tin necessary for 100 sq. ft. of roofing.

12. With tin costing \$16 a box, a plumber estimates the cost of the items necessary for 100 sq. ft. of tin roofing as follows; find the total cost:

Tin	\$4.50
Solder	0.80
Nails and cleats	0.18
Painting tin one coat on each side	1.60
Labor putting tin together	0.50
Labor putting tin on roof	1.25

13. Estimating the cost of 100 sq. ft. of tin roofing at \$8.90, find the cost of covering with tin a roof 36 ft. long and 24 ft. from ridge to eaves.

Solids

(For a fuller treatment of Solids, see p. 296.)

169. Anything that has length, breadth, and thickness is called a **solid.**

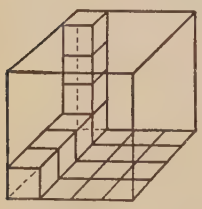
170. A **cube** is a solid bounded by six equal squares.

171. A **cubic unit** is a cube a unit long; that is, a cube of any desired length.

Thus, a *cubic inch* is a cube an inch long.

NOTE. The result obtained by using a number three times as a factor is called the **cube** of the number.

Thus, 8, which equals $2 \times 2 \times 2$, is the cube of 2. The cube of 2 may be written 2^3, read *2 cubed.*

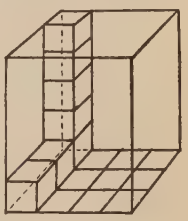


172. The number of cubic units in a cube equals the cube of the number of units in its length.

Thus, 4^3 is the number of cubic units in a cube 4 units long.

173. A **rectangular solid** is a solid bounded by six rectangles called **faces**.

174. The number of cubic units in a rectangular solid equals the number of units in its height times the number of units in its width times the number of units in its length.



Thus, $5 \times 3 \times 4$ is the number of cubic units in a rectangular solid 5 units high, 3 units wide, and 4 units long.

To the Teacher. The treatment of surfaces, pages 99–101, and of solids, pages 118–120, is simply a review of these subjects as presented in the lower grades. This review is given that the pupil may be better prepared to take up the work in Practical Measurements. In connection with this review, all tables relating to surfaces and solids as set forth on pages 360–363 should be thoroughly mastered.

Exercise 50

1. Find the combined lengths of the edges of a cube 2 ft. long.
2. How many cubic feet are there in a cube 3 ft. long?
3. How many cubic feet are there in a cube $2\frac{1}{2}$ ft. long?
4. Show why 27 cu. ft. equals 1 cu. yd.
5. Show why a cubic foot contains 1728 cu. in.
6. How many cubic yards are there in a cube 6 ft. long?
7. How many cubic inches are there in a cube 2 ft. long?
8. What part of a cubic foot does a cube contain that is $\frac{1}{2}$ ft. long?

9. What part of a cubic yard does a cube contain that is 1 ft. long?

10. A rectangular can 11 in. long, 7 in. wide, and 3 in. high holds 1 gal. How many cubic inches does the gallon contain?

11. A cord of wood is 8 ft. long, 4 ft. wide, and 4 ft. high. How many cubic feet does it contain?

12. Estimating that a load of earth contains 1 cu. yd., how many loads of earth must be removed in digging a cellar for a house 36 ft. long and 24 ft. wide, if the cellar is dug to the depth of 9 ft.?

13. How many cords of wood are there in a pile of wood 80 ft. long, 4 ft. wide, and 4 ft. high?

Masonry, Concrete Work, and Brickwork

175. The unit of measure for ordinary stone walls is the **perch**. It is a wall $16\frac{1}{2}$ feet long, $1\frac{1}{2}$ feet wide, and 1 foot high, and therefore contains $24\frac{3}{4}$ cubic feet.

176. When stone is laid in mortar, 22 cubic feet, or $\frac{8}{9}$ of a perch, of stone are required for a perch of wall, the remainder, $2\frac{3}{4}$ cubic feet, being allowed for mortar.

177. In making estimates for the cost of labor for rough stone walls, "mason's measure" is generally used; that is, the distance around the wall on the outside is measured, corners being thus counted twice, and no allowance is made for openings, such as doors and windows, unless they are unusually large. In making estimates for material, "quarry measurement" is generally used; that is, deductions are made for corners and all openings.

178. Concrete work is estimated by the cubic yard, except such as is less than 6 inches in thickness, which is usually estimated by the square foot.

179. In making estimates for the cost of labor and material for walls built of concrete, the same rules are followed as those used in making estimates for walls built of rough stone. This is frequently modified by special contract.

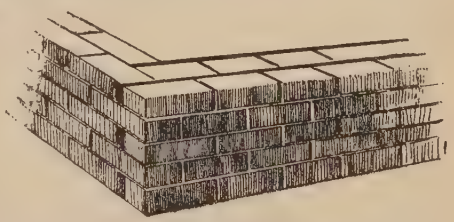
180. The unit of measure for walls built of cut stone is usually the **square foot.**

181. Brickwork is usually estimated by the **thousand bricks.**

182. In making estimates for the cost of labor and material for walls built of brick, allowance is made for corners and all openings, if no special contract is made to the contrary.

The dimensions of a common brick are 8 in. $\times$ 4 in. $\times$ 2 in.

Bricks are usually laid in walls as indicated in the diagram, the courses of mortar being $\frac{1}{4}$ in. thick.



It requires for each square foot in the estimated outside surface of a wall:

7 bricks, if the wall is 1 brick, or 4 in., thick.
14 bricks, if the wall is 2 bricks, or 9 in., thick.
21 bricks, if the wall is 3 bricks, or 13 in., thick.
28 bricks, if the wall is 4 bricks, or 18 in., thick.

From the above we have the following:

To find the number of bricks required for a wall, multiply the number of square feet by 7, 14, 21, or 28, according as the wall is 1, 2, 3, or 4 bricks thick.

Illustrative Examples

1. Estimate for cost of labor the masonry in the walls of the cellar of a house $42\frac{1}{2}$ ft. long, 40 ft. wide, the walls being $1\frac{1}{2}$ ft. thick and 9 ft. high.

$2 \times (42\frac{1}{2}$ ft. $+ 40$ ft.), or 165 ft. = the estimated perimeter of the walls.

9 ft. = the height of the walls.

$1\frac{1}{2}$ ft. = the thickness of the walls.

$\therefore 1\frac{1}{2} \times 9 \times 165$, or $2227\frac{1}{2}$ = the estimated number of cubic feet in the walls.

$2227\frac{1}{2} \div 24\frac{3}{4}$, or 90 = the number of perches required.

2. Find the number of bricks required to build to the height of 18 ft. the four walls of a house 40 ft. long, 36 ft. wide, outside measurement, the walls being two bricks, or 9 in., thick, and an allowance of 600 sq. ft. being made for doors and windows.

$2 \times (40 + 36) \times 18$, or 2736 = the estimated number of square feet of exposed surface in the walls.

$4 \times \frac{3}{4} \times 18$, or 54 = the number of square feet to be deducted for corners.

600 = the number of square feet to be deducted for openings.

$\therefore 2736 - (54 + 600)$, or 2082 = the estimated number of square feet to be built.

14 = the number of bricks required for each square foot.

$\therefore 2082 \times 14$, or 29,148 = the number of bricks required.

Exercise 51

1. How many perches of stone are there in a pile 33 ft. long, 4 ft. wide, and 6 ft. high?

2. At \$1.20 a perch, find the cost of building the walls of a cellar 40 ft. long and 36 ft. wide, if the walls are 18 in. thick and are built to a height of 10 ft.

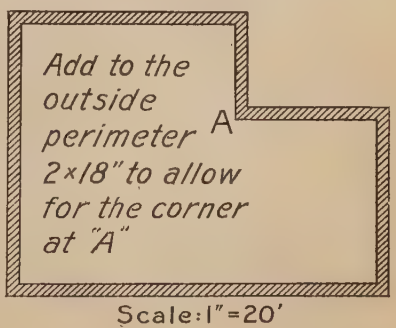
3. A factory 80 ft. long and 60 ft. wide is built on a concrete foundation 8 ft. deep and 3 ft. thick; find the cost of the concrete work at $6 per cubic yard.

4. Concrete for walls and pavements may be made by mixing 1 part of cement with 2 parts of sand with 4 parts of stone. If one barrel of cement contains 4 cu. ft., how many cubic feet of stone and how many of sand must be used with one barrel of cement?

5. Find the number of bricks required to build to the height of 16 ft. the four walls of a house 36 ft. long and 32 ft. wide, outside measurement, the walls being 2 bricks, or 9 in., thick, making allowance for 3 doors $7\frac{1}{2}$ ft. by 4 ft., and 16 windows 6 ft. by 4 ft.

6. It is estimated that the cost of the material and labor for each 1000 bricks in a wall is about $16. Find the cost, at this rate, of building to the height of 24 ft. the four walls of a house 36 ft. long, 30 ft. wide, outside measurement, the walls being 2 bricks, or 9 in., thick, and there being 4 doors $7\frac{1}{2}$ ft. by 4 ft., and 24 windows 6 ft. by 4 ft.

7. At $1.20 a perch, find the cost of building on the foundation here shown a wall 18 in. thick, to the height of 9 ft. Find the cost of excavating the cellar for this building to the depth of $7\frac{1}{2}$ ft., at 35¢ a cu. yd.



8. At $7.50 a cubic yard, find the cost of a concrete floor 50 ft. long, 30 ft. wide, and 9 in. thick.

9. Find the cost of building with concrete the walls for the cellar of a house 36 ft. long and 32 ft. wide, the walls being 9 in. thick and 9 ft. high, if the cost is $7.50 per cubic yard and if no deductions are made for corners or openings.

The following table gives the quantity of cement, sand, and crushed stone or gravel that may be used to make a certain quantity of a standard concrete. The table may be used to advantage in making small batches.

Portland Cement	Sand	Crushed Stone or Gravel	Concrete Obtained
2 bags	3 bu. (stricken)	6 bu. (stricken)	8½ cu. ft.

10. Find the quantity of each of the materials given in the above table required to make 85 cu. ft. of concrete.

Remark. The mixture given above is a 1:2:4 mixture; that is, a mixture of 1 part by measure of cement with 2 parts of sand and 4 parts of stone or gravel. A bag of cement is estimated to be 1 cu. ft.

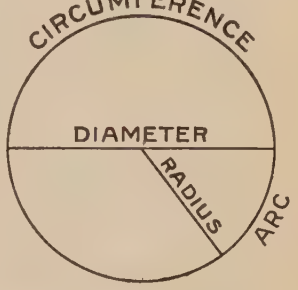
The Circle

183. A **circle** is a plane figure bounded by a curved line all points of which are equally distant from a point within called the **center.**

184. The boundary of a circle is called the **circumference.**

185. Any portion of the circumference is called an **arc**.

186. A straight line drawn from the center of a circle to any point in the circumference is called a **radius**.



187. A straight line drawn through the center of a circle and terminating in the circumference is called a **diameter**.

188. Two perpendicular diameters divide the circumference of a circle into four equal parts called **quadrants**.

$\frac{1}{90}$ of a quadrant = an arc of 1 degree (°)
$\frac{1}{60}$ of an arc of 1° = an arc of 1 minute (′)
$\frac{1}{60}$ of an arc of 1′ = an arc of 1 second (″)

189. **Table**

60″ = 1′	90° = 1 quadrant
60′ = 1°	360° = 1 circumference

Important Facts

190. 1. *The diameter equals two times the radius.*

2. *The circumference equals about 3.1416 times the diameter.*

3. *The area of a circle equals one half the number of units in the radius times the number of units in the circumference.*

4. *The area of a circle equals 3.1416 times the square of the number of units in the radius.*

NOTE 1. Results obtained by using 3.1416 are more nearly accurate than those obtained by using $\frac{22}{7}$, which is often used.

NOTE 2. For fuller treatment of the circle see page 293.

Exercise 52

Express as minutes:

1. 25° 2. 32° 3. 45° 4. 90° 5. 120°

Express as seconds:

6. 12′ 7. 24′ 8. 30′ 9. 50′ 10. 58′

Express as minutes:

11. 300″ 12. 420″ 13. 540″ 14. 720″

Express as degrees:

15. 240′ 16. 360′ 17. 840′ 18. 1170′

19. What part of a circumference is an arc of 90°?

20. What part of a circumference is an arc of 6°?

21. What part of a circumference is an arc of 45°?

22. If an arc of 30° is 6 in. long, what is the circumference of the circle?

23. How many minutes are there in $2\frac{1}{2}$°?

24. How many minutes are there in a circumference?

25. What part of a circumference does the point of the minute hand of a watch pass over in 25 minutes?

Using 3.1416, find the circumference of a circle whose diameter is:

26. 15 in. 27. 40 ft. 28. 65 ft. 29. 7.2 ft.

30. 1.75 rd. 31. 1.25 yd. 32. 4.6 ch. 33. 7.46 ch.

Using 3.1416, find the circumference of a circle whose radius is:

34. 9 in. 35. 24 ft. 36. 32.5 ft. 37. 16.25 yd.

38. 12 rd. 39. 16 ch. 40. 12.6 rd. 41. 14.25 ch.

Using 3.1416, find the diameter of a circle whose circumference is:

42. 15.7080 in. 43. 21.9912 ft. 44. 31.416 yd.

Using 3.1416, find the radius of a circle whose circumference is:

45. 12.5664 in. 46. 25.1328 ft. 47. 62.8320 yd.

48. Find the area of a circle whose radius is 2 in.

49. Find the area of a circle whose radius is 6 ft.

50. Find the area of a circle whose radius is 8 yd.

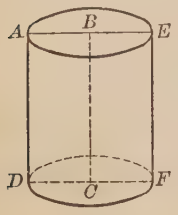
51. At $0.50 per square foot, find the cost of cementing the bottom of a cistern, the bottom being a circle whose radius is 4 ft.

52. Find, correct to the nearest hundredth, the number of acres in a piece of land in the form of a circle whose radius is 30 rd.

53. Find the area of a circle whose radius is 5 ch.

The Cylinder

191. The solid formed by revolving a rectangle about one of its sides as an axis is called a **cylinder of revolution, or a right circular cylinder.**



Thus, the solid formed by revolving the rectangle $ABCD$ about the side BC as an axis is a cylinder of revolution. The surface formed by the side AD is the **lateral surface** and its area is the **lateral area.** The circles formed by the sides AB and DC are the **bases.** The axis BC is the **altitude.**

Important Facts

192. 1. **The lateral area of a cylinder of revolution equals the number of units in the altitude times the number of units in the circumference of the base.**

2. **The number of cubic units in a cylinder of revolution equals the number of units in the altitude times the number of square units in the base.**

NOTE 1. In this book only cylinders of revolution are treated. They will be called merely cylinders.

NOTE 2. For fuller treatment of cylinders, see p. 296.

Exercise 53

1. Find the lateral area of a cylinder the diameter of whose base is 8 in. and whose altitude is 10 in.

SUGGESTION. 3.1416×8 in. = the circumference of the base.
$10 \times 3.1416 \times 8$ = the number of square inches required.

2. Find the area of the entire surface of a cylinder the radius of whose base is 4 ft. and whose altitude is 6 ft.

3. Find the lateral area of a cylinder the circumference of whose base is 12 ft. and whose altitude is 6 ft.

4. Find the lateral area of a cylinder whose diameter is 6 ft. and whose altitude is 10 ft.

5. Find the area of the entire surface of a cylinder the radius of whose base is 6 and whose altitude is 4.

6. Find the lateral area of a cylindrical vessel the diameter of whose base is 6 in. and whose altitude is 12 in.

7. How many square yards of metal sheeting, correct to the nearest hundredth of a square yard, are required for a cylindrical tank, inclusive of ends, if the circumference of the tank is 60 ft. and its altitude 12 ft.?

8. Find the number of cubic feet in a cylinder the radius of whose base is 6 ft. and whose altitude is 5 ft.

SUGGESTION. $3.1416 \times 6^2 =$ the number of square feet in the base.
$5 \times 3.1416 \times 6^2 =$ the number of cubic feet required.

9. How many cubic feet will a silo hold that is 10 ft. in diameter and 20 ft. deep, if the silo is in the form of a cylinder?

10. The standard bushel of the United States is in the form of a cylinder $18\frac{1}{2}$ in. in diameter and 8 in. deep; how many cubic inches does it contain?

11. Estimate, correct to the nearest gallon, the capacity of a tank in the form of a cylinder 8 ft. in diameter and 12 ft. high.

12. Estimate, correct to the nearest gallon, the capacity of a cistern 6 ft. in diameter and 20 ft. deep.

COMPOUND DENOMINATE AMOUNTS

193. A **compound denominate amount** is a combination of two or more simple denominate amounts of the same kind of quantity, but of different denominations.

Reduction

Illustrative Examples

1. Reduce 9 rd. 4 yd. 2 ft. to feet.

Explanation:

1 rd. $= 5\frac{1}{2}$ yd.

9 rd. $= 9 \times 5\frac{1}{2}$ yd., or $49\frac{1}{2}$ yd.

9 rd. 4 yd. $= 49\frac{1}{2}$ yd. $+ 4$ yd., or $53\frac{1}{2}$ yd.

1 yd. $= 3$ ft.

$53\frac{1}{2}$ yd. $= 53\frac{1}{2} \times 3$ ft., or $160\frac{1}{2}$ ft.

9 rd. 4 yd. 2 ft. $= 160\frac{1}{2}$ ft. $+ 2$ ft., or $162\frac{1}{2}$ ft.

The answer may be found thus:

$$\begin{array}{r} 9-4-2 \\ \hline 5\frac{1}{2} \\ \hline 49\frac{1}{2} \\ \hline 4 \\ \hline 53\frac{1}{2} \\ \hline 3 \\ \hline 160\frac{1}{2} \\ \hline 2 \\ \hline 162\frac{1}{2} \end{array}$$

2. Reduce 256 feet to a compound amount.

Explanation:

The number of feet $\div 3 =$ the number of yards.

$\therefore$ 256 ft. $= 85$ yd. 1 ft.

The number of yards $\div 5\frac{1}{2} =$ the number of rods.

$\therefore$ 85 yd. 1 ft., or 256 ft. $= 15$ rd. $2\frac{1}{2}$ yd. 1 ft.

$= 15$ rd. 2 yd. 2 ft. 6 in.

The answer may be found thus:

$$\begin{array}{r|l} 3 & 256 \\ 5\frac{1}{2} & 85^{1} \\ 2 & 2 \\ \hline 11 & 170 \\ \hline & 15\frac{2}{11} \end{array}$$

(read *85 and 1 over*)

$15\frac{2}{11}$ rd. $2\frac{1}{2}$ yd. 1 ft. $= 15$ rd. 2 yd. 2 ft. 6 in.

3. Reduce 4 T. 12 cwt. 25 lb. to hundredweight.

Explanation:

1 lb. = .01 cwt.
∴ 25 lb. = .25 cwt.
1 T. = 20 cwt.
∴ 4 T. = 80 cwt.
4 T. 12 cwt. 25 lb. = 80 cwt. + 12 cwt. + .25 cwt.
= 92.25 cwt.

4. Reduce $\frac{39}{64}$ bu. to a compound amount.

Explanation:

1 bu. = 4 pk.
∴ $\frac{39}{64}$ bu. = $\frac{39}{64}$ of 4 pk., or $2\frac{7}{16}$ pk.
1 pk. = 8 qt.
∴ $\frac{7}{16}$ pk. = $\frac{7}{16} \times 8$ qt., or $3\frac{1}{2}$ qt.
1 qt. = 2 pt.
∴ $\frac{1}{2}$ qt. = $\frac{1}{2}$ of 2 pt., or 1 pt.
∴ $\frac{39}{64}$ bu. = 2 pk. 3 qt. 1 pt.

5. Reduce .7 rd. to a compound amount.

Explanation:

1 rd. = 5.5 yd.
∴ .7 rd. = .7 × 5.5 yd., or 3.85 yd.
1 yd. = 3 ft.
∴ .85 yd. = .85 × 3 ft., or 2.55 ft.
1 ft. = 12 in.
.55 ft. = .55 × 12 in. or 6.6 in.
∴ .7 rd. = 3 yd. 2 ft. 6.6 in.

REMARK. Compound Denominate Amounts are rapidly going out of use, except in the measurement of *time, angles,* and *arcs.*

Exercise 54

Express:

1. 3 gal. 2 qt. 1 pt. as pints.
2. 4 T. 8 cwt. 5 lb. as pounds.
3. 4 bu. 3 pk. 2 qt. as quarts.
4. 4° 25′ 12″ as seconds.
5. 2 rd. 5 yd. 2 ft. as feet.
6. 2 sq. rd. 25 sq. yd. 2 sq. ft. as square feet.
7. 81 pt. as gallons.
8. 150 qt. as bushels
9. 4565 lb. as short tons.
10. 265 ft. as rods.
11. 5000″ as degrees.
12. 8740 sq. yd. as acres.
13. 12 hhd. 20 gal. 2 qt. as gallons.
14. 2 rd. 12 ft. 6 in. as feet.
15. 4 bu. 3 pk. 2 qt. as pecks.
16. 2 A. 20 sq. rd. 11 sq. yd. as square rods.
17. 3 T. 4 cwt. 30 lb. as hundredweight.
18. $\frac{3}{4}$ rd. as a compound amount.
19. $\frac{3}{5}$ da. as a compound amount.
20. $\frac{3}{4}$ sq. rd. as a compound amount.
21. $\frac{2}{11}$ mi. as a compound amount.
22. 1.4 A. as a compound amount.
23. .6 sq. rd. as a compound amount.
24. 2.07° as a compound amount.
25. .75 rd. as a compound amount.

Addition and Subtraction

Illustrative Examples

1. Find the sum of 5 rd. 1 yd. 2 ft., 8 rd. 5 yd. 1 ft., and 7 rd. 2 ft.

Explanation:

	rd.	yd.	ft.
	5	1	2
	8	5	1
	7	0	2
Sum	= 21	$1\frac{1}{2}$	2
	= 21	1	$3\frac{1}{2}$
	= 21	2	$0\frac{1}{2}$

The sum of the column of feet is 5 ft., which equals 1 yd. 2 ft. Write 2 under the column of feet and add 1 yd. to the yards. The sum of the column of yards is 7 yd., which equals 1 rd. $1\frac{1}{2}$ yd. Write $1\frac{1}{2}$ under the column of yards and add 1 rd. to the rods. The sum of the column of rods is 21 rd.; write 21 under the column of rods. $1\frac{1}{2}$ yd. 2 ft. = 1 yd. $3\frac{1}{2}$ ft. = 2 yd. $0\frac{1}{2}$ ft.

∴ the sum is 21 rd. 2 yd. $\frac{1}{2}$ ft. = 21 rd. 2 yd. 6 in.

2. From 4 rd. 2 yd. 1 ft. take 1 rd. 4 yd. 2 ft.

Explanation:

	rd.	yd.	ft.
	4	2	1
	1	4	2
Diff.	= 2	$2\frac{1}{2}$	2
	= 2	2	$3\frac{1}{2}$
	= 2	3	$0\frac{1}{2}$

We cannot take 2 ft. from 1 ft., so we take 1 yd. from 2 yd. in the minuend, leaving 1 yd., and add it to the 1 ft., making 1 yd. 1 ft., or 4 ft. 2 ft. from 4 ft. leaves 2 ft. We cannot take 4 yd. from 1 yd., so we take 1 rd. from the 4 rd. in the minuend, leaving 3 rd., and add it to the 1 yd., making 1 rd. 1 yd., or $6\frac{1}{2}$ yd. 4 yd. from $6\frac{1}{2}$ yd. leave $2\frac{1}{2}$ yd. 1 rd. from 3 rd. leaves 2 rd. $2\frac{1}{2}$ yd. 2 ft. = 2 yd. $3\frac{1}{2}$ ft. = 3 yd. $0\frac{1}{2}$ ft.

∴ the difference is 2 rd. 3 yd. $\frac{1}{2}$ ft. = 2 rd. 3 yd. 6 in.

Exercise 55

Add:

1.

yr.	mo.	da.
4	2	10
27	3	12
16	1	15

2.

bu.	pk.	qt.
2	1	4
5	3	2
1	2	7

3.

rd.	yd.	ft.	in.
103	4	1	8
75	3	2	5
43	1	1	3

4.

T.	cwt.	lb.
4	14	63
12	9	21
7	18	42

5.

A.	sq. rd.	sq. yd.
16	128	0
4	10	16
4	8	20

6.

°	′	″
24	15	40
75	35	18
80	18	45

Subtract:

7.

yr.	mo.	da.
1914	6	12
1888	9	27

8.

rd.	yd.	ft.
18	3	1
6	4	2

9.

°	′	″
180	0	0
75	30	45

10.

A.	sq. rd.	sq. yd.
10	40	4
2	80	5

11.

T.	cwt.	lb.
15	6	0
6	17	75

12.

yr.	mo.	da.
1892	3	15
63	7	29

Multiplication and Division

Illustrative Examples

1. Multiply 6 rd. 3 yd. 2 ft. by 5.

Explanation:

rd.	yd.	ft.
6	3	2
		5
33	$1\frac{1}{2}$	1

5×2 ft. $= 10$ ft., or 3 yd. 1 ft.

5×3 yd. $= 15$ yd.

15 yd. $+ 3$ yd. $= 18$ yd., or 3 rd. $1\frac{1}{2}$ yd.

5×6 rd. $= 30$ rd.

30 rd. $+ 3$ rd. $= 33$ rd.

$\therefore$ the product $= 33$ rd. $1\frac{1}{2}$ yd. 1 ft.

$= 33$ rd. 1 yd. $2\frac{1}{2}$ ft.

$= 33$ rd. 1 yd. 2 ft. 6 in.

2. Divide 18 bu. 3 pk. 4 qt. by 4.

	bu.	pk.	qt.
4)	18	3	4
	4	2	7

Explanation:

18 bu. ÷ 4 = 4 bu. and 2 bu. over.
2 bu. 3 pk. = 11 pk.
11 pk. ÷ 4 = 2 pk. and 3 pk. over.
3 pk. 4 qt. = 28 qt.
28 qt. ÷ 4 = 7 qt.
∴ the quotient = 4 bu. 2 pk. 7 qt.

Exercise 56

Multiply:

	bu.	pk.	qt.		yr.	mo.	da.		°	′	″			
1.	12	3	5	2.	27	10	15	3.	135	40	45			
			4				7				3			
	rd.	yd.	ft.		T.	cwt.	lb.		A.	sq. rd.	sq. yd.			
4.	120	5	2	5.	20	15	75	6.	6	30	3			
			9				6				5			

Divide:

	yr.	mo.	da.		bu.	pk.	qt.		°	′	″			
7.	4)123	11	16	8.	3)65	3	1	9.	9)175	32	21			
	T.	cwt.	lb.		A.	sq. rd.	sq. yd.		rd.	yd.	ft.			
10.	7)97	13	87	11.	8)11	143	5	12.	6)21	2	2			

Exercise 57

1. Washington was born Feb. 22, 1732, and died at the age of 67 yr. 9 mo. 22 da. Find the date of his death.

2. How long was it from sunrise to sunset on a day on which the sun rose at 6.02 A.M. and set at 5.36 P.M.?

3. Find the cost of shipping 6 cwt. 25 lb. of merchandise at $0.85 per hundredweight.

4. A wagon loaded with grain weighed 32 cwt. 36 lb. The wagon weighed 12 cwt. 25 lb. Find the weight of the grain.

5. A dealer bought a tract of land containing 12 A. 120 sq. rd. He divided it into building lots each containing 24 sq. rd. How many building lots did the tract make?

6. Find the circumference of a circle, if an arc of 60° measures 3 yd. 2 ft. 1 in.

7. A merchant shipped four invoices of goods. The first weighed 2 T. 17 cwt. 25 lb., the second 1 T. 18 cwt. 15 lb., the third 2 T. 1 cwt. 60 lb., and the fourth 18 cwt. 40 lb. Find the total weight of the invoices.

8. What time elapses from 6.20 P.M. Monday to 5.15 A.M. Thursday?

9. What was the duration of an eclipse that began at 7 hr. 37 min. 12 sec. A.M., and ended at 10 hr. 11 min. 42 sec. A.M.?

10. If an arc of 1° measures 4 yd. 2 ft. 6 in., what is the length of an arc of 8°?

11. If 15 times an arc is 62° 2′ 30″, what is the arc?

12. At 15¢ per hour, what did a laborer earn during the 26 working days of March of a certain year, if he began work at 7.00 A.M. and quit at 6.30 P.M., taking each day 30 min. for lunch?

13. The sides of a certain field measure 12 ch. 20 li., 9 ch. 42 li., 18 ch. 68 li., 10 ch. 56 li., and 14 ch. 9 li. Find the distance around the field.

14. How much larger is an arc of 90° than one of 72° 14′ 40″?

DENOMINATE AMOUNTS OF MINOR IMPORTANCE

Foreign Money

194. The standard units of value in some of the most important European countries and the equivalents in U. S. money are as follows:

Country	Standard	Approximate Equivalent	Exact Equivalent
Great Britain	Pound	$5.00	$4.8665
Germany	Mark	0.25	0.238
France	Franc	0.20	0.193
Russia	Ruble	0.50	0.515
Austria-Hungary	Crown	0.20	0.203
Italy	Lira	0.20	0.193
Spain	Peseta	0.20	0.193

195. **Table of English Money**

4 farthings (far.)	= 1 penny (d.)
12 pence	= 1 shilling (s.)
20 shillings	= 1 pound (£)
21 shillings	= 1 guinea
5 shillings	= 1 crown
2 shillings	= 1 florin

Note. In Canada the units of money are the same as in the United States.

Troy Weight

196. Troy weight is used in weighing gold, silver, platinum, and jewelry, except gems and precious stones. The standard unit is the **Troy pound.**

197. **Table**

24 grains (gr.)	**= 1 pennyweight (pwt.)**
20 pennyweights	**= 1 ounce (oz.)**
12 ounces	**= 1 pound (lb.)**

NOTE. The unit of weight for gems and precious stones is the **carat**, which equals 200 milligrams,* or about 3.08 Troy grains. The term *carat* is also used to denote the purity, or fineness, of gold, 24 carats denoting pure gold.

198. Medicines are sold in prescriptions by apothecaries' weight, which is as follows:

1 pound (℔) = 12 ounces (℥)
= 96 drams (ʒ)
= 288 scruples (℈)
= 5760 grains (gr.)

199. In mixing liquid medicines druggists use **apothecaries' fluid measure**, which is as follows:

60 minims (♏) = 1 fluid dram (*f* ʒ)
8 fluid drams = 1 fluid ounce (*f* ℥)
16 fluid ounces = 1 pint (O)
8 pints = 1 gallon (Cong.)

NOTE 1. A minim is about a drop.

NOTE 2. An ounce bottle holds a fluid ounce, or $\frac{1}{16}$ of a pint; a 2-ounce bottle, $\frac{1}{8}$ of a pint; a 3-ounce bottle, $\frac{3}{16}$ of a pint, and so on.

NOTE 3. 45 drops of water equal about a teaspoonful; 1 teaspoonful equals 1 fluid dram; 1 dessert-spoonful equals 2 teaspoonfuls; and 1 tablespoonful equals 2 dessert-spoonfuls.

* See Supplement, Section 35.

Exercise 58

Express:

1. £3 as pence.

2. £2 4*s.* as pence.

3. 240*s.* as pounds.

4. 2758*d.* as pounds, etc.

5. What is the exact value in United States money of 100 francs?

6. What is the exact value in United States money of 1000 marks?

7. What is the exact value in United States money of 200 pesetas?

8. What is the approximate value in marks of $1? in francs? in crowns? in liras? in pesetas?

9. What is the exact value in United States money of 200 rubles?

10. An English florin has a value nearly equal to what piece of United States money?

11. One crown is worth approximately how much United States money?

12. Express, correct to the nearest hundredth of a pound, the exact value in English money of $1000.

13. Find the exact value in French money, correct to the nearest franc, of $100.

14. Express, correct to the nearest mark, the value in German money of $100.

15. How many grains are there in a pound Troy?

16. The standard gold dollar of the United States weighs 25.8 gr.; .9 of it is pure gold, and the remainder copper. What is the weight of the copper in a $5 gold piece?

17. The standard gold dollar weighs 25.8 gr. and is .9 pure gold. What is the weight of the pure gold in 5 eagles?

18. The United States silver dollar weighs 412.5 gr. and is .9 pure silver. 1 pound Troy of pure silver will furnish silver enough to make how many silver dollars and how many grains over?

19. About what is the weight in Troy grains of a diamond weighing 12 carats?

20. How many 2-grain quinine pills can be made from an ounce of quinine?

21. How many grains of gold are there in a piece of jewelry 18 carats fine and weighing 4 pwt. 16 gr.?

22. A pint of medicine will fill how many 2-ounce bottles?

23. What is the weight in Troy pounds of a thousand gold dollars?

24. 1 pwt. 10 gr. of pure gold will be sufficient for how many grains of jewelry 12 carats fine?

25. A pearl weighing $9\frac{1}{2}$ carats will weigh about how many Troy grains?

REVIEW

Exercise 59

1. The output of hops in Oregon during a recent year was 93,000 bales of 185 lb. each; what was received for them at $12\frac{1}{2}$ ¢ per pound?

2. Slates that are 8 in. by 12 in. will cover about 36 sq. in. of exposed roof surface; how many slates of these dimensions will cover a roof surface 18 ft. long and 12 ft. wide?

3. Find the number of square inches in the entire surface of a common brick.

4. A bill of goods that cost 500 marks cost how much in United States money?

5. How many leap years are in the 20th century?

6. Wheat bran weighs 20 lb. to the bushel; how many bushels of wheat bran are there in a ton of 2000 lb.?

7. A man sold drawing paper at 10 ¢ per dozen sheets and gained $\frac{1}{4}$ ¢ per sheet; what did he pay for the paper per ream?

8. On what date does a debt fall due that is payable 60 days after December 31, 1913?

9. The annual cut of timber in Oregon is estimated at 2,000,000,000 feet, valued at $30,000,000; what is the estimated value per 1000 feet?

10. There are 11,000 sq. ft. in the exposed surface of the walls of a certain building. These walls were built of cut stone at a cost of 50 ¢ per square foot for labor and material; find the entire cost.

11. Find the cost of 68 roofing laths averaging 12 ft. long at 65¢ per 100 linear feet.

12. How many bricks laid on the broadest face are necessary to make a pavement 100 yd. long and 8 ft. wide?

13. A debt that was due two calendar months after Dec. 31, 1913, was due on what date?

14. How many acres are there in a tract of land in the form of a square one side of which measures 80 rd.?

15. How many feet of boards 1 in. thick can be sawed from a log 16 ft. long by 18 in. by 18 in.?

16. Find the weight of a block of Susquehanna granite 8 ft. long, 2 ft. 6 in. wide, and 1 ft. thick, the granite weighing 2.704 times as much as an equal bulk of water.

17. A dealer bought 800 tons of coal at $5 per long ton and retailed it at $6.75 per short ton; what was his profit?

18. A dealer who buys thread at 55¢ per dozen spools and retails it at 5¢ per spool gains how much on a gross of thread?

19. How many acres are there in a tract of land $\frac{1}{2}$ mile square?

20. How many dust cloths 14 in. square can be made from 7 yd. of material 28 in. wide?

21. Find the cost of 2856 lb. of corn at $0.65 per bushel (56 lb.).

22. Allowing 400 cu. ft. to the ton, estimate the weight of a mow of hay 24 ft. long, 16 ft. wide, and 12 ft. deep.

23. How many square inches of pasteboard are necessary to make a cube 6 in. long?

24. It is estimated that 1300 plastering laths are required for 100 sq. yd. of plastering; at this estimate, how many laths are required in plastering a room 24 ft. long, 18 ft. wide, and 10 ft. high?

25. An avoirdupois pound contains 7000 grains; how many grains are there in an avoirdupois ounce?

26. How many feet long is a line that is 8 chains long?

27. The total rainfall during the year in Minnesota is about 4 ft. Estimate, correct to the nearest gallon, the rainfall to each square yard.

28. Find the cost of 25,680 lb. of hay at \$15 per ton.

29. How much does a man gain who buys 5 T. (2240 lb.) of coal at \$6 per ton and retails it at 8¢ per bucket, 20 lb. to the bucket?

30. At \$2.50 per cord, what is the value of a pile of wood 48 ft. long, 6 ft. high, and 4 ft. wide?

31. Estimating a barrel ($31\frac{1}{2}$ gal.) at $4\frac{1}{4}$ cu. ft. is making the estimate too high by how many cubic inches?

32. How many feet of 2-inch planks are required to lay a barn floor 42 ft. long and 16 ft. wide?

33. What is the sum of the angles of a square? of a rectangle?

34. Estimating a bushel of wheat at $\frac{5}{4}$ of a cubic foot, how many bushels of wheat will a bin hold that is 6 ft. × 6 ft. × 4 ft. 6 in.?

35. Over how many minute spaces does the minute hand of a clock pass in a day?

36. How many shingles are required for 240 sq. ft. of roofing, if they are 4 in. wide and laid 6 in. to the weather?

37. A farmer sold 125 bu. of oats which weighed 3750 lb.; the standard weight of a bushel of oats being 32 lb., find the loss in weight per bushel.

38. A rectangular piece of land 120 rd. long requires 320 rd. of fencing to inclose it; how many acres does the tract contain?

39. Find the cost of shipping farm machinery weighing 2648 lb., if the rate charged was 48¢ per 100 lb.

40. When shingles are selling at $12.50 per 1000, what should be paid for 16 bundles of shingles, 200 to the bundle?

41. How many perches of masonry are there in a wall 33 ft. long, 4 ft. high, and 18 in. thick?

42. How many yards of carpet $\frac{3}{4}$ yd. wide must be bought for a floor 24 ft. long and 16 ft. wide, if the strips are to be laid lengthwise and $\frac{1}{2}$ yd. is allowed on each strip for waste in matching?

43. When hay is bought at $16 per ton and retailed at $1 per bale of 100 lb., what is the profit per ton?

44. A schoolroom should have not less than 15 sq. ft. of floor space per pupil; at this estimate, a room 40 ft. long and 33 ft. wide will provide sufficient floor space for how many pupils?

45. The annual yield of butter fat from 550 cows in Minnesota during a recent year was 102,569.50 lb.; what was the average annual yield per cow?

46. What is the value of 2,722,528 lb. of skimmed milk at 15¢ per hundredweight?

47. Allowing 1 T. of clover hay, 10 bu. of corn, and 10 bu. of oats as the ration for a cow for 120 da., find the cost of keeping a herd of 20 cows 6 mo. with hay at $15 per ton, corn $0.60 per bushel, and oats $0.42 per bushel.

48. What is the income for June from a cow that gives an average of 22 lb. of milk daily, if this milk tests .035 butter fat, which is sold at $0.32 per pound?

49. Including land rental, labor, and machinery value consumed, the cost of producing wheat in Minnesota was recently shown to be $7.249 per acre. Estimate a farmer's profit on 100 A. of wheat averaging 20.6 bu. per acre and sold at $0.90 per bushel.

50. It is estimated that 2 bu. of corn in the ear (measuring $\frac{2\,5}{1\,6}$ cu. ft. per bushel) will make 1 bu. of shelled corn. A quantity of corn in the ear which fills a box 10 ft. long, 3 ft. wide, and 25 in. deep will make how many bushels of shelled corn?

51. A plasterer estimates that 5 bu. of lime will be necessary for each coat in 100 sq. yd. of plastering. At this estimate, how many bushels of lime are required to give 2 coats of plaster to a room 24 ft. long, 18 ft. wide, and 9 ft. high?

GENERAL REVIEW

Exercise 60

1. The amount of timber standing in Oregon is estimated at 400,000,000,000 ft., or one sixth of the standing timber of the United States; estimate the amount of standing timber in the United States.

2. A piece of wire $\frac{7}{8}$ in. long was divided into 7 equal parts; how many of these parts together measure $\frac{3}{4}$ of an inch in length?

3. A cubic foot of water weighs $62\frac{1}{2}$ lb. and ice is $\frac{10}{11}$ as heavy as water; find the weight of a block of ice $5\frac{1}{2}$ ft. by 2 ft. by 18 in.

4. What is the value of 102,569.5 lb. of butter fat at 25.92 ¢ per pound?

5. Estimating a cubic foot of water at $62\frac{1}{2}$ lb., how many tons (2000 lb.) of water will a tank hold that is 10 ft. long, 4 ft. wide, and 5 ft. deep?

6. When coal is being retailed from house to house at 8 ¢ per bucket of 20 lb., how much more is being paid for it per ton (2000 lb.) than if it were bought for $6.75 per ton?

7. If a miller buys bran at $24 per ton and retails it at 28 ¢ per bushel (20 lb.), what is his profit per ton?

8. The annual output of grapes in a certain state is estimated at 4,500,000 lb., valued at $135,000; what is the estimated value per pound?

9. Simplify: $\dfrac{\frac{3}{4} \div \frac{1}{2} - \frac{1}{2} \div \frac{3}{4}}{62\frac{1}{2} \div 100}$.

10. A knot is 6086 ft. and a statute mile is 5280 ft.; estimate, correct to the nearest hundredth, the number of statute miles in a knot.

11. At $2.50 per cord, what is the value of a pile of wood 40 ft. long, 6 ft. high, and 4 ft. wide?

12. Find the cost of 20 planks, each 12 ft. by 6 in. by $1\frac{1}{2}$ in., at $30 per 1000 ft.

13. Estimating the amount of standing timber in the United States at 2,400,000,000,000 ft., and the average price on the stump at $2.44 per 1000 ft., what is the value of the standing timber in the United States?

14. If the length of an arc of 22° 30′ is 6 in., what is the circumference of the circle?

15. Show that there are 4 rods in one chain.

16. How much is saved by using a 100-ride ticket between two stations instead of tickets bought at the rate of 10 for $4, if the 100-ride ticket cost $22.80?

17. Each bushel of wheat raised is estimated to use up approximately the following amount of fertilizing substances from the soil: nitrogen, 20 oz.; phosphoric acid, 8 oz.; potash, 5 oz.; how many pounds of each of these substances are used up by 10 A. of wheat averaging 25 bu. per acre?

18. Simplify: $\dfrac{100 \div .01}{.01 \times 100} \div \dfrac{.1 \div .01}{.01 \div 2.5}$.

19. In an experiment made on a large potato farm it was found that the price of producing an acre of potatoes, including seed, labor, land rental, etc., was \$ 26.37. The average yield per acre was 127 bu., which were sold at 40 ¢ per bushel; what was the net income per acre?

20. The factors of the dividend are 100, .1, .01, and .125; the divisor is $.125 \div .005 \times .0025$. Find the quotient.

21. A tract of land 1 mile square contains how many acres?

22. Every public schoolroom built in Pennsylvania after May 18, 1911, must have not less than 200 cu. ft. of air space per pupil. A schoolroom 40 ft. long, 30 ft. wide, and 10 ft. high contains sufficient air space for how many pupils, if the room is built in accordance with the regulation stated?

23. In a certain experiment it was found that the cost of maintaining a herd of 261 cows was \$ 44.67 each per year, and that the value of the product per cow was \$ 60.05. Find the net profit from the herd.

24. When a Mexican dollar is quoted at \$0.4875 in United States money, what are 1000 Mexican dollars worth in United States money?

25. Show why each common year begins 1 day later in the week than the preceding common year.

26. In measuring distances, a pace is considered $\frac{1}{5}$ of a rod. How many feet is this?

27. Estimating £1 to equal $4.8665, what is the value in United States money of £5 4*s*.?

28. Simplify:

$$116 \times 4 \div 8 - 84 \div 12 \times 3 + 63 \div (9 \times 7).$$

29. If one machine can grind 10 bu. of grain in $2\frac{1}{2}$ hr. and another can grind 10 bu. in $1\frac{2}{3}$ hr., how long will it take both machines to grind 100 bu. of grain?

30. Express each of the following as a decimal part of a dollar and find their sum: 12.5 cents, $18\frac{3}{4}$ mills, $\frac{7}{8}$ of a dollar, 1.75 cents, $\frac{7}{8}$ of a dime, and 14 cents 6.5 mills.

31. How many 2-in. cubes can be cut from a block of wood in the form of a rectangular solid 6 in. by 5 in. by 4 in.?

32. If a lot having a frontage of 40 ft. and a depth of 60 ft. was sold for $25 per front foot, what was the price per acre?

33. Find, correct to the nearest pound, the amount of flour a miller should give in exchange for 2 bu. of wheat weighing 62 lb. to the bushel, if he gives 38 lb. of flour in exchange for 60 lb. of wheat.

34. When clover seed is selling at $9 per bushel, find the cost of sufficient of it for a 15-acre field, allowing 10 lb. of seed to an acre.

35. During a certain year (365 da.) which began on Sunday, a rural mail carrier covered his route of 30 miles each week day except February 22 and July 4. How many miles did he travel during that year in delivering his mail?

36. A tract of land 10 ch. long and 3.75 ch. wide was bought at $75 per acre. The purchaser had it inclosed by a fence at a cost of $0.62 per rod. Find the total cost.

37. The following table shows the quantity of corn by weight that remained after shrinkage at the close of each month from each 100 bu. by weight placed in the crib, as determined in a recent trial at an Experimental Station. In the column headed "Market Price" fill in the prevailing market price of corn in your community at the close of each month of the current year, and determine in what month the farmer would have received the greatest amount for the 100 bu. placed in the crib in November.

Date	Quantity	Market Price	Value
Nov. 30	100 bu.		
Dec. 31	97.9 bu.		
Jan. 31	97.6 bu.		
Feb. 28	97 bu.		
Mar. 31	97.8 bu.		
Apr. 30	96.9 bu.		
May 31	93.1 bu.		
June 30	92 bu.		
July 31	89.4 bu.		
Aug. 31	86.7 bu.		
Sept. 30	87.7 bu.		
Oct. 31	89.8 bu.		

THE EQUATION

Introductory Exercises, Definitions, and Principles

200. 1. If a stands for 3, what is the value of $2 \times a$?

$2 \times a$ may be written $2a$.

2. If a stands for 4, what is the value of $5a$? $a + 2a$? $3a - a$?

3. If a stands for 4 and b stands for 2, what is the value of $a + b$? $a - b$? ab? $a \div b$?

Any letter may stand for any number.

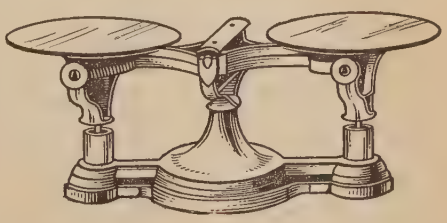
4. If x stands for 6, what is the value of $2x$? $x \div 3$? $3x + 2x$? $3x - x$?

201. When x stands for 4, $2x = 8$.

The expression $2x = 8$ is called an *equation*. $2x$ is the *first member* of the equation and 8 the *second member*.

202. An **equation** is a statement that two numbers or expressions are equal.

203. These scales are true; hence, they will balance when 6 lb. are placed on each plate. They will still balance if,



(1) 2 lb. additional are placed on each plate; or,
(2) 2 lb. are taken from each plate; or,
(3) 2×6 lb. are placed on each plate; or,
(4) 6 lb. $\div$ 2 are placed on each plate.

204. Section 203 illustrates the following:

(1) *If equals are added to equals, the results are equal.*
(2) *If equals are subtracted from equals, the results are equal.*
(3) *If equals are multiplied by equals, the results are equal.*
(4) *If equals are divided by equals, the results are equal.*

205. If x stands for 4,

$$(1)\quad 6x - 2 = 3x + 10.$$
$$(2)\quad 6x - 10 = 3x + 2.$$

In equations (1) and (2) observe that the 2 and also the 10 are in different members, and that the signs before each of these numbers are different in the two equations; hence:

206. A term may be transposed from one member of an equation to the other, if the sign before it is changed.

Exercise 61

1. If $x + 6 = 8$, for what number does x stand; that is, *what is the value of x?*

$$x + 6 = 8$$
$$x = 8 - 6 \quad [\S\ 206]$$
$$x = 2$$

In each of the following find the value of x:

2. $x+8=10$	3. $x-3=2$	4. $3=x+2$
5. $x-9=1$	6. $x-1=4$	7. $5=x-2$
8. $3+x=4$	9. $8+x=12$	10. $1=x-1$
11. $7+x=12$	12. $3+x=7$	13. $5=x+3$

14. If $5x=40$, what is the value of x?

$$5x=40$$

Dividing by 5, $x=8$ [§ 204, 4]

15. If $\frac{1}{3}x=4$, what is the value of x?

$$\tfrac{1}{3}x=4$$

Multiplying by 3, $x=12$ [§ 204, 3]

16. If $\frac{2}{3}x=4$, what is the value of x?

$$\tfrac{2}{3}x=4$$

Multiplying by 3, $2x=12$ [§ 204, 3]

Dividing by 2, $x=6$ [§ 204, 4]

In each of the following, find the value of x:

17. $2x=16$	18. $3x=7$	19. $4x=15$
20. $5x=21$	21. $2x=\frac{1}{2}$	22. $3x=1\frac{1}{2}$
23. $\frac{1}{2}x=4$	24. $\frac{1}{3}x=5$	25. $\frac{1}{4}x=\frac{1}{2}$
26. $\frac{1}{2}x=\frac{2}{3}$	27. $\frac{1}{5}x=2\frac{1}{2}$	28. $\frac{1}{8}x=12\frac{1}{2}$
29. $\frac{2}{3}x=6$	30. $\frac{3}{4}x=9$	31. $\frac{4}{5}x=12$
32. $\frac{2}{3}x=7$	33. $\frac{3}{4}x=10$	34. $\frac{4}{5}x=15$
35. $\frac{3}{4}x=\frac{1}{2}$	36. $\frac{2}{3}x=\frac{3}{4}$	37. $\frac{4}{5}x=2\frac{1}{2}$
38. $\frac{4}{5}x=3\frac{1}{3}$	39. $\frac{4}{3}x=12$	40. $2\frac{1}{2}x=15$

41. If $4x-3=2x+9$, find the value of x.

$4x-3=2x+9$

Transposing, $4x-2x=9+3$

Uniting terms, $2x=12$

Dividing by 2, $x=6$

In each of the following find the value of x:

42. $4x-2=x+7$

43. $7x+2=5x+6$

44. $5x-1=3x+7$

45. $7x+8=4x+11$

46. $10x+5=4x+8$

47. $12x+1=9x+3\frac{1}{4}$

48. $x+4=\frac{1}{2}x+5$

49. $\frac{3}{4}x+2=\frac{1}{2}x+6$

50. $2\frac{1}{2}x-5=1\frac{1}{4}x+15$

Illustrative Example

The sum of two numbers is 25, and the larger is 4 times the smaller; what are the numbers?

Explanation:

Let x = the smaller number.

Then $4x$ = the larger number,

and $5x$ = the sum.

25 = the sum.

$\therefore\ 5x=25$.

$x=5$, the smaller.

$4x=20$, the larger.

Check: $20+5=25$; $20=4\times 5$.

Exercise 62

1. The sum of two numbers is 30, and the larger is 4 times the smaller; what are the numbers?

2. The sum of two numbers is 15, and the smaller is $\frac{1}{2}$ of the larger; what are the numbers?

3. Two girls have 50¢; if one of them has 4 times as much as the other, how much has each?

4. A woman bought a certain number of yards of lining and twice as many yards of dress goods; if she bought 30 yd. of both, how many yards did she buy of each?

5. A and B together own 350 acres of woodland; if A owns 4 times as much as B, how much does each own?

6. An article was sold at a gain of \$ 12, which was $\frac{1}{4}$ of the cost; what was the cost?

7. An article was sold at a loss of \$ 2.50, which was $\frac{1}{5}$ of the cost; what was the cost?

8. A dealer sold an article for \$ 300, which was at a gain of $\frac{1}{2}$ of the cost; what was the cost?

9. An article was sold for \$ 75, which was at a loss of $\frac{1}{4}$ of the cost; what was the cost?

10. Two men together bought 65 head of cattle; if one of them bought 5 head more than the other, how many did each buy?

11. After a man had sold $\frac{1}{4}$ of his land to one man and $\frac{1}{5}$ of it to another, he had left 220 acres; how much had he at first?

PERCENTAGE

207. Per cent means *hundredths.*

Thus, 1 per cent of a number is .01, or $\frac{1}{100}$, of it.
10 per cent of a number is .10, or $\frac{10}{100}$, or $\frac{1}{10}$, of it.
$12\frac{1}{2}$ per cent of a number is .12$\frac{1}{2}$, or $\frac{12\frac{1}{2}}{100}$, or $\frac{1}{8}$, of it.

The sign % stands for per cent.

Thus, *6 per cent of 25* may be expressed *6 % of 25.*

208. The number or amount of which a given per cent is taken is called the **base.** The number of hundredths of the base taken is called the **rate per cent,** or the **rate,** and the result obtained by taking a given per cent of the base is called the **percentage.**

Exercise 63

What part of a number is:

1. 2 % of it?	2. 5 % of it?	3. 20 % of it?
4. 25 % of it?	5. 50 % of it?	6. 75 % of it?
7. 30 % of it?	8. 40 % of it?	9. 80 % of it?
10. 60 % of it?	11. 90 % of it?	12. 70 % of it?
13. $12\frac{1}{2}$ % of it?	14. $62\frac{1}{2}$ % of it?	15. $37\frac{1}{2}$ % of it?
16. $87\frac{1}{2}$ % of it?	17. $6\frac{1}{4}$ % of it?	18. $18\frac{3}{4}$ % of it?
19. $33\frac{1}{3}$ % of it?	20. $16\frac{2}{3}$ % of it?	21. $8\frac{1}{3}$ % of it?

How many times a number is:

22. 100 % of it? 23. 600 % of it? 24. 500 % of it?

25. 300 % of it? 26. 800 % of it? 27. 900 % of it?

28. 250 % of it? 29. 750 % of it? 30. 1000 % of it?

What per cent of a number is:

31. $\frac{1}{2}$ of it? Thus, $\frac{1}{2}$ of a number = $\frac{1}{2}$ of 100 % of it, or 50 % of it.

32. $\frac{1}{4}$ of it? 33. $\frac{1}{5}$ of it? 34. $\frac{1}{20}$ of it?

35. $\frac{3}{4}$ of it? 36. $\frac{1}{5}$ of it? 37. $\frac{2}{25}$ of it?

38. $\frac{2}{5}$ of it? 39. $\frac{7}{10}$ of it? 40. $\frac{7}{100}$ of it?

41. $\frac{1}{8}$ of it? 42. $\frac{3}{8}$ of it? 43. $\frac{5}{8}$ of it?

44. $\frac{7}{8}$ of it? 45. $\frac{1}{3}$ of it? 46. $\frac{2}{3}$ of it?

What per cent of a number is:

47. 2 times the number? 48. 6 times the number?

49. 5 times the number? 50. 8 times the number?

51. $2\frac{1}{2}$ times the number? 52. $1\frac{1}{2}$ times the number?

What per cent of a number is:

53. .2 of it? Answer thus: .2 of a number = .20 of it, or 20 % of it.

54. .7 of it? 55. .8 of it? 56. .5 of it?

57. .1 of it? 58. .9 of it? 59. .3 of it?

60. 1.2 times the number? 61. 2.56 times the number?

62. State instantly the per cent equivalent of each of the following:

.25;	.35;	.65;	.75;	.95;
.81;	.12;	.17;	.92;	.99;
1.50;	1.25;	2.05;	2.75;	3.45.

SUGGESTION. 1.50 is *one hundred fifty hundredths*, or 150%.

7.1;	8.3;	2.4;	2.7;	3.2;
$.12\frac{1}{2}$;	$.37\frac{1}{2}$;	$.62\frac{1}{2}$;	$.87\frac{1}{2}$;	$.33\frac{1}{3}$;
$\frac{1}{2}$;	$\frac{1}{4}$;	$\frac{1}{5}$;	$\frac{1}{3}$;	$\frac{1}{8}$;
$1\frac{3}{4}$;	$1\frac{2}{5}$;	$1\frac{1}{2}$;	$1\frac{3}{5}$;	2.

What per cent is:

63. 2 of 4?

Answer thus: $2 = \frac{1}{2}$ of 4, or 50% of 4.

64. 1 of 5?	65. 4 of 8?	66. 3 of 12?
67. 4 of 5?	68. 4 of 10?	69. 2 of 3?
70. 6 of 8?	71. 1 of 8?	72. 6 of 16?
73. 5 of 8?	74. 5 of 15?	75. 4 of 12?
76. 2 of 1?	77. 6 of 2?	78. 9 of 3?

TO THE TEACHER. Numerous simple exercises involving small numbers should be given in percentage, in order that pupils may become thoroughly grounded in reckonings involving per cent, before they are required to take up the more difficult problems of the subject or to study the applications of percentage. In the following exercise they should be able to name the result in each example instantly, and if they are not, they should receive more drill in these types of problems.

Exercise 64

Answer at sight:

1. What is 1% of $100?
2. What is 2% of $100?
3. What is $2\frac{1}{2}$% of $100?
4. 50% of a gallon is how many quarts?
5. 25% of a bushel is how many pecks?
6. How many are 5% of a score?
7. What is 10% of $1000?
8. $12\frac{1}{2}$% of a peck is how many quarts?
9. 25% of a day is how many hours?
10. 20% of a dime is how many cents?
11. 200% of one cent is what?
12. $37\frac{1}{2}$% of a pound (avoir.) is how many ounces?
13. 400% of $2 is what sum?
14. $66\frac{2}{3}$% of a yard is how many feet?
15. 100% of 2 quarts equals how many quarts?
16. 150% of 2 is what number?
17. 250% of 4 is what number?
18. $1 is 50% of what sum?
19. 5 yd. is 25% of what length?
20. 2 lb. is 20% of what weight?
21. 9 ft. is 300% of what length?
22. 25¢ is $33\frac{1}{3}$% of what sum?
23. $18 is $66\frac{2}{3}$% of what sum?

24. 5 min. is $12\frac{1}{2}\%$ of what time?

25. 80 words is 40% of how many words?

26. 15 lb. is 200% of what weight?

27. \$1.25 is $33\frac{1}{3}\%$ of what sum?

28. \$1.50 is 50% of what sum?

29. 6 lb. is $37\frac{1}{2}\%$ of what weight?

30. 14 is $87\frac{1}{2}\%$ of what number?

31. $\frac{1}{4}$ is 50% of what fraction?

32. What per cent is \$1 of \$100?

33. What per cent is \$2 of \$100?

34. What per cent is \$15 of \$100?

35. What per cent is \$7.50 of \$100?

SUGGESTION. The number of units estimated on 100 *of the same kind* is the rate; here 7.5 is the number estimated on 100.

36. What per cent is \$12.50 of \$100?

37. What per cent is \$1.25 of \$100?

38. What per cent is \$3 of \$100?

39. What per cent is \$0.50 of \$100?

40. What per cent is \$25 of \$200?

41. What per cent is 1¢ of 2¢?

42. What per cent is 5¢ of 2¢?

43. 1 qt. is what per cent of a peck?

44. 5 times any number is what per cent of it?

Illustrative Examples

1. Find $37\frac{1}{2}\%$ of \$ 24.80.

\$ 24.80
.375
12400
17360
7440
\$ 9.30000

Explanation:

$37\frac{1}{2}\%$ of \$ 24.80 = .375 × \$ 24.80, or \$ 9.30.
The result may also be found by taking $\frac{3}{8}$ of \$ 24.80.

2. \$ 74.76 is 42 % of what sum?

\$ 178
.42)\$ 74.76
42
327
294
336
336

Explanation:

.42 × the sum = \$ 74.76.
∴ \$ 74.76 ÷ .42, or \$ 178 = the sum.

3. \$ 240.20 is $66\frac{2}{3}\%$ of what sum?

Explanation:

$66\frac{2}{3}\%$, or $\frac{2}{3}$, of the sum = \$ 240.20.
∴ $\frac{1}{3}$ of the sum = $\frac{1}{2}$ of \$ 240.20, or \$ 120.10.
∴ $\frac{3}{3}$ of the sum = 3 × \$ 120.10, or \$ 360.30.

4. What per cent is \$ 12.50 of \$ 200?

Explanation:

\$ 12.50 is the same per cent of \$ 200 that \$ 6.25 is of \$ 100; that is, 6.25 %.

5. What per cent is \$ 200 of \$ 1600?

Explanation:

\$ 200 is $\frac{1}{8}$ of \$ 1600.
∴ $\frac{1}{8}$ of 100 %, or $12\frac{1}{2}\%$ = the required rate.

6. What per cent is $\frac{3}{4}$ of $\frac{4}{5}$?

Explanation:

$\frac{4}{5} = \frac{16}{20}$; $\frac{3}{4} = \frac{15}{20}$.

$\therefore \frac{3}{4}$ is $\frac{15}{16}$ of $\frac{4}{5}$.

$\therefore \frac{15}{16}$ of 100 %, or $93\frac{3}{4}$ % = the required rate.

7. What per cent of \$ 56.875 is \$ 3, correct to the nearest .01 per cent?

Explanation:

$3 \div .56875 = 56875\overline{)300000.000}$ (5.274+; etc.)

1 % of \$ 56.875 = .01 of \$ 56.875 = \$ 0.56875.

$3 \div .56875 = 5.274+$.

$\therefore 5.27$ % = the required rate.

NOTE. The nature of the problem should determine the method used in solving problems of percentage, especially those in which the rate per cent is to be found. Unless the relation between the numbers involved is a simple one, the method given in Example 7 will be found the most practical in finding the rate per cent. The equation having already been explained, those teachers desiring to use x and the equation to find the missing term will be able to do so.

Exercise 65

Find:

1. 6 % of 275.
2. 8 % of 390.
3. 12 % of 296.
4. $12\frac{1}{2}$ % of 650.
5. 15 % of \$ 785.
6. $12\frac{1}{2}$ % of \$ 460.
7. 36 % of 475 bu.
8. $66\frac{2}{3}$ % of 861 ft.
9. 75 % of \$ 3200.
10. $37\frac{1}{2}$ % of \$ 185.
11. $62\frac{1}{2}$ % of 864 pupils.
12. $16\frac{2}{3}$ % of \$ 600.
13. $6\frac{1}{4}$ % of 800 A.
14. 125 % of \$ 280.
15. 2.5 % of \$ 1000.
16. 4.7 % of \$ 2000.

Find the number or amount of which:

17. 54 is 12 %.
18. 135 is 18 %.
19. $175 is 20 %.
20. $700 is 25 %.
21. $240 is $33\frac{1}{3}$ %.
22. $75 is $12\frac{1}{2}$ %.
23. $240 is $66\frac{2}{3}$ %.
24. 162 bu. is $37\frac{1}{2}$ %.
25. 550 A. is $62\frac{1}{2}$ %.
26. $210 is $87\frac{1}{2}$ %.
27. 105 is 2.1 %.
28. 288 is 4.8 %.

What per cent is:

29. 90 of 360?
30. 126 of 280?
31. 63 of 350?
32. 150 of 750?
33. $130 of $390?
34. $68 of $425?
35. $494 of $741?
36. $324 of $5400?
37. 288 bu. of 3600 bu.?
38. $182 of $4550?
39. $95 of $4750?
40. $500 of $2000?

What per cent (correct to the nearest .01 per cent) is:

41. 4 of 15?
42. 7 of 75?
43. 14 of 115?
44. 35 of 237?
45. 89 of 589?
46. 97 of 971?
47. 128 of 4271?
48. 357 of 3764?
49. 25 of 5176?
50. 19 of 4768?

Illustrative Examples

1. An agent collected $87\frac{1}{2}\%$ of a debt of $832 and charged 5% of the sum collected; how much did he charge?

The answer may be found thus:

$832
.875
4160
5824
6656
$728.000
.05
$36.40 *Ans.*

Explanation:

$832 = the debt.
.875 × $832, or $728 = the amount collected.
.05 × the amount collected = the charges.
∴ .05 × $728, or $36.40 = the charges.

NOTE. $87\frac{1}{2}\%$ of $832 could also be found by taking $\frac{7}{8}$ of $832.

2. Milk yields butter to the amount of about 4% of its weight; what weight of milk will yield 240 lb. of butter?

Explanation:

.04 × the weight of the milk = the weight of the butter.
240 lb. = the weight of the butter.
∴ 240 lb. = .04 × the weight of the milk.
∴ 240 lb ÷ .04, or 6000 lb. = the required weight.

The answer may be found thus:

.04)240.00
6000 *Ans.*

3. When an investment of $65.75 earns $4.50, what is the rate per cent of earning, correct to the nearest .01 %?

The answer may be found thus:

```
        6.844
.6575)4.50000000
      39450
       55500
       52600
        29000
        26300
         27000
         26300
```

6.84 % *Ans.*

Explanation:

$65.75 = the investment.
$4.50 = the earnings.
$0.6575 = the earnings if they were 1 % of the investment.
4.50 ÷ .6575 = 6.844.
∴ 6.84 % = the required rate.

Exercise 66

(Solve as many of these as possible orally.)

1. 80 % of a class of 40 pupils were promoted; how many were promoted?

2. A farmer sold 60 % of his crop of 40 tons of hay; how many tons did he sell?

3. How many words does a boy miss out of 50, if he spells 90 % of them correctly?

4. A family that uses 2 % of a ton (2000 lb.) of coal a day uses how many pounds a day?

5. What per cent should a boy receive who spells correctly 40 words out of the 50 given?

6. When cloth is advanced from $1 to $1.10 per yard, what is the per cent of advance?

7. If poultry, in dressing, lose 10 % of their live weight, what should a dressed turkey weigh whose live weight is 15 lb.?

8. A boy who buys newspapers at $\frac{1}{2}$¢ each and sells them at 1¢ each is selling them at what per cent of cost?

9. A pupil who attends school 18 days out of 20 during a certain month should receive what per cent for attendance?

10. When $6 is paid for the use of $100, what rate per cent is paid?

11. $12\frac{1}{2}$ % of a pole 24 ft. long is in the ground; how far is the top of the pole from the ground?

12. When 2 bu. of wheat, sowed, yield 30 bu., what is the per cent of yield?

13. How much did an agent collect during a certain month if he received 5 % of the sum collected, and his earnings were $100?

14. After a farmer had sold 60 % of his crop of corn he had remaining 280 bu.; what was the amount of his crop? SUGGESTION. 280 bu. = 40 % of the required crop.

15. A newsboy sold papers at 5¢ a copy, which was 200 % of what he paid for them; what did he pay for them?

16. 30 % of a class failed in an examination; if 14 of the class passed, how many were in the class?

17. How many words did a boy spell correctly if he missed 10 words and was given an average of 75 %?

18. Calves, in dressing, lose about 40 % of their weight; what was the live weight of a calf whose dressed weight is 80 lb.?

19. When 10 lb. of green tea are mixed with 15 lb. of black tea, what per cent of the mixture is black tea and what per cent is green tea?

20. In a crate of eggs containing 6 doz., 9 eggs were found broken; what was the loss per cent from breakage?

21. All school buildings hereafter built or rebuilt in Pennsylvania must have the total light area equal to at least 20 % of the floor space. What is the least light area a school building may have whose floor space is 1200 sq. ft.?

22. If $37\frac{1}{2}$ % of the pupils in a school are boys and there are 40 girls in this school, how many pupils are there?

23. The standard gold coin of the United States is 9 parts pure gold and 1 part alloy. What per cent is alloy? What per cent is pure gold?

24. Pure gold is designated as 24 carats fine; what is the per cent of gold in jewelry that is 18 carats fine?

25. A man bought a horse for $275 and sold it for 20 % more than he paid for it; what was the selling price?

26. 10 % of all U. S. silver coin is copper alloy; find the weight of the copper and also of the silver in a silver dollar, the weight of which is 412.5 gr.

27. How much does a merchant lose of a bill of $275, if he is able to collect only 75 % of it?

28. Water expands 10 % of its volume in freezing; how many cubic feet of ice can be made from 100 cubic feet of water?

29. Wood ashes contain about 5 % of potash, 1.5 % of phosphoric acid, and 32.5 % of lime; how many pounds of each of these substances are in a ton of wood ashes?

30. If a miller takes 10 % of each bushel of corn ground for toll, and there is also a loss of about 1 % in grinding, how many pounds of feed should be returned to a farmer for 30 bu. (56 lb.) of corn?

31. Potatoes are about 75 % water, and of the remainder about 64.8 % is starch; how many pounds of starch are there in 200 lb. of potatoes?

32. After a piece of cloth had shrunk 5 % of its length, it measured 38 yd.; how many yards did it measure before shrinking?

33. After a laborer's wages had been increased 20 %, they were \$54 a month; what were they before the increase?

34. A merchant's sales for a certain day were \$714.35, which was 87½ % of his sales the preceding day. What were his sales for both days?

35. Corn is about 5.4 % fat; how many pounds of fat are there in 20 bu. (56 lb.) of corn?

36. A horse was sold for 12½ % less than it cost. If it was sold for \$175, what was the cost?

37. Last year a farmer's wheat crop yielded 20 bu. to the acre. This year it yielded $21\frac{1}{2}$ bu. to the acre. What was the per cent of increase?

38. If a teacher's salary is advanced from \$60 to \$65 per month, what is the per cent of increase?

39. If 1 cu. ft. of cement be mixed with 2 cu. ft. of sand and 4 cu. ft. of broken stone to make concrete, what per cent of the mixture is cement? What per cent of the mixture is sand? What per cent of the mixture is broken stone?

40. In the 8th grade of the schools of a certain city 210 minutes are assigned per week to the subject of arithmetic. If the school is in session 1500 minutes per week, what per cent of the total time is assigned to arithmetic?

41. When tomatoes are advanced in price from 10¢ to 12¢ per can, what is the per cent of advance?

42. What is 7% of \$600?

SUGGESTION. Move the decimal point two places to the left, and multiply the result by 7.

43. The amount of land surface in New Jersey is 7455 sq. mi., and the amount of water surface 360 sq. mi.; what per cent of the area of the state is land and what per cent water? Answer correct to the nearest .01%.

44. On an investment of \$85.38 a man gained \$4.75; find the rate per cent of gain, correct to the nearest .01%.

45. In ten years, recently, the population of Oklahoma increased from 398,331 to 1,637,155; find the per cent of the increase, correct to the nearest .01%.

46. In ten years, recently, the population of Indianapolis increased from 169,164 to 233,650. Find the per cent of increase, correct to the nearest .1%.

47. In ten years, recently, the population of Lansing, Mich., increased from 16,485 to 31,489. What was the per cent of increase, correct to the nearest .1 % ?

48. During ten years, recently, the population of a certain township decreased from 875 to 854. What was the per cent of decrease, correct to the nearest .1 % ?

49. A long ton is how many per cent more than a short ton ?

50. A man's investment earned him, during a certain year, $135. This was 4.5 % of the investment; what was the investment ?

COMMERCIAL DISCOUNT

209. Commercial Discount is any deduction made from the *list price*, the *time price*, or the *marked price* of goods; the **net price** is the remainder.

210. If but one discount is made on any price, a certain per cent of that price is deducted; if two or more discounts are made, the first discount is reckoned on the price, the second on the remainder, the third on the next remainder, and so on.

Thus, if an article is marked $ 0.80 and sold at a discount of 25 %, the discount is 25 % of $ 0.80, or $ 0.20. If it is sold at a discount of 25 % and 10 %, the second discount is 10 % of $ 0.60, or $ 0.06, and the total discount is $ 0.26.

Illustrative Example

What is the net cost of a bill of goods listed at \$240 and bought at 20 % and $12\frac{1}{2}$ % off?

Explanation:

20 %, or $\frac{1}{5}$, of \$ 240, or \$ 48 = the first discount.
∴ \$ 240 − \$ 48, or \$ 192 = the first remainder.
$12\frac{1}{2}$ %, or $\frac{1}{8}$, of \$ 192, or \$ 24 = the second discount.
And, \$ 192 − \$ 24, or \$ 168 = the net cost.

211. Business men generally convert two successive rates of discount into an equivalent single rate before they reckon the discount; to do so they use the following:

A single rate of discount equivalent to two successive rates equals their sum diminished by $\frac{1}{100}$ of their product.

Thus, a single rate of discount equivalent to

$$20\ \%\text{ and }10\ \% = 20\ \% + 10\ \% - \frac{20 \times 10}{100}\ \%$$
$$= 30\ \% - 2\ \%, \text{ or } 28\ \%.$$

Exercise 67

State at sight the missing discounts in Examples 1–10:

	Marked Price	Rate of Discount	Discount
1.	\$ 2.00	25 %	?
2.	\$ 1.60	25 %	?
3.	\$ 1.20	$33\frac{1}{3}$ %	?
4.	\$ 4.00	$12\frac{1}{2}$ %	?
5.	\$ 0.80	$37\frac{1}{2}$ %	?

	Marked Price	Rate of Discount	Discount
6.	\$ 5.00	25 %	?
7.	\$ 0.50	40 %	?
8.	\$ 3.00	25 %	?
9.	\$ 0.05	20 %	?
10.	\$ 0.32	$6\frac{1}{4}$ %	?

11. On a bill of goods amounting to \$ 410.20 a discount of 5 % was allowed. Find the cost.

12. Find the cost of a piano catalogued at \$ 600, bought subject to a discount of 20 %.

13. A merchant reduced gingham aprons, marked 25 ¢, 40 %. What was then the asking price?

14. Find the asking price of a robe that was marked \$ 4, but was reduced $18\frac{3}{4}$ %.

15. A dealer bought a bill of dry goods amounting to \$ 275, subject to a discount of 20 % and 10 %. Find the cost.

16. What was paid for a bill of goods amounting to \$ 250, and bought subject to a discount of 20 % and 5 %?

17. A wholesaler sold wall paper to a retailer, subject to a discount of 20 %, $12\frac{1}{2}$ %, and 5 %. The total bill amounted to \$ 160. How much should the retailer pay?

18. March 1, a dealer bought a bill of goods amounting to \$ 350, payable in 60 da., subject to a discount of 1 %, if paid within 30 da., or 3 % if paid within 10 da. How much would settle this bill Apr. 30? Mar. 8? Mar. 28?

19. After deducting a discount of 20 %, a dealer sent $ 12.60 in payment of a bill of hardware. What was the bill?

SUGGESTION. 100 % of the bill − 20 % of the bill, or 80 %, or $\frac{4}{5}$, of the bill = $ 12.60.

20. A dealer paid $ 276.50 for a bill of shoes which he bought subject to a discount of $12\frac{1}{2}$ %. What was the bill?

21. Find the single rate of discount equivalent to 50% and 20%.

22. Find the single rate of discount equivalent to 30 % and 20 %.

23. Find the single rate of discount equivalent to 40 % and 10 %.

24. Find the single rate of discount equivalent to 20 % and 20 %.

25. What single rate of discount is equivalent to 10 % and 10 %?

26. What single rate of discount is equivalent to 20 % and 15 %?

27. What single rate of discount is equivalent to 40 %, 20 %, and 10 %?

Find the rate of discount on:

28. A china dinner set marked down from $ 25 to $ 17.

29. Vests marked down from $ 2 to $ 0.50.

30. Trousers marked down from $ 10 to $ 7.50 a pair.

31. Afternoon dresses marked down from $25 to $11.50.

32. Muffs marked down from $35 to $28.

33. Serge suit marked down from $30 to $18.

In marking goods, a merchant generally uses a *key* consisting of a word or words. He writes the asking price above a horizontal line and the cost below it. If *bankruptcy* is the key chosen, the letters *b-a-n-k-r-u-p-t-c-y*, in order, would stand for 1, 2, 3, 4, 5, 6, 7, 8, 9, 0.

Thus, if an article cost $1.20 and is marked at $1.48 it would be marked $\frac{\$1.48}{B^{ay}}$, if the dealer wishes to conceal only the cost, and $\frac{B^{kt}}{B^{ay}}$, if he wishes to conceal both the asking price and the cost.

34. Read the costs of the following goods, *bankruptcy* being used as the key word: $\overline{K^{yy}}$; $\overline{B^{ty}}$; $\overline{N^{pr}}$; $\overline{U^{cy}}$.

Mark each at an advance of 25 %.

PROFIT AND LOSS

212. The **cost** of an article is the amount paid for it. The **selling price** is the amount received for it. The **profit** is the amount that the selling price exceeds the cost and the expenses. The **loss** equals the amount that the cost and the expenses exceed the selling price. When there are no expenses, the profit equals the selling price less the cost, and the loss equals the cost less the selling price.

213. *Gain or loss per cent is always reckoned on the cost.*

Illustrative Examples

1. A dealer bought shoes at $2.50 a pair and sold them at a profit of 20 %; what was the selling price?

The answer may be found thus:

$2.50
.20
.50
2.50
$3.00 *Ans.*

Explanation:

$2.50 = cost.
20 %, or $\frac{1}{5}$, of the cost = gain.
$\frac{1}{5}$ of $2.50, or $0.50 = gain.
$2.50 + $0.50, or $3 = selling price (s. p.).

Remark. The selling price may be found directly by taking $1.20 \times \$2.50$.

2. Coffee was sold at 40¢ a pound, which was at a profit of 25 %; find the cost.

The answer may be found thus:

1.25) $0.4000 | $0.32 *Ans.*
375
250
250

Explanation:

.25 × cost = gain.
∴ 1.25 × cost = selling price.
$0.40 = selling price.
$0.40 = 1.25 × cost.
∴ $0.40 ÷ 1.25, or $0.32 = cost.

Remark. For many problems the following is a preferable solution:

.25, or $\frac{1}{4}$, of cost = gain.
∴ $\frac{5}{4}$ of cost = selling price.
40¢ = selling price.
∴ $\frac{5}{4}$ of cost = 40¢.
cost = 32¢.

3. What was the gain per cent on an office desk bought for $40 and sold for $55?

Explanation:

$55 = selling price.
$40 = cost.
∴ $15 = gain.
∴ $\frac{3}{8}$ of cost = gain.
∴ $\frac{3}{8}$ of 100 %, or $37\frac{1}{2}$ % = the rate.

Another Explanation:

$55 = selling price.
$40 = cost.
∴ $15 = gain.
.01 × $40 or $0.40 = gain at 1 %.
$15 \div .40 = 37\frac{1}{2}$.
∴ $37\frac{1}{2}$ % = the rate.

Remark. The second explanation is preferable if the gain or loss does not bear some simple relation to the cost.

Exercise 68

To the Teacher. Encourage pupils to solve orally as many problems as possible. Much drill should be given in exercises involving simple numbers.

1. Name at sight each missing gain or loss:

Cost	Gain Per Cent	Loss Per Cent	Gain	Loss
$25	20 %		?	
$6		25 %		?
$100	$12\frac{1}{2}$ %		?	
$200		1 %		?
$0.02	200 %		?	

2. Name at sight each missing rate per cent of gain:

Cost	Gain	Rate Per Cent of Gain
$0.05	$0.01	?
$0.01	$0.03	?
$1	$0.05	?
$100	$2.50	?
$1	$0.50	?

3. Name at sight each missing rate per cent of loss:

Cost	Selling Price	Rate Per Cent of Loss
\$ 0.02	\$ 0.01	?
\$ 0.05	\$ 0.01	?
\$ 0.05	\$ 0.04	?
\$ 1	\$ 0.25	?
\$ 300	\$ 100	?

Find the loss and the selling price:

4. If the cost is \$ 10.50, and the rate of loss, $33\frac{1}{3}\%$.
5. If the cost is \$ 2.40, and the rate of loss, $12\frac{1}{2}\%$.
6. If the cost is \$ 0.75, and the rate of loss, 4 %.
7. If the cost is \$ 2000, and the rate of loss, $12\frac{1}{2}\%$.

Find the gain and the selling price:

8. If the cost is \$ 3.60, and the rate of gain is $66\frac{2}{3}\%$.
9. If the cost is \$ 45, and the rate of gain is $2\frac{1}{2}\%$.
10. If the cost is \$ 150, and the rate of gain is 75 %.
11. If the cost is \$ 2000, and the rate of gain is $7\frac{1}{2}\%$.

Find the cost and the gain or loss:

12. If the selling price is \$ 60, and the rate of gain is 25 %.
13. If the selling price is \$ 72, and the rate of gain is $12\frac{1}{2}\%$.
14. If the selling price is \$ 6.30, and the rate of gain is 200 %.

15. If the selling price is $3.20, and the rate of loss is 20%.

16. If the selling price is $2.50, and the rate of loss is 50%.

Find the selling price when:

17. The gain is $2.10, and the rate of gain is 10%.

18. The gain is $6.10, and the rate of gain is $66\frac{2}{3}$%.

19. The loss is $3.60, and the rate of loss is $37\frac{1}{2}$%.

20. The loss is $4.80, and the rate of loss is 40%.

Find the rate per cent of gain or loss when the cost is:

21. $5, and the selling price is $7.

22. $75, and the selling price is $25.

23. $200, and the selling price is $205.

24. $400, and the selling price is $392.

25. $1000, and the selling price is $750.

26. $100, and the selling price is $112.50.

27. Find the gain per cent on goods listed at $5, bought at a discount of 20%, and sold at $4.80.

28. Find the gain per cent on goods listed at $4.50, bought at a discount of 20%, and sold for $4.20.

29. Find the gain per cent on goods listed at $0.40, bought at a discount of 10%, and sold for 0.48.

30. Find the gain per cent on goods listed at $25, bought at a discount of 25%, and sold for $20.

31. Find the gain on goods listed at $6, bought at a discount of 10%, and sold at a gain of 20%.

32. Find the gain on goods listed at $48, bought at a discount of $12\frac{1}{2}$%, and sold at a profit of 8%.

33. Find the gain on goods listed at $200, bought at a discount of 20% and 10%, and sold at a gain of 5%.

34. Find the loss on an article listed at $250, bought at a discount of 20% and $12\frac{1}{2}$%, and sold at a loss of 5%.

35. What is the gain per cent on goods that are sold for 3 times their cost?

36. What per cent is lost on goods that are sold for half their cost?

37. Find the gain per cent on 12 doz. spools of thread, bought for 50¢ per dozen spools, and sold for 5¢ per spool.

38. By selling a horse for $220, a dealer gained 10%; what was the gain?

39. Shoes were sold at $3 a pair, which was at a profit of 20%; what was their cost?

40. Crochet hooks are sold at 2¢ each, which is at a gain of 44%; what did they cost per gross?

41. Belts that cost $9 per dozen were retailed at a profit of 20%; what was the selling price of each belt?

42. A sweater that cost $1.50 was marked to gain $33\frac{1}{3}$%, but it was sold out of season at a discount of 25%; what was the gain or loss?

43. Serge dresses that cost $7.50 each were marked

to gain 20 %, but they were sold at a discount of $33\frac{1}{3}$ %; find the gain or loss per cent.

44. Men's silk shirts that cost $ 2 each were marked to gain 75 %, but they were sold at a discount of 33 %; find the gain or loss per cent.

45. A dealer bought carpet at $ 1.20 a yard, subject to a discount of $16\frac{2}{3}$ %, and sold it for $ 1.50 a yard; find the gain per cent.

46. If chestnuts were bought for $ 2 a bushel and retailed at 10 ¢ a quart, what was the gain per cent if they shrunk one quart before they were sold?

47. $33\frac{1}{3}$ % was gained by selling 20 doz. eggs for $ 6.40; what did they cost a dozen?

48. A woman gains 50 % by selling 6 boxes of matches for 10 ¢; what per cent would she have gained by selling 2 boxes for 5 ¢?

49. The loss on an article sold for $ 9 was 25 %; what would have been the gain per cent had it been sold for $ 15?

50. A dealer bought English cheese at $ 0.20 a pound; how many ounces should he give for 10 ¢ in order to gain 20 %?

51. A dealer bought oranges at 20 ¢ a dozen; what should he charge for three of them in order that his rate of gain may be 50 %?

52. A chiffonier was sold at 120 % of its cost; if the gain on it was $ 8, what was the selling price?

53. A dealer bought hams at 15¢ a pound; at what price must he sell them to gain $33\frac{1}{3}$ %, if they lose 20 % of their weight in drying?

54. I invest a sum of money and sell the investment at a loss of 30 %; I invest the proceeds and sell at a gain of 50 %. Do I gain or lose, and what per cent of the original investment?

55. A dealer bought goods at a discount of 20 % and $12\frac{1}{2}$ % from the list price; he sold them at a discount of 16 % from the list price. Find his gain per cent.

56. A dealer marked a hat to gain 25 %, but he sold it for $1.75, which was at a loss of $12\frac{1}{2}$ %; find the marked price.

57. A dealer made a profit of $87\frac{1}{2}$ % by selling oranges at 30¢ a dozen. What per cent would he have gained had he sold them at 20¢ a dozen?

58. A dealer gained $33\frac{1}{3}$ % when he sold herring at $1 per hundred; what per cent did he gain on those that he sold at 15¢ per dozen?

59. What per cent profit does a dealer make on goods that he buys at 20 % and $12\frac{1}{2}$ % from the list price and sells at the list price?

60. Goods are marked to gain 20 %, but having become damaged, they are sold at a discount of 10 % from the marked price; find the gain per cent.

COMMISSION

214. An **agent** is a person who transacts business for another called the **principal.**

An agent that receives into his possession the property of the principal, and transacts in his own name the business relating to such property, is called a **commission merchant.** An agent that does business in the name of his principal, and without receiving the property of the principal into his possession, is called a **broker.**

215. Commission or **brokerage** is the sum charged by an agent for transacting business for a principal, and is usually a certain per cent of the amount involved in the transaction.

216. *When a sale is made, commission is reckoned on the amount of the sales, and when a purchase is made, on the amount of the purchase.*

217. A **consignment** is the goods sent by one person to another for sale. The party who sends the goods is the **consignor**, and the party who receives them is the **consignee.** The **net proceeds** of a consignment is the sum that is left after all expenses are paid.

Illustrative Examples

1. An agent sold fertilizer to the amount of $3450. He paid storage and other expenses amounting to $35; what were the net proceeds, if he deducted a commission of 1.5%?

The answer may be found thus:

$3450
.015
17250
3450
51.750
35
$86.75

Explanation:

$3450 = amount of sales.
.015 × $3450, or $51.75 = commission.
$35 = storage.
∴ $51.75 + $35, or $86.75 = total charges,
and $3450 − $86.75, or $3363.25 = net proceeds.

$3450 − $86.75 = $3363.25 *Ans.*

2. My agent purchased for me 500 bu. of corn at 75¢ per bushel, prepaid freight amounting to $25, and charged me a commission of 2%; find the entire cost of the corn.

The answer may be found thus:

$0.75
500
375.00
.02
7.50
25.00
375.00
$407.50 *Ans.*

Explanation:

500 × $0.75, or $375 = cost of the corn.
2% of $375, or $7.50 = commission.
$25 = freight charges.
∴ $407.50 = entire cost.

Exercise 69

In the following exercises supply at sight the missing amounts:

	Sales	Rate of Commission	Commission	Net Proceeds
1.	$80	5%	?	?
2.	$100	$12\frac{1}{2}$%	?	?
3.	$40	$37\frac{1}{2}$%	?	?

	Sales	Rate of Commission	Commission	Net Proceeds
4.	$10	5%	?	?
5.	$500	1%	?	?
6.	$100	$\frac{1}{2}$%	?	?

In the following exercises supply at sight the missing amounts:

	Amount of Purchase	Rate of Commission	Commission	Entire Cost
7.	$1	5%	?	?
8.	$100	$2\frac{1}{2}$%	?	?
9.	$40	$62\frac{1}{2}$%	?	?
10.	$5	100%	?	?
11.	$6.50	50%	?	?
12.	$30	8%	?	?

In the following exercises supply at sight the missing quantity or per cent:

	Amount of Sales	Rate of Commission	Commission	Other Expenses	Net Proceeds
13.	$50	10%	?	$5	?
14.	$100	?	?	$5	$80
15.	?	5%	$50	$100	?
16.	$400	?	$50	$50	?
17.	?	10%	?	$30	$150
18.	?	5%	$20	?	$370

In the following exercises supply the missing terms:

	Amount of Purchase	Rate of Commission	Commission	Other Expenses	Entire Cost
19.	$100	12½ %	?	$5	?
20.	?	5 %	$20	$10	?
21.	$200	?	?	$10	$226
22.	?	4 %	?	$30	$550

23. An agent was paid 12½ % commission on his sales and $50 a month. How much did he earn during a certain month in which his sales amounted to $1280?

24. A commission merchant sold 650 lb. of dressed turkey at $0.18 a pound. He charged a commission of 5 %, and his other expenses amounted to $2.50. Find the net proceeds of the sale.

25. Find the entire cost of 1020 bu. of wheat at $0.92 a bushel, if the purchasing agent charged a commission of 2 % and the freight charges amounted to $48.50.

26. A commission merchant sold 650 doz. eggs at $0.32 a dozen. After deducting a commission of 4 % and other expenses amounting to $1.75, how much should he remit to his principal?

27. How much money should I send my agent to pay for 50 T. of bran at $22 a ton, and pay his commission amounting to 2½ %?

28. An agent sold a consignment of 8 wagons at $85 each. He charged a commission of 15 %, and paid delivery charges amounting to $12.50. Find the net proceeds.

29. A real estate agent sold a tract of 216 A. of land at $40 an acre. He charged the owner a commission of 2 %, and the transfer cost $35.50 additional; how much should be paid to the owner?

30. My agent purchased oats for me at $0.50 a bushel, charging a commission of 2 %; how many bushels did he buy if the total cost was $510?

SUGGESTION. Find the commission on the cost of 1 bu.

31. An administrator is usually paid a commission of 5 % on all personal property sold, and $2\frac{1}{2}$ % on all real estate; what commission should be paid an administrator of an estate the personal property of which sold for $625, and the real estate for $3020?

32. Find the commission charged on a bill of goods the entire cost of which was $220, if the rate of commission was 10 %.

SUGGESTION. 10 % of the cost of goods = commission.
∴ 110 % of the cost of the goods = the entire cost, or $220.

33. An agent received $2000 with which to buy flour at $4.50 a barrel. If he charged a commission of 5 %, how many barrels did he buy and what was the surplus?

34. A lawyer collected for his client 80 % of a debt of $650; if the lawyer charged a commission of 25 %, how much should he pay over to his client?

35. A dealer in farm implements sent his principal $578 as the net proceeds of a sale after he had deducted his commission of $102. What rate of commission was charged?

36. I sent my agent $512.50 with which to purchase leaf tobacco after deducting his commission of $2\frac{1}{2}\%$; how much commission should he deduct?

37. The entire cost of a carload of 100 sheep, including the agent's commission and $150 freight charges, was $1500. The sheep cost $12 a head; what rate of commission was charged?

38. A lawyer charged me a commission of 10 % for collecting a debt. If he remitted to me $585, what commission did he charge?

39. I paid $483 for 500 bu. of wheat purchased through an agent at $0.92 a bushel. What rate of commission did the agent charge?

40. The cost of a bill of goods including a commission of $\frac{3}{4}\%$ and $50.70 other expenses was $1300; find the commission charged.

SIMPLE INTEREST

218. **Interest** is money paid for the use of money.

219. The **principal** is the money for the use of which interest is paid.

220. The **rate of interest** is the rate per cent of the principal paid as interest for a certain time, usually a year.

REMARK. The rate of interest is understood to mean the rate per cent of the principal paid as interest for *one year*, unless otherwise specified.

Thus, the interest on $200 at 5 % means that 5 % of $200 is one year's interest; but the interest on $200 at 2 % per quarter means that 2 % of $200 is 3 months' interest.

221. The **amount** is the sum of the principal and the interest.

222. **Simple interest** is interest charged on the principal only.

Each state has established a **legal rate** of interest for the settlement of interest accounts when no rate of interest has been agreed upon by the parties concerned. Many of the states have also a **maximum contract rate.**

223. **Usury** is the amount of interest charged above what is legally allowed; it is unlawful interest, and there are various penalties in the different states for receiving it.

224. The time for which interest is paid is usually reckoned in *years and days when it exceeds a year, and in days only when it is less than a year, the exact number of days being counted.*

Thus, if a sum of money is on interest from April 1, 1913, to June 20, 1914, the time is counted as follows: from April 1, 1913, to April 1, 1914, is 1 yr., and from April 1, 1914, to June 20, 1914, is 80 da., making in all 1 yr. and 80 da. Again, if a sum of money is on interest from March 1, 1913, to July 1, 1913, the time is 122 da.

Remark. Settlements of interest accounts are usually made at intervals of not more than a year, so that in practice it seldom becomes necessary to reckon interest on a sum of money for a time greater than a year.

225. Interest for a time less than a year is usually reckoned on a basis of 360 days to a year; that is, 12 months of 30 days each, in which case the interest is called **common interest.**

226. Interest for a time less than a year reckoned on a basis of 365 days to the year is called **exact interest.**

NOTE. In this book, unless otherwise specified, interest will be reckoned on the basis of 30 da. to the month; that is, 1 da. will be considered $\frac{1}{360}$ of a year.

227. The following table shows the number of days from any day of one month to the same day of any other month in the same year except when Feb. 29 intervenes.

Table

FROM ANY DAY OF	TO THE SAME DAY OF											
	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
January . .	365	31	59	90	120	151	181	212	243	273	304	334
February . .	334	365	28	59	89	120	150	181	212	242	273	303
March . . .	306	337	365	31	61	92	122	153	184	214	245	275
April	275	306	334	365	30	61	91	122	153	183	214	244
May	245	276	304	335	365	31	61	92	123	153	184	214
June. . . .	214	245	273	304	334	365	30	61	92	122	153	183
July	184	215	243	274	304	335	365	31	62	92	123	153
August . . .	153	184	212	243	273	304	334	365	31	61	92	122
September .	122	153	181	212	242	273	303	334	365	30	61	91
October . .	92	123	151	182	212	243	273	304	335	365	31	61
November .	61	92	120	151	181	212	242	273	304	334	365	30
December .	31	62	90	121	151	182	212	243	274	304	335	365

Exercise 70

State at sight how much is the interest on:

1. \$100 for 1 yr. at 5%.
2. \$100 for 2 yr. at 6%.
3. \$200 for 3 yr. at $4\frac{1}{2}$%.

4. $200 for $1\frac{1}{2}$ yr. at 6%.
5. $400 for 6 mo. at 4%.
6. $600 for 1 yr. 6 mo. at 6%.
7. $1000 for 9 mo. at 4%.
8. $200 for 3 mo. at $4\frac{1}{2}$%.
9. $1000 for 2 yr. 3 mo. at 6%.
10. $100 for 2 yr. at 3% semiannually.
11. $200 for 1 yr. 6 mo. at 2% semiannually.
12. $500 for 9 mo. at 1% quarterly.
13. $600 for 1 yr. 3 mo. at 2% quarterly.
14. $1000 for 1 yr. 9 mo. at 1% quarterly.

Methods in Use Explained and Applied

The General Method

Illustrative Example

Find the interest on $360 from Jan. 6, 1913, to Sept. 3, 1915, at 6%.

(1) *Solution with the time reckoned in years and days.*

$360	The time from Jan. 6, 1913, to Jan. 6, 1915 = 2 yr.
.06	The time from Jan. 6, 1915, to Sept. 3, 1915 =
$21.60	(25 + 28 + 31 + 30 + 31 + 30 + 31 + 31 + 3) da.,
$2\frac{2}{3}$	or 240 da.; the principal is on interest 2 yr. 240
43.20	da., or $2\frac{2}{3}$ yr.
14.40	.06 × $360, or $21.60 = the interest for 1 yr.
$57.60	$2\frac{2}{3}$ × $21.60, or $57.60 = the required interest.

(2) *Solution with the time reckoned in years, months, and days.*

	yr.	mo.	da.	
$360				
.06	1915	9	3	
$21.60	1913	1	6	
$2\frac{237}{360}$	2	7	27	= 2 yr. 237 da., or $2\frac{237}{360}$ yr.
43.20				.06 × $360, or $21.60 = the interest for 1 yr.
14.22				$2\frac{237}{360}$ × $21.60, or $57.42 = the required interest.
$57.42				

The Six Per Cent Method

228. (1) $0.06 = the interest on $1 for 1 yr. (12 mo.) at 6%.

(2) 0.00\frac{1}{2}$ = the interest on $1 for 1 mo. (30 da.) at 6%.

(3) 0.000\frac{1}{6}$ = the interest on $1 for 1 da. at 6%.

Observe that in (1), (2), and (3) the principal is $1, and the rate is 6%; that in (2) $\frac{1}{2}$ of the number of months is the number of cents in the interest; that in (3) $\frac{1}{6}$ of the number of days is the number of mills in the interest. Therefore:

229. To find the interest on $1 at 6%, reckon $0.06 for each year in the time; 0.00\frac{1}{2}$ for each month in the time; 0.000\frac{1}{6}$ for each day in the time.

Illustrative Example

Find the interest on $360 from Jan. 6, 1913, to Sept. 3, 1915, at 6%.

(1) *Solution with the time reckoned in years and days.*

The time from Jan. 6, 1913, to Sept. 3, 1915, is 2 yr. 240 da.

$0.12	= the interest on $1 for 2 yr. at 6%.
0.04	= the interest on $1 for 240 da. at 6%.
$0.16	= the interest on $1 for 2 yr. 240 da. at 6%.
360	
$57.60	= the required interest.

(2) *Solution with the time reckoned in years, months, and days.*

yr.	mo.	da.	
1915	9	3	
1913	1	6	
2	7	27	= the required time.

$0.12 = the interest on $1 for 2 yr. at 6 %.
$0.035 = the interest on $1 for 7 mo. at 6 %.
$0.0045 = the interest on $1 for 27 da. at 6 %.
$0.1595 = the interest on $1 for the required time at 6 %.

360
95700
4785
$57.42 = the required interest.

230. To find the interest at:

2 %, take $\frac{1}{3}$ of the interest at 6 %.

3 %, take $\frac{1}{2}$ of the interest at 6 %.

4 %, take $\frac{2}{3}$ of the interest at 6 % (subtract $\frac{1}{3}$ of the interest at 6 %).

4$\frac{1}{2}$ %, take $\frac{3}{4}$ of the interest at 6 % (subtract $\frac{1}{4}$ of the interest at 6 %).

5 %, take $\frac{5}{6}$ of the interest at 6 % (subtract $\frac{1}{6}$ of the interest at 6 %).

7 %, take $\frac{7}{6}$ of the interest at 6 % (add $\frac{1}{6}$ of the interest at 6 %).

7$\frac{1}{2}$ %, take $\frac{5}{4}$ of the interest at 6 % (add $\frac{1}{4}$ of the interest at 6 %).

The 60-Day Method

231. (1) .06 of the principal = the interest for 1 yr. (360 da.) at 6 %.

(2) .01 of the principal = the interest for 60 da. at 6 %.

(3) .001 of the principal = the interest for 6 da. at 6 %.

From (2) and (3) we have the following:

232. To find the interest on any principal at 6 %, reckon .01 of the principal as the interest for each 60 da. in the time; and .001 of the principal as the interest for each 6 da. in the time.

Illustrative Example

Find the interest on $360 from Jan. 6, 1913, to Sept. 3, 1915, at 6 %.

(1) *Solution with the time reckoned in years and days.*

The time from Jan. 6, 1913, to Sept. 3, 1915, is 2 yr. 240 da.

$3.60 = the interest for 60 da. at 6 %.
16
2160
360
$57.60 = the interest for 2 yr. 240 da. (16 × 60 da.).

(2) *Solution with the time reckoned in years, months, and days.*

yr.	mo.	da.	
1915	9	3	
1913	1	6	
2	7	27	= the required time.

$3.60 = the interest for 60 da. at 6 %.
15
1800
360
$54.00 = the interest for 2 yr. 6 mo. (15 × 60 da.).
1.80 = the interest for 1 mo. (½ of 60 da.).
1.44 = the interest for 24 da. (4 × 6 da.).
0.18 = the interest for 3 da. (½ of 6 da.).
$57.42 = the interest for 2 yr. 7 mo. 27 da. at 6 %.

REMARK. Observe that by each of the three methods explained the interest is $57.60, when the time is counted in years and days; and $57.42 when it is counted in years, months, and days.

Method of Exact Interest

233. Exact interest is used by the United States Government, by some state governments, and by many trust companies.

234. As exact interest is based on a reckoning of 365 da. to a year, neither the six per cent method nor the 60-day method can be employed in finding the exact interest, as they are based on a reckoning of 360 da. to a year.

Illustrative Example

Find the interest on \$240 from June 1 to Dec. 3 of the same year, at 6 %.

Explanation:

The time from June 1 to Dec. 3,

$= (29 + 31 + 31 + 30 + 31 + 30 + 3)$ da. $= 185$ da., or $\frac{37}{73}$ yr.

\$240
.06
\$14.40 = the interest for 1 yr.

37
10080
4320
73)\$532.800(\$7.298+ = the interest for 185 da.

That is, \$7.30 = the interest, correct to the nearest \$0.01.

Exercise 71

NOTE. Teachers who desire to do so may use only one method of reckoning common interest throughout this exercise. They should employ the method in use in their state and count time in accordance with the practice prevailing therein. In this book the time is reckoned in years and days unless otherwise specified.

Find by the general method the interest on:

1. \$ 378 from May 1 to Dec. 31, at 4 %.

2. \$ 200 from Feb. 1, 1913, to June 20, 1915, at 6 %.

3. \$ 1000 from Sept. 1, 1913, to Dec. 1, 1914, at $4\frac{1}{2}$ %.

4. \$ 60.25 from July 15, 1913, to Jan. 5, 1915, at $5\frac{1}{2}$ %.

5. \$ 230 from Apr. 1, 1913, to Mar. 1, 1914, at $4\frac{1}{2}$ %.

6. \$ 278 from July 8, 1913, to Oct. 6, 1914, at 3 % semiannually.

Suggestion. The time is 2 half years, 90 da., or $2\frac{1}{2}$ semiannual periods.

7. \$ 500 from July 15, 1913, to Aug. 20, 1914, at $2\frac{1}{2}$ % semiannually.

8. \$ 200 from May 1 to Dec. 18, at $1\frac{1}{2}$ % per quarter.

9. \$ 285 from June 1, 1913, to Dec. 31, 1913, at $4\frac{1}{2}$ %.

10. \$ 1000 from June 1, 1913, to Sept. 15, 1914, at $2\frac{1}{2}$ % semiannually.

Find by the six per cent method the interest on:

11. \$ 450 from Mar. 15 to Dec. 31, at 6 %.

12. \$ 1000 from Mar. 1, 1913, to Oct. 16, 1914, at 6 %.

13. \$ 1250 from July 1 to Dec. 31, at $4\frac{1}{2}$ %.

14. \$ 2000 from Feb. 15, 1913, to Apr. 1, 1915, at $4\frac{1}{2}$ %.

15. \$ 25 from May 1 to Dec. 15, at $4\frac{1}{2}$ %.

16. \$ 600 from May 1, 1913, to Oct. 31, 1914, at 5 %.

17. \$ 275 from Apr. 1, 1913, to July 1, 1914, at $4\frac{1}{2}$ %.

18. \$ 480 from Jan. 15, 1916, to Dec. 1, 1916, at $4\frac{1}{2}$ %.

Find by 60-day method the interest and amount of:

19. $350 for 60 da. at 6%.
20. $480 for 120 da. at 6%.
21. $85.60 for 90 da. at 6%.
22. $1000 for 6 da. at 6%.
23. $250 for 12 da. at 6%.
24. $680 for 30 da. at 6%.
25. $478 for 90 da. at 6%.
26. $340 for 66 da. at 6%.
27. $350 for 300 da. at 6%.
28. $278 for 240 da. at 4%.
29. $39.60 for 300 da. at $4\frac{1}{2}$%.
30. $82.94 for 126 da. at 3%.
31. $100 for 200 da. at 4%.
32. $98 from Oct. 1, 1913, to May 24, 1914, at 6%.
33. $207.85 from Dec. 12, 1914, to Sept. 5, 1915, at $3\frac{3}{4}$%.

Find the exact interest on:

34. $300 for 73 da. at 6%.
35. $300 for 150 da. at 6%.
36. $375 from June 1 to Dec. 20, at 5%.
37. $850 from May 1 to Dec. 31, at $4\frac{1}{2}$%.
38. $85 from June 1, 1913, to July 15, 1914, at $4\frac{1}{2}$%.

Indirect Problems in Simple Interest

Finding the Principal

Illustrative Example

1. What principal will amount to $595.20 in 2 yr. 200 da. at 4%?

$0.04
$2\frac{5}{9}$
$0.10\frac{2}{9}$
1.00
$1.10\frac{2}{9}$) $595.20
9 9
$9.92) $5356.80 (540

Explanation:

$0.04 = the int. on $1 for 1 yr.

2 yr. 200 da., or $2\frac{5}{9}$ yr. = the time.

$\therefore 2\frac{5}{9} \times \$0.04$, or $\$0.10\frac{2}{9}$ = the interest on $1 for the time.

And $\$1.10\frac{2}{9}$ = the amount of $1 for the time.

$595.20 = the entire amount.

$\therefore \$595.20 \div \$1.10\frac{2}{9}$, or 540 = the number of dollars in the principal.

$\therefore$ $540 = the principal.

Exercise 72

1. What principal will produce $24 interest in 2 yr. at 6%?

2. What principal will produce $6 interest in 6 mo. at 4%?

3. What principal will produce $25 interest in 1 yr. 3 mo. at 4%?

4. What principal will amount to $336 in 3 yr. at 4%?

5. What principal will amount to $132 in 2 yr. 6 mo. at 4%?

6. What principal will earn $13.50 interest from Mar. 15 to Dec. 10, at 4%?

7. What principal will earn $15 interest from Apr. 1, 1913, to July 30, 1914, at 5%?

8. What principal will amount to $300 from June 1, 1912, to Dec. 31, 1913, at 6%?

9. What principal will amount to $1000 from May 1 to Dec. 2, at 6% exact interest?

235. The **present worth** of a debt due at some future time is such a sum as placed at interest will amount to the debt when due. The **true discount** is the difference between the debt and the present worth.

236. *To find the present worth of a debt is to find the principal.*

Thus, the illustrative example on page 197 might be stated thus: Find the *present worth* of $595.20 due 2 yr. 200 da. hence at 4%. The debt would then be $595.20, the present worth $540, and the true discount $595.20 − $540, or $55.20.

Finding the Rate

Illustrative Example

At what rate will $300 earn $24 in 2 yr. 60 da.?

Explanation:

.01 of $300, or $3 = the interest for 1 yr. at 1%.

2 yr. 60 da., or $2\frac{1}{6}$ yr. = the time.

$\therefore 2\frac{1}{6} \times \$3$, or $6.50 = the interest for the time at 1%.

$24 = the given interest.

$24 \div 6.50 = 3.692^{+}$.

$\therefore 3.69\%$ = the required rate, correct to the nearest .01%.

$$\begin{array}{r} \$300 \\ .01 \\ \hline \$3.00 \\ 2\frac{1}{6} \\ \hline \$6.50)\$24.00(3.692^{+} \end{array}$$

Exercise 73

At what rate will:

1. $320 earn $17.28 in 1 yr. 72 da.?

2. $1000 earn $138 in 2 yr. 315 da.?

3. $456 earn $4.56 from Aug. 1 to Oct. 12?

4. $100 amount to $104.60 from July 20, 1913, to Sept. 12, 1914?

SUGGESTION. $104.60 − $100, or $4.60 = the interest.

5. $425 amount to $446.25 from Jan. 1, 1914, to Apr. 1, 1915?

6. $400 amount to $416.80 from Feb. 8 to Dec. 20?

7. $500 amount to $508 at exact interest from Oct. 1 to Feb. 24?

8. Any sum double itself in 20 yr.?

SUGGESTION. 100 % of principal = the interest.
20 % of principal = the interest for 20 yr. at 1 %.

9. Any sum double itself in 15 yr.?

Finding the Time

Illustrative Example

In what time will $400 amount to $438.40 at 4 %?

$400 $438.40
.04 400.00
$16.00) $38.400(2.4

Explanation:

.04 of $400, or $16 = interest for 1 yr.
$438.40 − $400, or $38.40 = the entire interest.
∴ 38.40 ÷ 16, or 2.4 = the number of years.
.4 yr. = .4 of 360 da., or 144 da.
∴ 2 yr. 144 da. = the required time.

Exercise 74

In what time will:

1. \$500 earn \$72 interest at $4\frac{1}{2}\%$?

2. \$1000 earn \$138 interest at $4\frac{4}{5}\%$?

3. \$200 amount to \$208 at 4%?

4. \$425 amount to \$446.25 at 4%?

5. \$250 amount to \$300 at 6%?

6. \$200 amount to \$300 at $4\frac{1}{2}\%$?

7. On what date will \$200 have amounted to \$208.10, if loaned Jan. 1 at 6%?

8. On what date will \$200 have amounted to \$203.50, if loaned Mar. 1 at $4\frac{1}{2}\%$?

9. In what time will any sum double itself at 4%?

SUGGESTION. 100% of the principal = the interest.
4% of the principal = the interest for 1 yr. at 4%.

10. In what time will any sum double itself at 5%?

11. In what time will any sum earn $\frac{1}{2}$ of itself at $4\frac{1}{2}\%$?

COMPOUND INTEREST

237. If interest is added to the principal at the end of each period as it becomes due, to form a new principal for the next period, the entire interest, that is, the difference between the final amount and the original principal, is the **compound interest** of the original principal.

238. The **compound amount** is the final amount when compound interest is reckoned.

239. Table of Compound Interest

AMOUNT OF $1 FOR EACH OF 20 PERIODS AT RATES FROM ¾% TO 6%

PERIODS	¾ PER CENT	1 PER CENT	1¼ PER CENT	1½ PER CENT	2 PER CENT
1	1.00750000	1.01000000	1.01250000	1.015000	1.020000
2	1.01505625	1.02010000	1.02515625	1.030225	1.040400
3	1.02266917	1.03030100	1.03797070	1.045678	1.061208
4	1.03033919	1.04060401	1.05094533	1.061364	1.082432
5	1.03806673	1.05101005	1.06408215	1.077284	1.104081
6	1.04585223	1.06152015	1.07738318	1.093443	1.126162
7	1.05369612	1.07213535	1.09085047	1.109845	1.148686
8	1.06159884	1.08285670	1.10448610	1.126493	1.171660
9	1.06956083	1.09368527	1.11829217	1.143390	1.195093
10	1.07758254	1.10462212	1.13227082	1.160541	1.218994
11	1.08566441	1.11566834	1.14642421	1.177949	1.243374
12	1.09380689	1.12682503	1.16075451	1.195618	1.268242
13	1.10301044	1.13809328	1.17526389	1.213552	1.293607
14	1.11027552	1.14947421	1.18995469	1.231756	1.319479
15	1.11860259	1.16096895	1.20482913	1.250232	1.345868
16	1.12699211	1.17257864	1.21988949	1.268985	1.372786
17	1.13544455	1.18430443	1.23513811	1.288020	1.400241
18	1.14396038	1.19614747	1.25047734	1.307341	1.428246
19	1.15254009	1.20810895	1.26610830	1.326951	1.456811
20	1.16118414	1.22019003	1.28193466	1.346855	1.485947

Table of Compound Interest — *Continued*

PERIODS	2½ PER CENT	3 PER CENT	3½ PER CENT	4 PER CENT	5 PER CENT	6 PER CENT
1	1.025000	1.030000	1.035000	1.040000	1.050000	1.060000
2	1.050625	1.060900	1.071225	1.081600	1.102500	1.123600
3	1.076891	1.092727	1.108718	1.124864	1.157625	1.191016
4	1.103813	1.125509	1.147523	1.169859	1.215506	1.262477
5	1.131408	1.159274	1.187686	1.216653	1.276282	1.338226
6	1.159693	1.194052	1.229255	1.265319	1.340096	1.418519
7	1.188686	1.229874	1.272279	1.315932	1.407100	1.503630
8	1.218403	1.266770	1.316809	1.368569	1.477455	1.593848
9	1.248863	1.304773	1.362897	1.423312	1.551328	1.689479
10	1.280085	1.343916	1.410599	1.480244	1.628895	1.790848
11	1.312087	1.384234	1.459970	1.539454	1.710339	1.898299
12	1.344889	1.425761	1.511069	1.601032	1.795856	2.012197
13	1.378511	1.468534	1.563956	1.665074	1.885649	2.132928
14	1.412774	1.512590	1.618695	1.731676	1.979932	2.260904
15	1.448298	1.557967	1.675349	1.800944	2.078928	2.396558
16	1.484506	1.604706	1.733986	1.872981	2.182875	2.540352
17	1.521618	1.652848	1.794676	1.947901	2.292018	2.692773
18	1.559659	1.702433	1.857489	2.025817	2.406619	2.854339
19	1.598650	1.753506	1.922501	2.106849	2.526950	3.025600
20	1.638616	1.806111	1.989789	2.191123	2.653298	3.207136

Exercise 75

[In solving the examples of this exercise use table, page 201.]

1. Find the compound amount and the compound interest of $600 for 4 yr. at 3%.

SUGGESTION. $1.125509 = the compound amount of $1 for 4 yr. at 3%.

2. Find the compound amount and the compound interest of $500 for 2 yr. 6 mo. at 6%, interest payable semiannually.

SUGGESTION. 2 yr. 6 mo. = 5 semiannual periods. The semiannual rate = 3%. Find by the table the compound amount of $1 for 5 periods at 3%.

3. Find the compound amount and the compound interest of $500 for 2 yr. at 5%, interest payable quarterly.

SUGGESTION. 2 yr. = 8 quarterly periods. The quarterly rate is $1\frac{1}{4}$%.

Find the compound amount and the compound interest of:

4. $400 for 3 yr. at 6%.
5. $300 for 4 yr. at 4%.
6. $250 for 5 yr. at $3\frac{1}{2}$%.
7. $3000 for 6 yr. at $2\frac{1}{2}$%.
8. $2000 for 3 yr. 6 mo. at 5%, payable semiannually.
9. $750 for 2 yr. 6 mo. at 5%, payable semiannually.
10. $2000 for 2 yr. 3 mo. at 8%, payable quarterly.

TAXES

240. A **tax** is money paid by the people for the expenses of the government.

241. **Local taxes** are those paid to counties, townships, cities, or towns, such as school taxes or road taxes.

242. **State taxes** are paid to a state for its expenses; but some states divide a part of the state taxes among the counties, townships, cities, and towns.

243. **National taxes** are paid to the Treasurer of the United States at Washington to maintain the army and navy, and to pay pensions and other expenses of the national government.

244. A **poll tax** (head tax) is a low tax collected in some states from every male citizen over twenty-one years of age.

245. An **assessor** is an officer who assesses, that is, places a value upon property for taxation. This assessment, or tax value, is often less than the real value of the property.

246. A **tax rate** is usually a certain number of mills on each dollar of the assessed value of property.

247. A tax rate is sometimes expressed as a certain per cent of the assessed value of property, or as a certain amount on each $100 of its assessed valuation.

248. Most local and state taxes are paid by the owners of property at a rate or percentage on the value of their property. The tax rate is so fixed by the state or local officers as to raise the amount of money needed by the state or local governments.

249. National taxes are of two classes, **tariff** or **duties**, which are laid upon articles imported from other countries; and **internal revenue taxes**, which are levied upon a few things made in this country, usually upon alcoholic liquors and tobacco.

250. Duties on imports and internal revenue taxes are **indirect taxes**, because the importer or manufacturer who pays them adds that to the cost of the article imported or made, so that the people who buy and use these articles pay these taxes *indirectly*.

251. **Direct taxes** are taxes which cannot be shifted upon somebody else. Local taxes and state taxes are almost always direct taxes.

252. Duties are of two kinds: **specific duties**, which are those paid on the *quantity* of the imports; and **ad valorem duties**, which are those paid on the *value* of the imports.

253. Some goods are imported free of duty, some are subject to specific duty only, some to ad valorem duty only, and some to both kinds of duty.

Illustrative Example

In a town whose property is assessed at $350,000 and in which there are 420 persons who pay a poll tax of $1.50 each, $1855 is to be raised by taxation. What rate of taxation must be levied, and what tax must George Fox pay on property assessed at $7500?

Explanation:

$420 \times \$1.50$, or $\$630$ = the poll tax.
$\$1855 - \$630$, or $\$1225$ = the property tax.
$\$1225 \div 350{,}000$, or $\$0.0035$ = the tax on $\$1$.
$\therefore$ 3.5 mills = the tax rate.
$7500 \times \$0.0035$, or $\$26.25$ = George Fox's property tax.

Exercise 76

Find the tax on property the tax rate and valuation of which are as follows:

	Valuation	Tax rate		Valuation	Tax rate
1.	$1000	2 mills	2.	$5000	3 mills
3.	$2700	5 mills	4.	$7500	7 mills
5.	$2250	8 mills	6.	$8750	$4\frac{1}{2}$ mills
7.	$1400	12 mills	8.	$7000	$8\frac{1}{2}$ mills
9.	$8500	$12\frac{1}{2}$ mills	10.	$10,000	$1\frac{1}{2}$ mills

In the following find the tax rate expressed in mills on each dollar of valuation:

	Valuation	Tax		Valuation	Tax
11.	$1000	$1	12.	$10,000	$60
13.	$2000	$8	14.	$2500	$40
15.	$1500	$7.50	16.	$1400	$12.60
17.	$2400	$14.40	18.	$2200	$17.60
19.	$25,000	$40	20.	$5000	$7.50

Find the duty on the following

	Imports	Duty
21.	$600 worth of cut glassware	60% ad valorem
22.	300 cu. ft. of marble	65¢ per cubic foot
23.	$250 worth of sponges	20% ad valorem
24.	4 T. (2240 lb.) maple sugar	4¢ per pound
25.	1200 bu. oats	15¢ per bushel
26.	A $5000 painting	15% ad valorem

27. Find the duty on 420 lb. of drugs valued at $1.20 per pound, on which there is a duty of $1\frac{1}{4}¢$ per pound and 10 % ad valorem.

28. Find the duty on 420 yd. of carpet, $\frac{3}{4}$ yd. wide and valued at $1.20 per yard, on which there is a duty of 10¢ per square foot and 40 % ad valorem.

29. Find a farmer's total taxes for a year on 75 A. of land assessed at $20 per acre, if the rate of county tax levied is $2\frac{1}{2}$ mills, the rate of school tax $7\frac{1}{2}$ mills, and the rate of road tax 5 mills.

30. The assessed valuation of the property of a certain town is $480,000. If the school tax amounting to $2160 is to be raised, what must be the rate of taxation?

31. The assessed valuation of the property of a certain town is $420,000, upon which a tax of $3150 is to be raised; what must be the rate of taxation, and how much tax must B. F. Hess pay on property assessed at $7500?

32. When the tax rate is 12 mills, and a man pays a tax of $37.50 including $1.50 poll tax, what is the assessed value of his property?

33. A tax of $17,700 is to be raised in a county whose property is assessed at $3,650,000 and 850 polls. If the polls are assessed at $1.50 each, what must be the rate of taxation?

34. A tax collector paid over to the school authorities of a certain township $7095.20 school tax, after he had taken out his commission of 2 % for collecting; what amount of tax did he collect, and what was his commission?

35. Find the duty on an importation of goods from Paris, dutiable value 1850 francs, at 60 % ad valorem.

NOTE. All values of imported merchandise are reduced to U. S. money on the basis of the exact money equivalents given in the table under Section 194, before the duty is reckoned. Amounts less than 50 ¢ are rejected, and those equal to or greater than 50 ¢ are considered one dollar in the total sum upon which duty is charged.

36. Find the duty on a shipment of German toys, dutiable value 18,750 marks, at 55 % ad valorem.

37. Find the duty on 6 cwt. 3 qr. 20 lb. of wool, whose dutiable value is 30 cents per pound, and on which is charged a specific duty of $27\frac{1}{2}$ ¢ per pound and 40 % ad valorem.

NOTE. See table, page 361.

PROPERTY INSURANCE

254. Property insurance is a contract by which, for a consideration, one party agrees to indemnify another in case of loss.

255. Insuring property is called **taking a risk**. The party taking the risk, which is usually a company, is called the **insurer,** or **underwriter**, and the party whose property is insured is called the **insured**. Risks are usually taken for periods of one, two, three, four, or five years, or for an unlimited number of years, in which case the risk is called a **perpetual risk**.

256. The written contract between the insured and the insurance company is called the **policy**. The amount of

indemnity named in the **policy** is called the **face of the policy**. The amount charged for insuring property is called the **premium**.

257. The **annual rate of premium** is the number of cents charged for insuring property to the amount of one hundred dollars for one year.

258. Insurance rates are sometimes expressed in per cent. Rates for 2 yr., 3 yr., 4 yr., and 5 yr. are generally $1\frac{3}{4}$, $2\frac{1}{2}$, $3\frac{1}{4}$, and 4 times the annual rate.

259. Short rates are rates for less than a period, and are usually expressed as certain per cents of the periodic rates. Short rates on perpetual risks are usually 10 per cent of the premium paid on such risks.

260. A policy may be cancelled at any time at the request of the insured, in which case the company retains the customary short rate for the time the policy has been in force.

261. Property insurance is of various kinds, such as **fire insurance**, which is insurance against loss by fire; **marine insurance**, which is insurance against loss of property at sea; **hail insurance**, which is insurance against loss of property by hail.

Exercise 77

1. A barn was insured for \$ 2800 at 40¢ per \$ 100 for 1 year; find the premium.

2. A house was insured for \$ 2400 at 20¢ per \$ 100 for 1 year; find the premium.

3. A house was insured for \$ 2200 for 3 years, the rate for 3 years being $2\frac{1}{2}$ times the annual rate; what was the premium, if the annual rate was $12\frac{1}{2}$¢ per \$ 100?

4. A house was insured for $3200 for 4 years, the rate for 4 years being $3\frac{1}{4}$ times the annual rate; what was the premium if the annual rate was 16¢ per $100?

5. A company that charges 75¢ per $1000 for insuring brick houses should charge how much for insuring for a period of 5 years a brick house valued at $5000, the rate for 5 years being 4 times the annual rate?

6. A man had his library insured for each of 5 years at 40¢ per $100 per year; how much would he have saved had he had it insured at first for a period of 5 years, the insured value being $2000 and the rate for 5 years being 4 times the annual rate?

7. A man had a quantity of wheat insured for $1000 for 1 year, but at the end of 60 days he sold the wheat and cancelled the policy; how much should the company return, if the annual premium charged was 85¢ per $100 and the company retains as the short rate for 60 days 30% of the annual premium?

8. A company that is charging 16¢ per $100 insurance for 1 year is charging what rate per cent? What rate per $1000 is it charging?

9. If a building is insured for $4500 and the annual premium is $18, find the annual rate per $100.

10. A church is insured for $10,000 on the perpetual plan at $4 per $100; what premium was paid and how much would the insurance company retain if the policy should be cancelled, in which case the company would retain 10% of the premium paid?

LIFE INSURANCE

262. Life insurance is a contract whereby a life insurance company agrees to pay, in consideration of a certain payment or payments, a sum of money to a certain party called the *beneficiary*.

The beneficiary may be the insured, his estate, or any other party named in the policy.

Life insurance policies are payable at the death of the insured or at the expiration of a certain period.

263. There are three general kinds of life insurance policies issued, and several modifications of each; namely, *Ordinary Life*, *Limited Payment Life*, and *Endowment*.

264. Ordinary life policies provide for the payment of the insurance at the death of the insured, and require premiums to be paid during the entire life.

265. Limited payment policies provide for the payment of the insurance at the death of the insured or for the cancelling of the policy at the end of the insurance period in accordance with a plan or plans provided for in the policy, on condition that the insured outlives the insurance period.

266. Endowment policies provide for the payment of the insurance at the end of a stated period, should the insured live so long; or for the payment thereof at the death of the insured, if it happens within the endowment period. As in the "Limited Life," the premiums cease at the end of a stipulated period, usually in 10, 15, 20, 25, or 30 years.

267. Life insurance premiums are given in dollars and cents per $1000 insurance for one year, and are uniform throughout the insurance period.

The age of the insured is his age at his nearest birthday.

268. The following table shows the annual premiums charged by a leading company for $ 1000 insurance by the three plans in general use.

Age	Ordinary Life	Limited Payment Life			Endowment		
		10-Payment	15-Payment	20-Payment	20-Year	25-Year	30-Year
21	18.76	46.98	34.73	28.73	48.45	38.03	31.36
22	19.18	47.69	35.26	29.18	48.54	38.14	31.48
23	19.61	48.43	35.82	29.65	48.64	38.25	31.62
24	20.07	49.19	36.39	30.13	48.75	38.37	31.76
25	20.55	49.97	36.98	30.63	48.86	38.50	31.92
26	21.05	50.78	37.60	31.15	48.98	38.65	32.09
27	21.57	51.62	38.23	31.69	49.12	38.80	32.27
28	22.12	52.49	38.89	32.25	49.26	38.96	32.48
29	22.70	53.39	39.57	32.83	49.41	39.15	32.70
30	23.31	54.31	40.28	33.44	49.57	39.34	32.94
35	26.88	59.46	44.23	36.85	50.64	40.64	34.56
40	31.56	65.58	49.03	41.10	52.33	42.76	37.18
45	37.82	72.94	54.99	46.57	55.15	46.25	41.45
50	46.36	81.96	62.62	53.86	59.87	52.02	48.28

Exercise 78

[In answering questions in this exercise refer to table, Sec. 268.]

Find the annual premium on a $ 1000 policy, when the age of the insured and the kind of policy are as follows:

Age of Insured	Kind of Policy
1. 21	10-Payment Life
2. 28	20-Year Endowment
3. 30	Ordinary Life

Age of Insured	Kind of Policy
4. 40	15-Payment Life
5. 28	25-Year Endowment
6. 25	Ordinary Life
7. 30	20-Payment Life
8. 25	20-Payment Life
9. 22	30-Year Endowment

10. Find the annual premium on a $2000 ordinary life policy taken out when the insured was 30 years old.

11. Find the annual premium on a $5000 twenty-year endowment policy taken out when the insured was 29 years old.

12. Find the annual premium that a man must pay on a twenty-year endowment policy for $5000 taken out Dec. 1, 1912, if the insured was born July 15, 1882.

REVIEW

Exercise 79

1. When spices are bought at $1.20 per pound and sold at 15¢ per ounce, what is the gain per cent?

2. A retailer buys oilcloth at 75¢ per square yard, subject to a discount of $16\frac{2}{3}\%$ and 4%, and sells it at 80¢ per square yard; find his gain per cent.

3. An agent charged $12.40 for selling a consignment of poultry at a commission of 4%; how much should he return to the consignor after taking out his commission and paying storage amounting to $4.80?

4. A dealer marks goods at 50¢ per yard, but he throws off 10% and still makes a profit of 50%; find the cost of the goods per yard.

5. A tradesman's prices are 25% above cost; if he allows a discount of $2 on a bill of $20, find his per cent of profit.

6. A dealer marked an article at 12¢, which was 50% more than the cost; if he sold it at a discount of 2¢, what was his gain per cent?

7. When an article is bought at a discount of 25% from the marked price and sold at the marked price, what is the gain per cent?

8. A chafing dish was marked at $5, which was an advance of 56¼% of the cost. If it was sold for $3.60, find gain or loss per cent.

9. What inheritance tax must be paid in Michigan on $7500 which a man left to his wife and children, on which legacy there is a tax of 1%; and on $2000 left to a charitable institution, on which legacy there is a tax of 5%?

10. When shoes marked at $3.50 per pair are bought at a discount of 20% and sold at the marked price, what is the gain per cent?

11. When 40¢ interest is charged per month for the use of $100, what is the annual rate?

12. A bushel of clover seed (60 lb.) costing $9 was found to be 25% weed seed; what price was really paid per bushel for the pure clover seed?

13. If the weight of a dressed turkey is 75 % of its live weight, which is the more profitable to the farmer, to sell a 16-lb. turkey, live weight, at 16 ¢ per pound, or at 25 ¢ per pound dressed?

14. What rate per cent of tax is levied when $1.40 is levied to the $100?

15. If a piece of cloth 6 yd. long and 54 in. wide shrunk $16\frac{2}{3}$ % in length and in width on being sponged, how many square yards did the piece then contain?

16. Find the duty on a painting whose dutiable value is 60,000 lire, at 15 % ad valorem.

17. What must the marked price of shirts be apiece that cost $7.20 per dozen, so that a dealer may allow a discount of 25 % and still make a profit of 25 %?

18. A piece of gold jewelry that is 18 carats fine is what per cent pure gold?

19. What per cent does a dealer gain on nails that he buys at $3.20 per keg (100 lb.) and retails at 5 ¢ per pound, if they lose 2 lb. in the retailing?

20. Find the exact interest on $250 from June 1 to December 15 at 4.5 %.

21. How much will a man pay in premiums who takes out at the age of 28 yr. a 20-year endowment policy for $5000, provided that he lives out the endowment period? (See table, page 211.)

22. A farmer calls his four cows Red, Spot, Bess, and Beauty. Red gives 22 lb. of milk daily which tests 4.2 %;

Spot gives 24 lb. of milk daily which tests 4.1 %; Bess gives 28 lb. of milk daily which tests 3.5 %; Beauty gives 30 lb. of milk daily which tests 3.0 %. With butter fat selling at 36¢ per pound, what is the income from each cow during the month of June? Which is the most profitable cow for the farmer to keep, provided that he feeds them equal amounts?

23. A barrel of 42 gal. of vinegar was bought for $4.20; what must it be retailed for per gallon to gain 20 %?

24. What is the simple interest on $200 from June 1, 1912, to Oct. 15, 1913, at 4.5 %?

25. When buttons are bought at 60¢ per gross and sold at 10¢ per dozen, what is the gain per cent?

26. What per cent does a merchant gain who buys coal at $6 per ton of 2240 lb. and retails it at $7.50 per ton of 2000 lb.?

27. A man borrowed $50 from a loan company in Pennsylvania March 1, and paid back $55 August 1; how much of this interest was usury? (See table, page 254.)

28. When goods are bought at a discount of 20 % and 10 % from the marked price and sold at 8 % from the marked price, what is the gain per cent?

29. I invest and sell at a loss of $37\frac{1}{2}$ %; I invest the proceeds and sell at a gain of 80 %; did I gain or lose, and what per cent?

30. If a mail clerk in distributing 12,000 pieces of mail matter makes 7 errors, what is the per cent of error, correct to the nearest .01 %?

31. How much fire tax should be paid on an insurance of $ 3250, when the rate is 20¢ per $ 100 of insurance?

32. The assessed valuation, in a recent year, of the taxable property of Denver, Col., was $ 132,909,000, which was 50 % of its actual value. The tax rate levied for the year was $ 3.50 per $ 100 of assessed value. What amount of tax was levied, and what was the actual value of the taxable property?

33. Find the commission charged on a bill of goods the entire cost of which was $ 230, if the rate of commission was 10 % and freight charges amounted to $ 10.

34. A dealer gained $33\frac{1}{3}$ % when he sold bananas at 20¢ per dozen; what per cent did he gain on those that he sold at 2¢ each?

35. After deducting a commission of $\frac{3}{4}$ % and $ 25.80 for other expenses, an agent sent his principal $ 4500 as the net proceeds of a sale; find the commission charged.

36. When the duty on goods of a certain kind is 10¢ per square foot and 40 % ad valorem, find the duty on a piece of these goods 6 yd. long and 4 yd. wide and valued at $ 120.

37. A laborer saved $ 152.50 in three years; his savings the second year were 25 % more than those of the first, and those of the third, 25 % more than those of the second; how much did he save each year?

STOCKS AND BONDS

269. A **corporation** is a body doing business as a single person under a general law or a special charter.

270. The capital of a corporation is called **stock.** The stock of a corporation is divided into a number of equal parts called **shares.** The face or original value of a share is its **par value.**

271. The par value of a share of stock is usually $50 or $100. Unless a different par value is expressed, $100 is the par value used in the examples of this book.

A Certificate of Stock

272. A **certificate of stock** is a document specifying the number of shares to which the holder is entitled, and the par value of each.

273. The price at which stock sells in the market is the **market value.**

274. The market price of stock is usually quoted at a certain per cent of its par value, but sometimes the market price of a share is quoted.

Thus, on a certain day, Reading R.R. stock, par $50, was quoted in the New York market at $170\frac{1}{4}$, which means that it was sold on that day at $170\frac{1}{4}\%$ of $50, or $85\frac{1}{8}$ per share. The same stock selling at the same price was quoted in the Philadelphia market at $85\frac{1}{8}$, meaning that it was sold at $85\frac{1}{8}$ per share.

275. A **dividend** is that part of the profits of a corporation which is divided among the stockholders. It is usually a certain per cent of the par value of the stock.

Thus, if a 6% dividend is declared on stock, par value $50, the owners of the stock are paid 6% of $50, or $3, on each share owned.

276. **Preferred stock** is stock on which dividends, at a fixed rate, are payable before those on the **original** or **common stock** are paid.

277. A **bond** is a written obligation of a corporation or a government to pay a specified sum of money at a certain time, together with interest at regular intervals at a fixed rate per cent per annum.

278. Government bonds are of two classes, **coupon** and **registered.**

279. A **coupon bond** is a bond made payable to the bearer, and, when sold, passes by delivery without assignment. It has attached to it interest coupons payable to the bearer

No. 99

STATE OF PENNSYLVANIA
COUNTY OF CHESTER
Borough of West Chester

500

GENERAL LOAN BOND

SERIES TWO

This is to Certify, That the Borough of West Chester, County of Chester, and State of Pennsylvania, is indebted to bearer, or if registered to the registered holder hereof, in the sum of Five Hundred Dollars, payable in lawful money of the United States, on the **First Day of January, A D. 1942,** at the office of the Treasurer of the Borough of West Chester, in said Borough, upon surrender hereof, with interest thereon at the rate of four per centum per annum, payable semi-annually, upon the first days of January and July, until said principal sum is paid at the Treasurer's office upon the presentation and surrender of the annexed coupon, free from all taxes levied or to be levied upon this bond under any law of the State of Pennsylvania, now enacted or hereafter to be enacted, and for the payment of the said principal and interest the faith, honor and credit of the said Borough of West Chester are solemnly pledged.

This obligation is one of a series of one hundred and sixty obligations, of like tenor and date, of an authorized loan of Sixty Thousand Dollars, Bonds numbered from one to one hundred and ten inclusive, being for the sum of Five Hundred Dollars each, and bonds numbered from one hundred and eleven to one hundred and sixty inclusive, being for the sum of One Hundred Dollars each, issued in pursuance of an Ordinance duly enacted by the Council of said Borough and approved by the Burgess thereof on the 30th day of August, A.D. 1912, signifying the desire of the Borough of West Chester to increase its indebtedness by the sum of Sixty Thousand Dollars, and a Resolution of said Council likewise approved, passed on the 27th day of September, A.D. 1912, providing for the holding of an election at the time of the holding of the general election in said Borough on the Fifth day of November, A.D. 1912, for the purpose of obtaining the consent of the qualified electors of the Borough, to such increase of indebtedness, and another Resolution of said Council likewise duly approved, passed on the 27th day of November, A.D. 1912, after the holding of said election at which a majority of said electors consented to the increase of said indebtedness; said indebtedness being incurred for the purpose of providing funds for the improvement of the equipment of the fire department, for the extension and improvement of the water system, and for street improvements and the removal of garbage. All of said bonds being valid and binding obligations of the said Borough and in the issuing thereof all requirements of law have been complied with.

The said Borough of West Chester reserves the right to redeem all bonds of said issue, maturing at and after ten years from date at par, at any interest paying period, all accrued interest having been paid thereon, upon giving personal notice to the holder four weeks prior to the time of redemption, if the bond is registered, and if not registered, either by personal notice as aforesaid, or by an advertisement inserted once a week for four successive weeks, immediately prior to the date of said redemption, in a newspaper of general circulation published in said Borough, having respect to the lowest number in both denominations, in numerical order in making such redemption.

This bond shall pass by delivery, but any holder may have the principal thereof registered in the Office of the Treasurer of said Borough, and after registration no transfer except upon the books of the Borough shall be valid unless the last preceeding transfer shall have been restored, but no registry shall restrain the negotiability of the coupon by delivery merely.

IN WITNESS WHEREOF, the said "BOROUGH OF WEST CHESTER, COUNTY OF CHESTER, PENNSYLVANIA," has caused these presents to be sealed with its corporate seal attested by its Secretary, and its Chief Burgess has hereunto set his hand this First day of January, A.D. 1913.

Borough of West Chester, County of Chester, Pennsylvania.

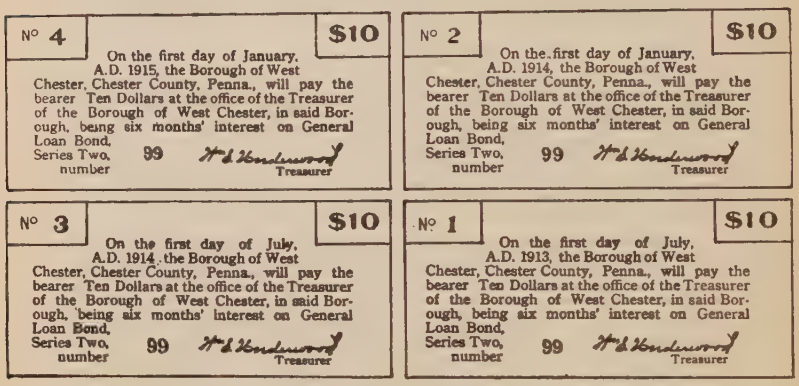
Flemmer E. Jefferis
Chief Burgess

Attest.

Wm S Underwood
Secretary

A Coupon Bond

at specified times. When interest is due, a coupon is cut off and presented to a bank for payment.


No 4
$10
On the first day of January, A.D. 1915, the Borough of West Chester, Chester County, Penna., will pay the bearer Ten Dollars at the office of the Treasurer of the Borough of West Chester, in said Borough, being six months' interest on General Loan Bond, Series Two, number 99
Treasurer
No 2
$10
On the first day of January, A.D. 1914, the Borough of West Chester, Chester County, Penna., will pay the bearer Ten Dollars at the office of the Treasurer of the Borough of West Chester, in said Borough, being six months' interest on General Loan Bond, Series Two, number 99
Treasurer
No 3
$10
On the first day of July, A.D. 1914, the Borough of West Chester, Chester County, Penna., will pay the bearer Ten Dollars at the office of the Treasurer of the Borough of West Chester, in said Borough, being six months' interest on General Loan Bond, Series Two, number 99
Treasurer
No 1
$10
On the first day of July, A.D. 1913, the Borough of West Chester, Chester County, Penna., will pay the bearer Ten Dollars at the office of the Treasurer of the Borough of West Chester, in said Borough, being six months' interest on General Loan Bond, Series Two, number 99
Treasurer

Interest Coupons

280. A **registered bond** is a bond issued without interest coupons, filled up in the name of the registered owner, made payable to him or his assigns, and registered in his name.

NOTE. Interest on registered bonds is sent to the registered owner or to his attorney as it becomes due.

281. Bonds may sell above or below their face value, and are usually quoted in the market at a certain per cent of their face value. Bonds are usually named from the rate of interest they pay and the year in which they are payable.

Thus, "U. S. 4s reg., 1925" means U. S. Government bonds registered, bearing 4 % interest and payable in 1925.

282. **Brokerage** is the commission charged by a broker for buying or selling stocks or bonds for another person. It is reckoned on the par value, and the usual rate is $\frac{1}{8}$ %.

283. Stock is usually bought and sold in a place called the stock exchange, which is merely an auction room where brokers having stocks to sell endeavor to dispose of them to other brokers having orders to buy such stocks. $\frac{1}{8}\%$ of the par value is usually charged by the broker selling the share and by the broker buying the share.

Illustrative Examples

1. Find the cost of 10 shares of Penn. R.R. stock ($50) at $61\frac{5}{8}$ a share, brokerage $\frac{1}{8}\%$.

Explanation:

$50 = the par value of 1 share.
$\frac{1}{8}\%$ of $50, or $0.0625 = the brokerage on 1 share.
$61.625 = the market value of 1 share.
$61.6875 = the entire cost of 1 share.
$\therefore 10 \times \$61.6875$, or $616.88 = the entire cost of 10 shares.

2. Find the proceeds of 8 shares of Reading R.R. stock ($50) quoted at $171\frac{1}{4}\%$, brokerage $\frac{1}{8}\%$.

Explanation:

$50 = the par value of 1 share.
$171\frac{1}{4}\%$ of $50, or $85.625 = the market value of 1 share.
$\frac{1}{8}\%$ of $50, or $0.0625 = the brokerage on 1 share.
$85.5625 = the net proceeds of 1 share.
$\therefore 8 \times \$85.5625$, or $684.50 = the net proceeds of 8 shares.

3. A sent his broker $6000 to be invested in Southern Pacific R.R. stock at $111\frac{1}{8}\%$, brokerage $\frac{1}{8}\%$. How many shares did this sum buy, and what was the surplus?

Explanation:

$100 = the par value of 1 share.
$\frac{1}{8}$ % of $100, or $0.125 = the brokerage on 1 share.
$111.125 = the market value of 1 share.
∴ $111.25 = the entire cost of 1 share.
$6000 = the amount A sent for investment.
∴ 6000 ÷ 111.25, or 53 = the number of shares bought, with a surplus of $103.75.

4. Find the annual income on 25 shares of United Gas Improvement stock ($ 50), which pays a quarterly dividend of 2 %.

Explanation:

$50 = the par value of 1 share.
2 % of $50, or $1 = the quarterly dividend on 1 share.
$4 = the annual income on 1 share.
∴ 25 × $4, or $100 = the annual income on 25 shares.

5. What rate per cent of income from his investment does a man receive on 6 % stock, par value $ 50, if he purchased it when its market value was $ $56\frac{3}{4}$ per share, brokerage $\frac{1}{4}$ %? Answer correct to the nearest .01 per cent.

Explanation:

6 % of $50, or $3 = the income per share.
$\frac{1}{4}$ % of $50, or $0.125 = the brokerage per share.
∴ $56.75 + $0.125, or $56.875 = the cost per share.
∴ $3 = the income on $56.875 invested.
$0.56875 = the income per share if the rate were 1 %.
3 ÷ .56875 = 56875)300000.000(5.274 +.
284375
etc.
∴ 5.27 % = the required rate.

6. What must be the market value of 5 % stock in order that it may yield 4 % on the investment, brokerage $\frac{1}{8}$ %?

Explanation:

$100 = the par value of 1 share.
.05 × $100, or $5 = the annual dividend on 1 share.
4 % of the investment = the annual dividend.
∴ 4 % of the investment = $5.
100 % of the investment = $125.
$0.125 = the brokerage on 1 share.
∴ $125 − $0.125, or $124.875 = the market value of 1 share.

Exercise 80

1. Find the cost of 20 shares of Chesapeake & Ohio R.R. stock at 81$\frac{1}{2}$ %, brokerage $\frac{1}{8}$ %.

2. Find the cost of 25 shares of Illinois Central R.R. stock at 127$\frac{1}{4}$ %, brokerage $\frac{1}{8}$ %.

3. Find the cost of 8 shares of Lehigh Coal & Navigation Co. stock ($50) at 87\frac{1}{2}$ a share, brokerage $\frac{1}{4}$ %.

4. Find the cost of 200 shares of United States Steel stock at $68 a share, brokerage $\frac{1}{8}$ %.

5. Find the cost of 120 shares of Lehigh Valley R.R. stock ($50) at 175 %, brokerage $\frac{1}{8}$ %.

6. Find the proceeds from the sale of 200 shares of Canadian Pacific R.R. stock at 226$\frac{1}{2}$ %, brokerage $\frac{1}{8}$ %.

7. Find the proceeds of 20 shares of Cambria Iron Co. stock ($50) at 44\frac{1}{4}$ per share, brokerage $\frac{1}{8}$ %.

8. Find the proceeds of 12 shares of Consolidated Coal Co. stock at 106$\frac{1}{4}$%, brokerage $\frac{1}{8}$ %.

9. Find the proceeds of 125 shares of mining stock, par \$ 1 per share, if its market value is \$ $10\frac{1}{2}$ per share and the rate of brokerage $\frac{1}{8}$ %.

10. Find the proceeds of 300 shares of F. W. Woolworth Co. stock (\$ 10) at $115\frac{1}{4}$ %, brokerage $\frac{1}{8}$ %.

11. Find the cost of five \$ 100 U. S. bonds at $101\frac{1}{2}$ %, brokerage $\frac{1}{8}$ %.

12. Find the cost of four \$ 50 U. S. bonds at $103\frac{1}{4}$ %, brokerage $\frac{1}{8}$ %.

13. Find the proceeds from the sale of \$ 10,000 Baldwin Locomotive Co. bonds at $102\frac{3}{4}$ %, brokerage $\frac{1}{8}$ %.

14. Find the proceeds from the sale of \$ 6000 Northern Pacific R.R. bonds at $98\frac{1}{2}$ %, brokerage $\frac{1}{8}$ %.

15. How many shares of bank stock at 112 % can be bought for \$ 1121.25, brokerage $\frac{1}{8}$ %?

16. How many shares of stock (\$ 50) at $109\frac{1}{2}$ % can be bought for \$ 3000, brokerage $\frac{1}{8}$ %, and what will be the surplus?

17. How many shares of stock (\$ 50) at \$ $81\frac{1}{2}$ per share can be bought for \$ 1750, brokerage $\frac{1}{4}$ %, and what will be the surplus?

18. How many shares of Penn. R.R. stock (\$ 50) at \$ $61\frac{5}{8}$ per share can be bought for \$ 1200, and what will be the surplus, brokerage $\frac{1}{8}$ %?

19. How many \$ 100 U. S. bonds at $113\frac{3}{4}$ %, brokerage $\frac{1}{8}$ %, can be bought for \$ 10,000, and what will be the surplus?

20. How many \$1000 U. S. bonds at $102\frac{1}{2}\%$ can be bought for \$12,315, brokerage $\frac{1}{8}\%$?

21. How many shares of stock (\$50) at $\$62\frac{1}{2}$ per share, brokerage $\frac{1}{4}\%$, must be sold to realize \$998?

22. I wish to sell sufficient U. S. 2s at $101\frac{1}{2}\%$, brokerage $\frac{1}{8}\%$, to pay a debt of \$3800; how many \$100 bonds must I sell, and what will be the surplus?

23. Find my annual income from 10 shares of Boston & Maine R.R. stock, the semiannual dividend being 6%.

24. Find the annual income from 50 shares of Penn. R.R. stock (\$50), which pays a quarterly dividend of $1\frac{1}{2}\%$.

25. Find the annual income from 20 shares of Denver & Northwestern Railway Co. stock, which pays a quarterly dividend of 2%.

26. Find the annual income from 25 shares of Utah Copper Co. stock (\$10), during a year in which it paid quarterly dividends of $7\frac{1}{2}\%$.

27. Find the annual income from 700 shares of mining stock, par \$1 per share, if it pays a quarterly dividend of 25%.

28. Find the annual income from 20 shares of Standard Oil of N. J. stock, which pays a quarterly dividend of 5%.

29. When mining stock, par \$1 per share, pays 40% quarterly dividends, what income does a man receive each year from 40 shares of this stock?

30. How much is gained by buying 25 shares of Amalgamated Copper Co. stock at $60\frac{1}{2}$ % and selling it at $92\frac{3}{4}$ %, brokerage $\frac{1}{8}$ % on the sale and purchase?

31. What profit is made by buying 1300 shares of Chicago Great Western R.R. stock at 17 % and selling it at $18\frac{1}{8}$ %, brokerage $\frac{1}{8}$ % on the sale and purchase?

32. What profit is made by buying 20 shares of R.R. stock at $98\frac{1}{2}$ % and selling it at $112\frac{1}{2}$ %, brokerage $\frac{1}{8}$ % on the sale and purchase?

33. Find the annual income from $2310 invested in Northern Pacific R.R. stock at $115\frac{3}{8}$ %, brokerage $\frac{1}{8}$ %, if this stock pays a quarterly dividend of $1\frac{3}{4}$ %.

34. Find the annual income from $2584.50 invested in Adams Express Co. stock at $215\frac{1}{4}$ %, brokerage $\frac{1}{8}$ %, if this stock pays a quarterly dividend of 3 %.

35. Find the annual income from six $100 U. S. 4s, 1925.

36. Find the annual income from eight $1000 U. S. 4s, 1925.

37. Find the annual income from six $1000 Panama 2s, 1936.

38. Find the annual income from four $1000 Reading R.R. Co. 4 % bonds.

39. Find the annual income from eight $1000 Baldwin Locomotive Co. 5 % bonds.

40. How many shares of stock paying 5 % dividends must be purchased that the annual income derived therefrom may be $220?

41. How much must I pay for sufficient 8 % stock ($ 50), market value $ 89½ per share, that my annual income may be $ 120, brokerage ¼ % ?

42. What must I pay for sufficient United Gas Improvement Co. stock ($ 50) that my income from it may be $ 240 per year, if I buy this stock when its market value is $ 89½ per share, brokerage ¼ %, this stock paying a quarterly dividend of 2 % ?

43. If I purchase 6 % stock at 124¼ %, brokerage ⅛ %, what rate of interest does my investment pay? Answer to nearest hundredth per cent.

44. If I purchase Penn. R.R. stock ($ 50) at $ 61½ per share, brokerage ⅛ %, what rate of interest does my investment pay me, my annual income from each share of this stock being $ 3 ?

45. If I purchase 5 % stock at 119⅞ %, brokerage ⅛ %, what rate of interest do I receive? Answer to nearest .01 %.

46. What rate of interest does 4 % stock ($ 50) pay an investor, if it is bought at 112½ %, brokerage ⅛ % ? Answer to nearest .01 %.

47. What must be the market value of 6 % stock in order that it may yield 5 % on the investment (which includes brokerage), if the rate of brokerage is ⅛ % ?

48. What must be the market value of 5 % stock to yield an investor 4 % interest on his investment, brokerage ⅛ % ?

NEGOTIABLE PAPER AND BANKING

284. A **bank** is an institution which cares for, lends, exchanges, and issues money.

285. National banks are authorized by the United States, and controlled by its laws.

286. State banks are authorized by the various states, and controlled by their respective state laws.

287. Savings banks are banks established to take care of people's savings. They will receive very small deposits, and they pay interest on these deposits, but usually at a low rate.

288. Trust companies are state institutions which settle estates, take care of the property of children, and execute other trusts. They also do a banking business.

289. Private banks are individuals or partners who do a banking business.

290. Deposits are money put into banks by people for safe keeping. Some banks pay a small rate of interest on part or all these deposits; others do not.

291. All banks lend most of their deposits, keeping in their safes only as much as the laws require and experience shows will be asked for. Banks make most of their profits by lending their deposits.

292. All banks receive deposits and lend money. Only national banks issue **bank notes**, and they must deposit in the United States Treasury bonds of the United States whose par value is equal to the notes they issue.

293. Besides national bank notes there are two other kinds of paper money in common use: **United States notes,** or **greenbacks,** which are issued by the United States; and gold and silver certificates which are substitutes for **gold** and **silver** coin kept in the government vaults at Washington.

NOTE. Read carefully both sides of all these kinds of paper money.

OPENING AN ACCOUNT AT A BANK

294. To open an account at a bank, a person should fill out a deposit slip and deliver it, with the deposit, to the *receiving teller* or *cashier* of the bank.

295. After opening an account at a bank, a depositor is given a bank book with the amount of his deposit credited therein, and also a blank check book.

He is generally required to sign his name on a card or in a book of signatures.

South End National Bank
OF BOSTON

DEPOSITED FOR ACCOUNT OF

James L. Price

BOSTON, June 2 1913

All checks are credited subject to payment.
Boston Items—State name of BANK
Foreign Items—State name of CITY or TOWN

	DOLLARS	CENTS
BILLS	160	
COIN	22	75
CHECKS		
Boylston Nat	227	48
Philadelphia	332	16
	742	39

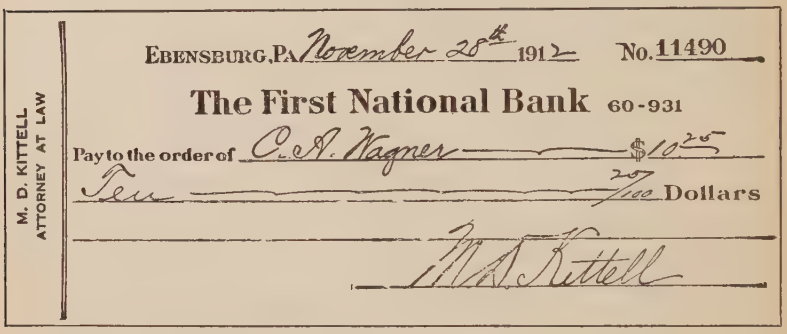
Deposit Slip

CHECKING OUT MONEY

296. When a depositor wishes to use any or all of the money he has in a bank, he fills out a check for the amount desired.

297. A **check** is a written order by which a bank is directed by a depositor to pay out money belonging to him.

Check


M. D. KITTELL
ATTORNEY AT LAW
EBENSBURG, PA. November 28th 1912 No. 11490
The First National Bank 60-931
Pay to the order of C. A. Wagner $10.25
Ten 25/100 Dollars
M. D. Kittell

298. The **drawer**, or **maker**, of a check is the party who signs it. The **payee** is the party to whom the check is payable. The **face** of a check is the amount named therein.

NOTE. The holder of a check should present it for payment on the day he receives it or on the next business day. Banks credit checks as cash.

299. Checks are usually made payable *to the order of* the payee.

300. If a check is made payable to the order of the payee, it may be transferred by him to another. This is done by the payee's writing his own name across the back of the check, which process is called **indorsing** the check. The person receiving the check then becomes the holder, and may again transfer it by indorsement to another person, and so on.

1. An indorsement containing the name of the indorser only is called an **indorsement in blank.**

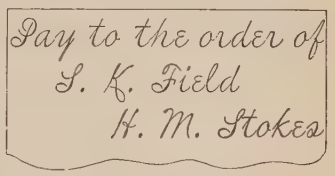
2. An indorsement directing payment to be made to the order of a certain person is an **indorsement in full,** or a **special indorsement.**

3. An indorsement which limits the responsibility of the indorser is a **qualified indorsement.**

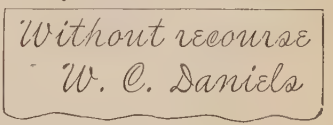
4. A check payable to *bearer,* or to a *payee* or *bearer,* is transferable without indorsement.

Indorsement in Blank

Indorsement in Full



Qualified Indorsement



Exercise 81

In Examples 1–4 make out deposit slips for the following deposits to be made in your local bank and in your name :

1. Bills, $215; gold, $15; silver, $10.90; check on Boylston National Bank, Boston, $316.25.

2. Bills, $480; gold, $62.50; silver, $26.50; check on Third National Bank, Boston, $98.75.

3. Bills, $365; gold, $25; silver, $20.60; check on Farmer's National Bank of Lancaster, Pa., $112.63.

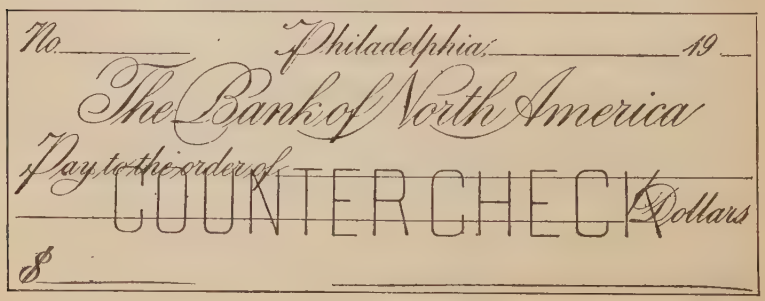
4. Bills, $170; gold, $30; silver, $36.20; check on First National Bank of Bangor, Me., $65.

5. In the check on page 230 name the drawer, or maker; the payee. When is this check payable? Which of the parties named must be a depositor in the First National Bank of Ebensburg, Penn.? Which party named receives the sum mentioned in the face? Must he receive it direct from the bank named in the check? Which party will finally hold the check? Must it be indorsed before it is cashed? Who must first indorse it?

6. Suppose you have money deposited in the nearest bank; write the check which you would draw to pay G. M. Brown $20.

7. Suppose that P. J. Welsh draws a check in your favor for $10 on the Continental & Commercial Bank of Chicago, and you have W. P. Bolton cash the check for you; write the check and the indorsement it bears.

8. This form, called "Counter Check," is frequently used by depositors in drawing cash for their own use from their bank of deposit:



Copy the "Counter Check," page 232, and fill it out to draw $20 cash.

9. With a "Counter Check" draw $5 from your nearest bank, supposing that you have deposits therein.

10. A man had a balance in bank of $756.25 on May 1. Since that time he has drawn checks for $25.20, $18.75, and $9.85, and deposited $25, $50, and $75. What is his present balance?

11. A merchant had a balance in bank of $375.80 on June 30. Since that time he has drawn checks for $37.50, $18.75, $6.25, and $32.50, and has deposited $50, $18.90, and $13.20. What is his present balance?

SAVINGS BANK ACCOUNTS

301. Interest paid depositors in savings banks is compounded monthly, quarterly, or semiannually. Quarterly interest periods usually begin Jan. 1, Apr. 1, July 1, and Oct. 1; semiannual periods usually begin Jan. 1 and July 1.

302. Depositors in savings banks are given bank books which must be presented when deposits are made and when money is drawn out; they should also be presented at the end of each interest period.

303. Savings banks usually credit interest at the end of each interest period on the *smallest amount* on deposit during the entire period.

304. The practices prevailing among savings banks governing the length of the interest period, withdrawals, and deposits are so varied that no fixed rules can be laid down.

305. The following is an illustrative statement of deposits and withdrawals with interest compounded semiannually at 3 % per annum:

Date	Deposited	Drawn Out	Interest	Balance
1911				
Nov. 13	400			400
1912				
Jan. 1				400
Feb. 10		60		340
Apr. 26	25			365
June 14	10			375
July 1		15	5 10	365 10
Sept. 21		34		331 10
Nov. 27	40			371 10
Dec. 5		100		271 10
1913				
Jan. 1			4 07	275 17
May 7		10		265 17
July 1			3 98	269 15

Exercise 82

Arrange the following as in Section 305.

1. Deposits: Oct. 16, 1911, $200; March 2, 1912, $125; Nov. 6, 1912, $250. Withdrawals: Nov. 13, 1911, $100; Dec. 20, 1912, $225. Find the balance due Jan. 1, 1913, interest compounded semiannually, Jan. 1 and July 1, at 3 % per annum.

2. Deposits: Feb. 13, 1911, $300; Nov. 20, 1911, $100; Jan. 6, 1912, $275; Dec. 2, 1912, $50; Feb. 9,

1913, $25. Withdrawals: June 23, 1911, $75; Sept. 16, 1911, $150; Apr. 21, 1912, $100; June 11, 1912, $60. Find the balance due Jan. 1, 1914, interest compounded semiannually Jan. 1 and July 1, at 3% per annum.

3. Deposits: Nov. 30, 1911, $600; Apr. 20, 1912, $100; Aug. 10, 1912, $100; Oct. 3, 1912, $150; Jan. 25, 1913, $200. Withdrawals: Dec. 20, 1911, $200; Apr. 29, 1912, $50; Sept. 5, 1912, $300; June 20, 1913, $175. Find the balance due July 1, 1913, interest compounded quarterly Jan. 1, Apr. 1, July 1, and Oct. 1, at 3% per annum.

POSTAL SAVINGS SYSTEM

(This information is from a circular distributed from the Post Office Department.)

306. Object. The Postal Savings System is established for the purpose of providing facilities for depositing savings at interest with the security of the United States Government for repayment.

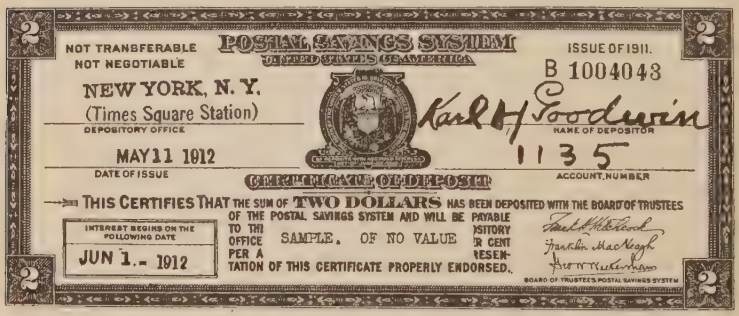
307. Who may Deposit. Accounts may be opened and deposits made by any person of the age of 10 years or over in his or her own name, and by a married woman in her own name and free from any interference or control by her husband. No person may open a postal savings account at any post office who is not a patron of that office. Deposits will be accepted only from individuals, and no account will be opened in the name of any corporation, association, society, firm, or partnership, or in the names of two or more persons jointly.

308. Service Free. The service of the Postal Savings System is free, and no charge or fee is collected or required in connection with the opening of an account or the withdrawal of money deposited.

309. How to open an Account. When a person applies to open an account, he must furnish the necessary information for the postmaster or his representative to fill out an application, which he will then be required to sign. If the applicant signs by mark, his signature must be witnessed by a disinterested person.

310. Deposits. Deposits are evidenced by postal savings certificates issued in fixed denominations of $1, $2, $5, $10, $20, $50, and $100, each bearing the name of the depositor, the number of his account, the date of issue, the name of the depository office, and the date on which interest begins. The postmaster or his representative will make out a duplicate of each certificate issued, which the depositor will be required to sign and which the postmaster will retain in his records.

Postal Savings Certificate


NOT TRANSFERABLE
NOT NEGOTIABLE
POSTAL SAVINGS SYSTEM
OF THE UNITED STATES OF AMERICA
ISSUE OF 1911.
B 1004043
NEW YORK, N. Y.
(Times Square Station)
DEPOSITORY OFFICE
Karl H. Goodwin
NAME OF DEPOSITOR
MAY 11 1912
DATE OF ISSUE
1135
ACCOUNT NUMBER
CERTIFICATE OF DEPOSIT
THIS CERTIFIES THAT THE SUM OF TWO DOLLARS HAS BEEN DEPOSITED WITH THE BOARD OF TRUSTEES OF THE POSTAL SAVINGS SYSTEM AND WILL BE PAYABLE TO THE DEPOSITORY OFFICE PER CENT PER ANNUM UPON PRESENTATION OF THIS CERTIFICATE PROPERLY ENDORSED.
INTEREST BEGINS ON THE FOLLOWING DATE
JUN 1 1912
SAMPLE. OF NO VALUE
BOARD OF TRUSTEES POSTAL SAVINGS SYSTEM

No account may be opened for less than $1, nor will fractions of a dollar be accepted for deposit.

No person is permitted to deposit more than $100 in any one calendar month nor to have a total balance to his credit at one time of more than $500, exclusive of accumulated interest.

Savings certificates cannot be transferred or negotiated and will be payable only to the person to whom issued.

On opening an account a depositor is supplied with an envelope in which he may keep his savings certificates. On this envelope is printed information for his guidance, and also a blank ledger record on which to keep an account of his deposits and withdrawals.

In case a savings certificate is lost or destroyed the depositor should notify the postmaster.

311. Savings Cards and Stamps. Amounts less than $1 may be saved for deposit by the purchase of 10-cent postal savings cards and adhesive 10-cent postal savings stamps. Each postal savings card contains blank spaces to which savings stamps may be affixed from time to time as purchased, and a postal savings card with nine 10-cent savings stamps thus affixed will be accepted as a deposit of $1 either in opening an account or in adding to an existing account.

Savings Card with One Postal Savings Stamp Affixed

U.S. POSTAL SAVINGS CARD
THIS CARD WITH NINE 10-CENT POSTAL SAVINGS STAMPS AFFIXED WILL BE ACCEPTED BY THE POSTMASTER AT ANY POSTAL SAVINGS DEPOSITORY OFFICE IN THE UNITED STATES AS A DEPOSIT OF ONE DOLLAR AND PLACED TO THE CREDIT OF THE DEPOSITOR, EITHER IN OPENING AN ACCOUNT OR IN ADDING TO AN EXISTING ACCOUNT.
10
1
2
3
4
5
6
7
8
9
SAMPLE. OF NO VALUE

312. Interest. Interest will be allowed on all deposits at the rate of 2 per cent per annum, computed on each savings certificate separately, and payable annually. No interest will be paid on money which remains on deposit for a fraction of a year only.

Deposits will bear interest from the first day of the month next following that in which deposited.

Interest will continue to accrue on a savings certificate as long as it remains outstanding, certificates being valid until paid, without limitation as to time.

Compound interest is not allowed on an outstanding certificate, but a depositor may withdraw interest payable and include it in a new deposit, which will bear interest at the regular rate.

313. Withdrawals. A depositor may at any time withdraw the whole or any part of his deposits to his credit with any interest payable by surrendering savings certificates, properly indorsed, for the amount desired.

When a depositor desires to withdraw merely the interest payable on any certificate, instead of indorsing and surrendering the certificate as in case of full payment he will be required to give his receipt in duplicate for the amount of the interest paid.

314. Postal Savings Bonds. A depositor will be permitted to exchange the whole or any part of his deposits in sums of $20, $40, $60, $80, $100, or multiples of $100 up to and including $500, for United States registered or coupon bonds bearing interest at the rate of $2\frac{1}{2}$ per cent per annum, payable semiannually, and redeemable at the pleasure of the United States after one year from date of issue, both principal and interest payable 20 years from such date in United States gold coin. Such exchange may be made under date of January 1 and July 1 of each year, provided such bonds are then available.

A depositor desiring to convert his savings deposits into bonds on January 1 and July 1 of any year must make application at least 15 days before either of the dates named to the postmaster.

Exercise 83

1. If I open a postal savings account and deposit $3, what certificates will be given me?

2. What will be given you at the post office if you deposit in postal savings $1.50?

3. How much money may one person deposit in one month in postal savings?

4. If I open a postal savings account Nov. 11, when does interest begin on my first deposit?

5. If I add a deposit of $25 to my postal savings account July 15 of the current year, when does interest begin thereon? What is the rate of interest? When may I draw the first year's interest on this deposit?

6. If I add a deposit of $50 to my postal savings account Feb. 20 of the current year, when does interest begin thereon? When may I draw my first year's interest on this deposit, and how much?

7. A savings certificate for $10, one for $20, and one for $50 were issued to me Nov. 20 of last year. On what date each year may I receive interest on these, and how much?

8. On Jan. 1 of the current year I exchange postal certificates to the amount of $400 for United States bonds. When may I draw the first interest on these bonds, and how much?

9. On July 1 of the current year I exchange postal certificates to the amount of $300 for United States bonds. When will I receive the first interest thereon, and how much?

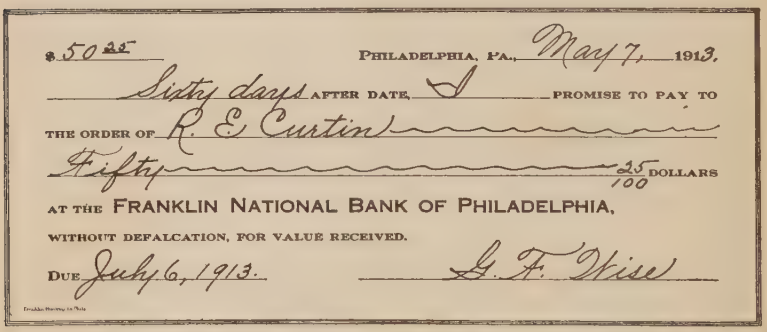
10. After having opened a postal savings account, what is the least amount that I may deposit? Explain how I may save 20¢ for deposit.

PROMISSORY NOTES

315. A **promissory note** is a written promise made by a party called the **maker**, to pay to a certain party, called the **payee**, or to his order, or to bearer, a specified sum of money at a certain time.

316. The **face** of a promissory note is the amount named in the note.

Individual Time Note


$50.25 PHILADELPHIA, PA., May 7, 1913.

Sixty days AFTER DATE I PROMISE TO PAY TO

THE ORDER OF R. E. Curtin

Fifty 25/100 DOLLARS

AT THE FRANKLIN NATIONAL BANK OF PHILADELPHIA,

WITHOUT DEFALCATION, FOR VALUE RECEIVED.

DUE July 6, 1913. G. F. Wise

In the above note *G. F. Wise* is the *maker*, *R. E. Curtin* is the *payee*, and *$50.25* is the *face*.

317. If the maker of a promissory note is unable to write, he may sign it by making his mark and having it witnessed, after his name has been written. Thus, if G. F. Wise cannot write his own name, he may become the maker by having the note executed thus:

Witness: G. L. Brown. G. F. X Wise.
his
mark

318. An **individual note** is a note signed by one person.

319. A **joint and several note** is a note signed by two or more persons, each making himself responsible for the payment thereof.

If a joint and several note is written "I promise to pay," or "We or either of us promise to pay," and signed by two or more persons, each is responsible for the payment thereof; but if it is written "We jointly promise to pay," or "We promise to pay," and signed by two or more persons, they are jointly responsible for the payment thereof. This form of note is commonly used at public sales, where goods are sold on credit and settled for by notes payable at a bank at the end of the credit period. Satisfactory security is usually demanded of the purchaser, which is obtained by requiring the security to join with the purchaser in making the note. The security may write after his name "surety." The note is then a **principal and surety** note. This form of note may also be used by joint partners in a transaction when no security is required.

320. A **collateral note** is used when a person borrows money and deposits as security with the holder some valuable collateral, as government bonds, certificates of stock, or bonds of some reliable corporation. The holder is authorized to sell the collateral in case the maker fails to pay the note when it becomes due.

321. An **accommodation note** is chiefly used in borrowing money from a bank.

Accommodation Note

$100.00 Wilmington, Del. March 1, 1913.

Sixty days after date I promise to pay to the order of R. P. Miller at the Farmers Bank at Wilmington One hundred Dollars without defalcation, value received.

CREDIT THE DRAWER
R. P. Miller

G. L. Smith.

322. Banks generally require from a borrower, besides his own name as the maker of a note, the name of some responsible party as security. This is usually obtained by making the note payable to the security and having him indorse it. The phrase "Credit the Drawer," subscribed to by the security, is not essential to the legality of an accommodation note, but it shows definitely the relation of the maker to the security.

Thus, if G. L. Smith desires to borrow one hundred dollars from the Farmers' National Bank and have R. P. Miller for his security, they may write the note as shown in Section 321, and R. P. Miller will indorse it. G. L. Smith may then present the note at bank and, after indorsing it, receive the face less the interest charged by the bank.

Important Facts concerning Promissory Notes

323. A note should be dated; that is, it should state the place *where*, and the time *when*, given.

324. A note may be made payable *on demand*, *a certain number of days after date*, *a certain number of months after date*, or upon *a certain date*.

1. The day on which a note becomes due is called the **day of maturity.**

2. A note payable on demand is due whenever the holder demands payment.

3. A note payable a certain number of days after date is due at the expiration of that number of days, not counting the date.

Thus, a note dated March 1 payable in 60 days is due April 30.

4. A note payable a certain number of months after date is due so many calendar months after date.

Thus, a note dated April 30 payable in 6 months is due October 30; a note dated July 31 payable in 3 months is due October 31.

5. In some states 3 days, called **days of grace**, are allowed for the payment of a note after the expiration of the time specified in the note.

325. A note may be made payable at a certain place.

1. If a note is made payable at a particular place, as at a certain bank, it should be presented at that place for payment, when due, during business hours.

2. When no place of payment is designated in a note, it should be presented for payment at the maker's place of business during business hours or at his residence.

326. A promissory note may be made payable to the order of the *payee*, to the *payee or bearer*, or to the *payee* only. It is indorsed or otherwise transferred in the same manner as is a check.

327. A note may be written so as to include the payment of interest.

1. When no rate is mentioned in an interest-bearing note, it bears interest at the legal rate of the state in which it is payable.

2. When a non-interest-bearing note is not paid when due, it will bear interest at the legal rate from the day it is due to the day it is paid.

3. Some interest-bearing notes have a condition stated in them that, if not paid when due, they will thereafter bear a higher rate of interest than the one named in the main note; when no such condition is stated, they will continue to bear interest at the original rate until paid.

Exercise 84

1. When is the note, page 240, due? Who pays this note when it is due? Where will the note be paid? Who holds the note after it has been paid?

2. Name the maker and the payee of the note, page 242. How long does this note run before it is due? Who is expected to pay the note when it is due? Who holds the note after it has been paid?

3. When is a note due that is dated December 31, 1914, payable in 90 days?

4. When is a note due that is dated December 31, 1913, payable in 2 months?

5. Write a promissory note for $20 with yourself as maker and George B. Smith as the payee. Date the note September 30 of the current year, make it payable in 60 days, and find when it is due.

6. If R. E. Curtin wishes to transfer the note on page 240 to W. J. Davis, in what ways may he write the indorsement?

7. Suppose you buy a horse from A. S. Harkness for $150 payable in 9 months. Write the joint and several note that you could give him in payment if B. J. French

signed the note with you. Date the note December 1 of the current year, make it payable at your nearest bank with interest at 5 %, and find what this note is worth at maturity. (Reckon interest for 9 mo.)

8. June 1 of the current year W. H. Jones accommodates you by becoming your security on a 60-day note to enable you to borrow $ 50 from your nearest bank; write the note.

9.

CLEVELAND, O., April 3, 1912.

Ninety days after date I promise to pay to the order of William F. Davis at

The First National Bank of Cleveland

Two hundred twenty ---------- Dollars

Value received, with interest at $4\frac{1}{2}$ %.

L. J. DRIGGS.

Name the parties to the above note. When is it due? How much is due at maturity? Who pays the note when due? Who receives the note when paid?

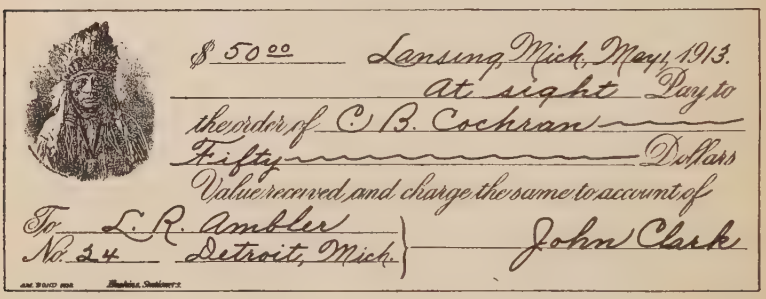
10. How much interest is due at maturity on a note for $ 500 dated Indianapolis, Ind., June 2, of the current year, and payable in 6 months with interest at the legal rate? (Reckon interest for 6 mo.)

11. When is a note due that is dated December 31, 1915, payable in 2 months?

DRAFTS

328. A **draft**, or **bill of exchange**, is a written order whereby one party directs another to pay a specified sum of money to the order of a certain party at a certain time.

Sight Draft


$ 50 00
Lansing, Mich, May 1, 1913.
At sight Pay to
the order of C. B. Cochran
Fifty Dollars
Value received and charge the same to account of
To L. R. Ambler
No. 24 Detroit, Mich.
John Clark

329. The **drawer** of a draft is the party who signs it and thereby orders its payment. The **drawee** is the party who is ordered to pay. The **payee** is the party to whom it is made payable. The **face** is the amount named in the draft.

Thus, in the above draft, *John Clark* is the *drawer*, *L. R. Ambler* the *drawee*, and *C. B. Cochran* the *payee*. *Fifty dollars* is the *face*.

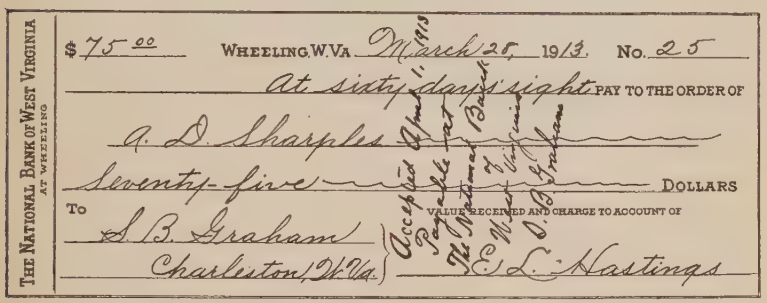
330. A sight draft is a draft that is payable whenever presented to the drawee for payment.

331. The purpose of the sight draft in Section 328 may be explained thus: John Clark, desiring to pay C. B. Cochran fifty dollars, orders L. R. Ambler, who owes him at least that amount, to pay the

same to C. B. Cochran, or his order, at sight, and sends the draft to C. B. Cochran, who may collect it from L. R. Ambler.

332. A **time draft** is a draft payable at a certain time after sight or after date.

Time Draft


THE NATIONAL BANK OF WEST VIRGINIA
AT WHEELING
$75 00
WHEELING, W. VA. March 25, 1913. No. 25
At sixty days sight PAY TO THE ORDER OF
A. D. Sharples
Seventy-five DOLLARS
To
S. B. Graham
Charleston, W. Va.
VALUE RECEIVED AND CHARGE TO ACCOUNT OF
E. L. Hastings
Accepted March 25, 1913
Payable at The National Bank of West Virginia
S. B. Graham

333. All time drafts must be presented to the drawee for *acceptance*.

334. To accept a draft, the drawee writes across the face the word "accepted" or "seen," together with the date of acceptance and his name, unless the draft is payable "after date," in which case the date of acceptance may be omitted.

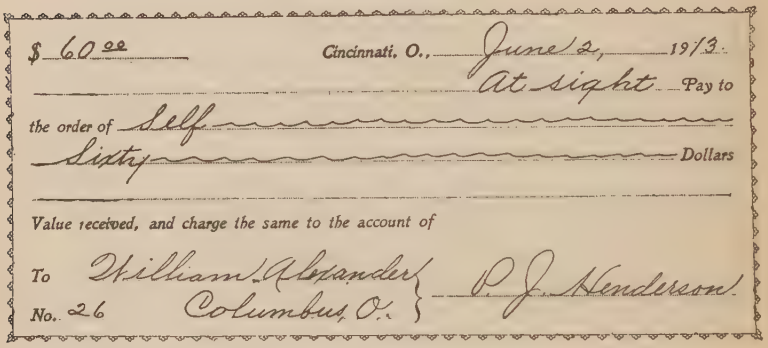
A time draft made payable a certain number of days after *sight* becomes due that number of days after acceptance; but when made payable at a certain time after date, it becomes due at the expiration of that time after date.

335. The purpose of the draft in Section 332 may be explained thus: E. L. Hastings desires to pay A. D. Sharples $75. He orders S. B. Graham, who owes him at least that sum, to pay the same to the order of A. D. Sharples, to whom the draft is sent. A. D.

Sharples presents it to S. B. Graham, who accepts it, as shown in the draft.

336. A draft, used merely for the purpose of collecting a debt, is sometimes called a **two-party** draft.

Two-Party Draft



337. The purpose of this two-party draft may be explained thus: P. J. Henderson wishes to collect a debt of $60 from William Alexander of Columbus, O. He makes himself the payee, then indorses the draft, and deposits it in a Cincinnati bank for collection. The Cincinnati bank forwards it to a bank in Columbus, and this bank collects it from William Alexander.

NOTE 1. P. J. Henderson might also collect the above sum by making the bank in which he is a depositor the payee.

NOTE 2. Two-party drafts may also be time drafts.

Bank Draft

338. A **bank draft** is a draft drawn by one bank upon another in which it has deposits. It is used mainly to transmit money from one place to another.

Bank Draft

339. The purpose of the draft in Section 338 may be explained thus: N. C. Cameron of West Chester, Penn., desiring to send $5 to C. E. Karlson of Cleveland, O., purchases the draft of the First National Bank of West Chester, Penn., indorses it "*pay to the order of C. E. Karlson, N. C. Cameron*" and sends it to C. E. Karlson, who then indorses it and presents it for payment to a bank in Cleveland, which bank cashes it and collects it from the Fourth National Bank of New York City.

Exercise 85

1. In the sight draft, page 246, who directs that payment be made? Who is ordered to pay? When is the draft due? Who is expected to pay it when due?

2. In the time draft, page 247, name the drawer; the drawee; the payee. When is the draft due? Who pays it when due? Who receives it when paid?

3. Write a sight draft for $ 5, making yourself drawer, E. W. Hayes drawee, and William Brown payee; date the draft July 15 of the current year.

4. Suppose Frank Jones owes you $20 and you owe George Robinson a like sum; write the sight draft that you could use both to collect from Frank Jones and to pay George Robinson.

5. Write a 30-day sight draft for $25, making yourself drawer, William Mack drawee, and N. C. Walton payee. Date the draft October 5 of the current year, and have it accepted October 7. When is the draft due? Who pays it when due? Who holds it when paid?

6. Suppose you owe G. R. French $50 and H. L. Beaver owes you at least that sum. Write the draft that you could draw on H. L. Beaver in favor of G. R. French for the sum you owe the latter; date the draft November 1 of the current year, making it payable 60 days after date. Write the acceptance and find when the draft is due.

7. Write the sight draft that you could draw on G. W. Boyd to collect a debt of $40. Explain the steps that you would take to collect the draft.

8. You wish to remit $20 to C. J. Carroll of Charleston, S.C. To do this you purchase a draft for this amount at your local bank, which has deposits in the Fourth National Bank, New York City. Write the draft and explain how you can use it to pay the debt.

PARTIAL PAYMENTS

340. When a note is paid in part, such payment is called a **partial payment**, and it is receipted on the back of the note. This acknowledgment is called an **indorsement.**

341. The following is the rule most generally used for determining the balance due on a note with indorsements:

United States Rule

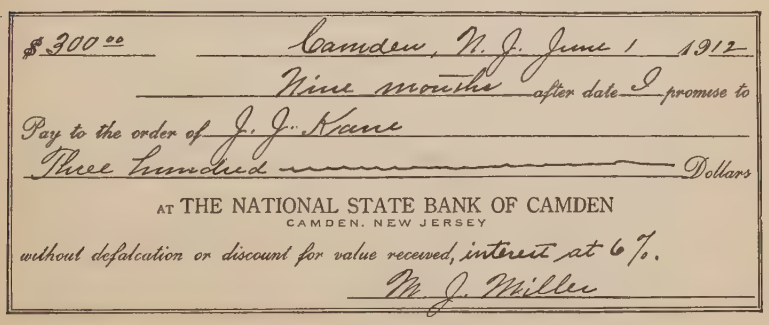
The rule adopted by the Supreme Court of the United States, known as the "United States Rule of Partial Payments," has been adopted as the legal rule by nearly all the states.

Find the amount of the given principal to the time when the payment, or the sum of the payments, is equal to or exceeds the interest due; subtract from this amount the payment or the sum of the payments. Treat the remainder as a new principal, and proceed as before.

NOTE. New Hampshire, Vermont, and Connecticut have their own rules, which, being limited to these states, will not be considered here.

Illustrative Example

Find the balance due Dec. 15, 1913, on the following note:


$300.00 Camden, N. J. June 1 1912

Nine months after date I promise to

Pay to the order of J. J. Kane

Three hundred Dollars

AT THE NATIONAL STATE BANK OF CAMDEN
CAMDEN, NEW JERSEY

without defalcation or discount for value received, interest at 6%.

M. J. Miller

Indorsements

Received on the within note.
1913, Mar. 1, *$120*
J. J. Kane.

1913, Sept. 15, *$5*
J. J. Kane.

1913, Nov. 1, *$50*
J. J. Kane.

Solution by United States Rule

Principal, June 1, 1912,	$ 300.00
Interest on principal from June 1, 1912, to Mar. 1, 1913 (273 da.),	13.65
Amount, Mar. 1, 1913,	$ 313.65
First payment,	120.00
New principal, Mar. 1, 1913,	$ 193.65
Interest on principal from Mar. 1, 1913, to Sept. 15, 1913 (198 da.), $ 6.39.	
(Interest due, Sept. 15, 1913, exceeds payment then made and no settlement is made.)	
Interest on principal from Mar. 1, 1913, to Nov. 1, 1913 (245 da.),	7.91
Amount, Nov. 1, 1913,	$ 201.56
Payments applied, Nov. 1, 1913 ($ 5 and $ 50),	55.00
New principal, Nov. 1, 1913,	$ 146.56
Interest on new principal from Nov. 1, 1913, to Dec. 15, 1913 (44 da.),	1.07
Amount due, Dec. 15, 1913,	$ 147.63

Exercise 86

Find the balance due on the date of settlement by the U. S. Rule:

1. On a note dated Albany, N.Y., Sept. 1, 1911, for $500 with interest at the legal rate, containing the following indorsements:

July 1, 1912, $50. Mar. 4, 1913, $50.
Aug. 31, 1912, $100. Aug. 31, 1913, $5.
Settled, Mar. 2, 1914.

2. On a note dated Harrisburg, Penn., June 1, 1910, for $1000 with interest at $4\frac{1}{2}\%$, containing the following indorsements:

Apr. 20, 1911, $200. Oct. 2, 1912, $50.
July 16, 1911, $100. Jan. 31, 1913, $200.
Settled, Apr. 1, 1913.

3. On a note dated Boston, Mass., Mar. 1, 1911, for $400 with interest at 4%, containing the following indorsements:

Aug. 13, 1911, $200. Feb. 29, 1912, $100.
Oct. 13, 1912, $50.
Settled, Mar. 1, 1913.

4. On a note dated Richmond, Va., Aug. 1, 1911, for $600 with interest at $4\frac{1}{2}\%$, containing the following indorsements:

Nov. 20, 1911, $200. Dec. 15, 1912, $200.
May 31, 1912, $10. Jan. 18, 1913, $100.
Settled, Mar. 3, 1913.

States and Territories	Interest Laws				Statutes of Limitation		
	Legal Rate	Maximum Contract Rate	Grace	Payable	Judgments	Notes	Open Accounts
					Years	*Years*	*Years*
Alabama	8	8		S	20	6	3
Arkansas	6	10	G	P	10	5	3
Arizona	6	Any		S	5	4	3
California	7	Any		S	5	4	2
Colorado	8	Any		S	20	6	6
Connecticut	6	6		S			6
Delaware	6	6		P*	10	6	3
Dist. of Columbia	6	10		S	12	3	3
Florida	8	10		S	20	5	2
Georgia	7	8		P	7	6	4
Idaho	7	12		S	6	5	4
Illinois	5	7		S	20	10	5
Indiana	6	8		P	20	10	6
Iowa	6	8		S	20	10	5
Kansas	6	10		P	5	5	3
Kentucky	6	6		P	15	15	2
Louisiana	5	8		S	10	5	3
Maine	6	Any		S	20	6	6
Maryland	6	6		P	12	3	3
Massachusetts	6	Any		S	20	6	6
Michigan	5	7		S	6	6	6
Minnesota	7	10		P	10	6	6
Mississippi	6	10	G	P	7	6	3
Missouri	6	8		S	10	10	5
Montana	8	Any		S	10	8	5
Nebraska	7	10		S	5	5	4
Nevada	7	Any	G	P	6	4	4
New Hampshire	6	6		S	20	6	6
New Jersey	6	6		S	20	6	6
New Mexico	6	12		S	7	6	4
New York	6	6		S	20	6	6
North Carolina	6	6	G	S	10	3	3
North Dakota	7	12		S	10	6	6
Ohio	6	8		S	15	15	6
Oklahoma	7	12		S	5	5	3
Oregon	6	10		S	10	6	6
Pennsylvania	6	6		S	5	6	6
Rhode Island	6	Any		S	20	6	6
South Carolina	7	8	G	S	20	6	6
South Dakota	7	12	G	S	10	6	6
Tennessee	6	6		S	10	6	6
Texas	6	10	G	P	10	4	2
Utah	8	Any		S	8	6	4
Vermont	6	6		S	8	6	6
Virginia	6	6		S	20	5	2
Washington	6	12		S	6	6	3
West Virginia	6	6		S	10	10	3
Wisconsin	6	10		S	20	6	6
Wyoming	8	12		P	5	5	3

*Unless due on Sunday or Monday holiday, in New Castle Co.

INTEREST LAWS AND STATUTES OF LIMITATION

342. The table on page 254 gives for each state and territory of the United States and the District of Columbia the legal rate, the maximum contract rate, the statute of limitation, the time for payment of a note or draft falling due on Sunday or on a legal holiday, and whether or not grace is allowed.

The following notation is used in the table:

P represents previous business day; S, the succeeding business day; and G, three days of grace.

NOTE 1. By the statute of limitation is here meant the period of time from the date of the last acknowledgment of the debt, within which suit must be brought to collect a debt overdue.

NOTE 2. By maximum contract rate is meant the highest rate of interest that the law allows to be contracted for and collected.

BANK DISCOUNT

343. Bank discount is the amount deducted by a bank for paying a note or a draft before it is due. It is the simple interest on the face of the note or draft, generally from the day of discount to the day of maturity, which time is called the **term of discount.** The **proceeds** is the sum discounted less the discount.

NOTE 1. In Pennsylvania, Delaware, Maryland, Missouri, and the District of Columbia, *the day of discount* as well as *the day of maturity* is counted in reckoning the term of discount.

NOTE 2. Some banks occasionally discount interest-bearing notes. The discount is then reckoned on the amount of the note at maturity instead of on the face.

NOTE 3. The rate of discount charged by a bank is usually the legal rate of interest prevailing in the state.

Illustrative Examples

1. A note dated January 15, 1913, payable in 6 months, was discounted at a Pennsylvania bank February 2, 1913. Find the discount and the proceeds if the face of the note was $150.

Explanation:

6 months after January 15, 1913, or July 15, 1913, was the date of maturity.
February 2 was the date of discount.
$\therefore$ 164 days = the term of discount (sec. 343, note 1).
6 %, the legal rate in Pennsylvania, is the rate of discount.
$\$0.027\frac{1}{3}$ = the bank discount on $1 for 164 days.
$150 \times \$0.027\frac{1}{3}$, or $4.10 = the required bank discount.
$150 − $4.10, or $145.90 = the proceeds.

2. Find the proceeds of a note of $250 dated Sept. 1, 1912, payable in 6 mo. with interest at 5 %, if this note was discounted at a New Hampshire bank Dec. 31, 1912.

Explanation:

$250.00 = the face of the note.
6.25 = the interest for 6 mo. at 5 %.
$256.25 = the amount of the note at maturity.
2.56 = the bank discount for 60 da. at 6 %.
$253.69 = the proceeds.

Exercise 87

Find the proceeds of the following notes:

	Date	Time	Face	Discounted
1.	June 1	4 mo.	$600	June 4, in New York
2.	May 15	90 da.	$200	May 16, in Maryland
3.	Mar. 12	60 da.	$250	Mar. 13, in Pennsylvania

	Date	Time	Face	Discounted
4.	Jan. 15	4 mo.	$225	Jan. 16, in Ohio
5.	Feb. 1	90 da.	$150	Feb. 1, in Michigan
6.	Feb. 1	4 mo.	$120	Feb. 5, in Idaho
7.	Oct. 20	90 da.	$500	Oct. 20, in Nebraska
8.	Sept. 1	3 mo.	$200	Sept. 1, in Massachusetts
9.	July 1	4 mo.	$318	July 1, in Maine
10.	Oct. 2	90 da.	$150	Oct. 9, in Indiana
11.	Sept. 1	3 mo.	$100	Sept. 15, in Oregon
12.	Sept. 30	90 da.	$155	Oct. 1, in West Virginia
13.	Nov. 1	3 mo.	$462	Nov. 2, in Washington
14.	Jan. 15	90 da.	$268	Jan. 16, in Vermont

Find the proceeds of the following notes:

	Face	Date	Time	Interest	Date of Discount	Rate of Discount
15.	$200	Feb. 1	6 mo.	5 %	June 1	6 %
16.	$350	June 1	6 mo.	4½ %	Oct. 15	6 %
17.	$500	Mar. 1	9 mo.	4½ %	June 15	6 %
18.	$275	Apr. 1	9 mo.	4 %	Nov. 1	6 %
19.	$250	Aug. 1	6 mo.	5 %	Jan. 2	6 %
20.	$218	Aug. 1	6 mo.	4½ %	Nov. 1	6 %

21. Find the proceeds of a note dated Mar. 10, payable in 60 da., if the face of the note was $600 and it was discounted Mar. 10 at a New Jersey bank.

22. Find the proceeds of a note of $268 dated May 2 and payable in 30 da., if it was discounted at an Illinois bank on May 10.

23. Find the proceeds of a note of $240 dated Sept. 19, 1912 and payable 6 mo. after date with interest at 5 %, if the note was discounted at a Pennsylvania bank Jan. 2, 1913.

24. What should be the face of a note payable 60 da. after date to yield proceeds of $350, if discounted at a New York bank on the day the note was made?

SUGGESTION. $0.01 = the discount on $1 for 60 da.
$0.99 = the proceeds of $1 of the note.
$350 = the entire proceeds.
$350 ÷ .99 = the required face.

25. What should be the face of a note dated June 1, and payable in 60 days, to yield $100 proceeds, if discounted at a Pennsylvania bank on the date the note was made?

26. What should be the face of a 3 months' note dated Jan. 12 to yield proceeds of $300 when discounted at an Indiana bank on the date the note was made?

DOMESTIC EXCHANGE

344. Exchange is the process of making payments by drafts. Exchange between two places in the same country is **domestic exchange.**

345. When the merchants and banks of one city, as Chicago, owe those of another city, as New York, to such an extent that gold must be sent from Chicago to New York to settle the balances, drafts drawn on merchants and banks in New York by those of Chicago will sell at a **premium** in Chicago; and drafts drawn by merchants and banks in New York on those of Chicago will sell at a **discount** in New York.

346. Premiums and discounts on drafts are usually reckoned as a certain per cent of their face value.

Thus, if a sight draft of $200 is sold at a premium of $\frac{1}{8}\%$, each dollar of the face is sold for $1.00\frac{1}{8}$; but if sold at a discount of $\frac{1}{8}\%$, each dollar of the face is sold for $0.99\frac{7}{8}$.

347. When exchange is at a premium, a time draft is worth its face minus the bank discount for the time the draft has yet to run, plus the exchange premium. When exchange is at a discount, a time draft is worth its face, minus the bank discount for the time the draft has yet to run, minus the exchange discount.

Thus, the cost in New York of a $500 draft due in 60 days, exchange $\frac{1}{4}\%$ premium, may be found as follows:

$0.01 = the bank discount on $1 for 60 da. at the legal rate.
0.99 = the cost of $1 of the face, if exchange were at par.
0.0025 = the premium on $1 of the face.
$0.9925 = the cost of $1 of the face.
500 × $0.9925, or $496.25 = the cost of the draft.

REMARK. If exchange were at $\frac{1}{4}\%$ discount, the cost of each $1 of the face would be $0.9875.

Exercise 88

1. Find the cost of a sight draft for $250 when exchange is at a premium of $\frac{1}{2}\%$.

2. Find the cost of a sight draft for $318 when exchange is at a discount of $\frac{1}{2}\%$.

3. What is the cost of a 60 days' sight draft for $150 when exchange is at a discount of $\frac{3}{4}\%$?

4. What is the cost of a 30 days' sight draft for $275 when exchange is at a premium of $\frac{3}{8}\%$?

5. Find the proceeds of a draft for $220 payable 90 days after sight, if discounted at a Richmond bank on the day of acceptance, exchange being $\frac{5}{8}\%$ premium.

6. What is the cost in Cincinnati on June 1 of a draft for $200 payable 30 days after sight and accepted May 24, exchange being $\frac{1}{2}\%$ discount?

RATIO, PROPORTION, AND PARTNERSHIP

RATIO

348. The **ratio** of one number or amount to another is the result obtained by finding how many times the first, called the **antecedent**, contains the second, called the **consequent**, as a measure.

349. The **sign** of ratio is the colon (:).

Thus, the expression 12 yd. : 3 yd. denotes that the number of times that 12 yd. contains 3 yd. as a measure is to be found. 12 yd. is the antecedent and 3 yd. is the consequent. The expression is read, *the ratio of 12 yd. to 3 yd.*

REMARK. The expression 12 yd. : 3 yd. is called *a ratio*, and may also be read *12 yd. measured by 3 yd.*

350. The antecedent and the consequent of a ratio are called its **terms**.

351. It is evident from Section 43 that:

(1) Finding the ratio is simply finding the quotient in comparative division.

(2) The antecedent corresponds to the dividend and the consequent to the divisor.

(3) If one term of a ratio is an amount, the other must be a similar amount.

(4) The ratio is always a number.

352. A ratio may be written as a fraction whose numerator is the antecedent and whose denominator is the consequent.

353. The **inverse** of a ratio is the ratio resulting from an interchange of terms.

Thus, the inverse of 4 : 5 is 5 : 4.

354. It is evident that 12 bu. contains 3 bu. as many times as 12 contains 3; hence,

12 bu. : 3 bu. = 12 : 3, or 12 ÷ 3. [§ 43]

The ratio of two similar amounts equals the number of units in the antecedent divided by the number of units in the consequent.

355. From Sections 86 and 87 it follows that:

Multiplying or dividing both terms of a ratio by the same number does not change the value of the ratio.

356. A ratio is in its *simplest form* when its terms are prime to each other.

Exercise 89

1. Find the ratio of $4\frac{1}{2}$ bu. to 2 bu.

Thus, $4\frac{1}{2}$ bu. : 2 bu. = $4\frac{1}{2} \div 2$, or $2\frac{1}{4}$.

2. Find the ratio of 4 qt. to $2\frac{1}{2}$ gal.

Thus, 4 qt. : $2\frac{1}{2}$ gal. = 4 qt. : 10 qt.
= 4 ÷ 10, or $\frac{2}{5}$.

Find the ratio of:

3. $2 : 5$ 4. $2\frac{1}{2} : 5$ 5. $4 : 3\frac{3}{4}$

6. $3\frac{1}{2}$ yd. : 15 yd. 7. $\frac{2}{3}$ hr. : $1\frac{1}{4}$ hr. 8. $\$\frac{3}{4} : \$\frac{7}{8}$

Find the missing antecedent (a), consequent (c), or ratio (r):

9. $\frac{10}{4} = r$ 10. $\frac{a}{4} = 12$ 11. $\frac{15}{c} = 5$ 12 $\frac{\frac{3}{4}}{4} = r$

13. $\frac{a}{\frac{1}{2}} = 2$ 14. $\frac{3}{c} = \frac{1}{4}$ 15. $\frac{\frac{1}{2}}{c} = 4$ 16. $\frac{a}{5} = .25$

17. In a period of eight years recently, certain foodstuffs advanced in price as given below; state at sight the ratio of the first price to the second:

(1) Tomatoes per can from 8¢ to 12¢.
(2) Peas per can from 10¢ to 12¢.
(3) Corn per can from 6¢ to 10¢.
(4) Apricots per pound from 20¢ to 25¢.
(5) Lamb chops per pound from 20¢ to 28¢.

18. Find the ratio of a short ton to a long ton.

19. A cubic foot of water weighs 62.5 lb. and a cubic foot of platinum weighs 1345.625 lb.; find, correct to the nearest hundredth, the ratio of the weight of platinum to that of water.

20. The ratio of a cubic foot to a stricken bushel is about $\frac{4}{5}$. How many cubic feet are there in a bin that holds 160 stricken bushels?

PROPORTION

357. An expression of equality between two equal ratios is called a **proportion.**

Thus, $2 : 3 = 6 : 9$ is a proportion, and may be read, *the ratio of 2 to 3 equals the ratio of 6 to 9.*

358. Since each of the two ratios in a proportion has two terms, a proportion has four terms; the first and fourth are called the **extremes**, the second and third the **means**, the first and third the **antecedents**, and the second and fourth the **consequents.**

359. The ratio of two similar amounts may equal the ratio of two other similar amounts; and the two ratios together form a proportion.

Thus, 12 yd. : 3 yd. = \$ 8 : \$ 2, since each ratio is 4.

360. From Section 354 we have:

$$12 \text{ yd.} : 3 \text{ yd.} = 12 : 3, \text{ and } \$8 : \$2 = 8 : 2;$$

therefore, the proportion 12 yd. : 3 yd. = \$ 8 : \$ 2 may be written,

$$12 : 3 = 8 : 2,$$

which is called a **numerical proportion**; that is, a proportion whose terms are all numbers.

361. The proportion $8 : 12 = 2 : 3$ may be written $\frac{8}{12} = \frac{2}{3}$; multiplying both members of the equation by 12×3, we have

$$\frac{\cancel{12} \times 3 \times 8}{\cancel{12}} = \frac{12 \times \cancel{3} \times 2}{\cancel{3}};$$

that is, $3 \times 8 = 12 \times 2$; therefore:

In any numerical proportion the product of the means equals the product of the extremes.

362. From $8 : 12 = 2 : 3$
we have $8 \times 3 = 2 \times 12$;
also, $8 = \frac{2 \times 12}{3}$, and $2 = \frac{8 \times 3}{12}$; therefore:

1. **In a numerical proportion one extreme equals the product of the means divided by the other extreme.**

2. **In a numerical proportion one mean equals the product of the extremes divided by the other mean.**

Exercise 90

In these examples x stands for the missing term; find the value of x:

1. $x : 8 = 4 : 16$
2. $9 : x = 12 : 8$
3. $32 : 16 = x : 2$
4. $20 : 25 = 100 : x$
5. $x : \frac{1}{2} = 6 : 12$
6. $\frac{3}{4} : x = 2 : \frac{2}{3}$
7. $x : 3\frac{1}{3} = 10\frac{1}{2} : 15$
8. $37\frac{1}{2} : 62\frac{1}{2} = 5 : x$
9. $\$x : \$5\frac{1}{2} = \$12 : \$18$
10. 10 yd. $: 3\frac{1}{3}$ yd. $= x : 2$
11. $\$x : \$87\frac{1}{2} = 6$ yd. $: 7$ yd.
12. 1 bu. $: 2\frac{1}{2}$ bu. $= \$x : \$10$

Types of Proportion Problems Considered

363. First Type. If 6 lb. of sugar cost 30¢, 12 lb. at the same rate would cost 60¢.

The statement may be abridged thus:

Weight	Cost
6 lb.	30¢
12 lb.	60¢

Observe:

(1) That when the weight is 6 lb., the corresponding cost is 30¢.

(2) That when the weight is 12 lb., the corresponding cost is 60¢.

(3) That $6 : 12 = 30 : 60$.

(4) That the ratio of the two values of weight equals the ratio of the corresponding values of cost.

(5) If any term of (3) were missing, it could be represented by x and found.

364. Two quantities are said to be directly proportional when the ratio of any two values of the one equals the ratio of the corresponding values of the other.

365. Second Type. If 6 lb. of sugar can be bought for a certain sum when sugar is 5¢ per pound, 3 lb. can be bought for the same sum when sugar is 10¢ per pound.

The statement may be abridged thus:

Weight	Cost
6 lb.	5¢
3 lb.	10¢

Observe:

(1) That when the weight is 6 lb., the corresponding price per pound is 5¢.

(2) That when the weight is 3 lb., the corresponding price per pound is 10¢.

(3) That $6 : 3 =$ the inverse of $5 : 10$; that is, $6 : 3 = 10 : 5$.

(4) That the ratio of the two values of weight equals the inverse of the ratio of the corresponding values of the price per pound.

(5) If any term of (3) were missing, it could be represented by x and found.

366. Two quantities are inversely proportional when the ratio of any two values of the one equals the inverse of the ratio of the corresponding values of the other.

367. Test to determine to which type a problem of proportion belongs:

(1) If when the second quantity is increased the first is also increased, the first is directly proportional to the second.

(2) If when the second quantity is increased the first is decreased, the first is inversely proportional to the second.

Illustrative Examples

1. Find the cost of 20 yd. of cloth, if 5 yd. cost $15.

Explanation:

Cost	Length
x	20 yd.
$15	5 yd.

The cost is *directly* proportional to the length; for if the length be increased, the cost will be increased; therefore:

$$x : 15 = 20 : 5.$$

$$x = \frac{15 \times 20}{5}, \text{ or } 60.$$

$$\therefore \$60 = \text{the cost.}$$

2. If a train running 40 mi. per hour runs a certain distance in 4 hr., in what time will it run the same distance if it runs 30 mi. per hour?

Explanation:

Time	Rate
x hr.	30 mi.
4 hr.	40 mi.

The time is inversely proportional to the rate; for if the rate be increased, the time will be decreased; therefore:

$$x : 4 = 40 : 30.$$

$$x = \frac{4 \times 40}{30}, \text{ or } 5\tfrac{1}{3}.$$

$\therefore 5\tfrac{1}{3}$ hr. = the time.

Exercise 91

1. In the following tell whether the first quantity named is directly or inversely proportional to the second:

(1) The cost of land and the quantity bought.

(2) The quantity of land bought for a certain price and the cost per acre.

(3) The principal on interest and the interest earned.

(4) The time required to go a certain distance and the speed.

(5) The time required to do work and the force.

(6) The quantity of work done in a day and the force working.

(7) The force required to do a piece of work and the time.

2. Steers should be fed 2.84 lb. of hay, or its equivalent, for each 100 lb. of their live weight. How much hay, or its equivalent, should be fed per day to a steer which weighs 1050 lb.?

3. If a 10-acre field of corn produces 600 bushels, what should a 6-acre field produce at the same rate?

4. When coal is retailing at $0.38 per 100 lb., how much is that per ton of 2000 lb.?

5. When hay is selling for $24 per ton, what should be charged for 1750 lb. of hay?

6. When hair for plastering is sold at the rate of \$ 0.25 per 7 lb., how much can be bought at the same rate for \$ 1.50 ?

7. When $2\frac{1}{2}$ in. on a certain scale represent 100 mi., what distance is represented on the same scale by 15 in. ?

8. It is estimated that 100 lb. of hay possesses the same feeding value as 58 lb. of barley; how much barley has the same feeding value as 1 T. of hay ?

9. If an arc of 30° measures 6 in., what should an arc of 100° on the same circle measure ?

10. If \$1.50 will buy a certain quantity of sugar when it sells for \$0.05 per pound, what would the same quantity cost when sugar sells at \$ 0.08 per pound ?

11. If a train running 40 mi. an hour runs a certain distance in 5 hr., in what time will it run the same distance at the rate of 25 mi. per hour ?

12. A certain sum will buy 100 lb. of sugar at \$ 0.05 a pound; what weight will the same sum buy if the sugar is sold for \$ $0.06\frac{1}{4}$ a pound ?

13. An Edison incandescent lamp burning 4 hr. each night will last about 200 nights; how many nights would it last if it were burned 3 hr. each night ?

14. In a plan of a farm 6.25 in. is made to represent a side 1.05 rd. long; what would represent a side 8.4 rd. long?

15. If 40 bu. of grain will last a herd of cows a week when each is fed 4 qt. per day, how long would the same quantity last them, if each were fed 5 qt. per day ?

16. A farmer's crop of 240 bu. of oats weighs 30 lb. to the bushel; if he sells it to a miller who takes 32 lb. to the bushel, for how many bushels should the farmer be paid?

PROPORTIONAL PARTS

Illustrative Examples

1. Divide 1260 into parts proportional to 2, 3, and 4.

Explanation:

The number is to be divided into $2 + 3 + 4$, or 9 equal parts.
Then two of these parts, or $\frac{2}{9}$ of 1260, or $280 =$ the first part.
And 3 of these parts, or $\frac{3}{9}$ of 1260, or $420 =$ the second part.
And 4 of these parts, or $\frac{4}{9}$ of 1260, or $560 =$ the third part.
Check: $280 + 420 + 560 = 1260$.

2. Two numbers are to each other as 7 is to 2, and their difference is 75. What are the numbers?

Explanation:

Let l represent the larger and s the smaller.

Then $l : s = 7 : 2$.

$2\,l = 7\,s$.

$l = \frac{7}{2}\,s$.

$s = s$.

$l - s = \frac{5}{2}\,s$, or 75.

$\frac{1}{2}\,s = 15$.

$s = 30$.

$\frac{7}{2}\,s$, or $l = 105$.

Check: $105 - 30 = 75$.

Exercise 92

1. Divide $1250 into parts proportional to 3 and 2.

2. Divide 500 into parts proportional to 2, 3, and 5.

3. The sum of 2 numbers is 480, and the larger is to the smaller as 9 is to 7. What are the numbers?

4. The difference of two numbers is 50, and the larger is to the smaller as 6 is to 5. What are the numbers?

5. A and B together invested $225. A invested $7 as often as B invested $3. How much did each invest?

6. Divide 20 into parts proportional to $\frac{1}{2}$ and $\frac{1}{3}$.

SUGGESTION. 1st part : 2d part $= \frac{1}{2} : \frac{1}{3} = 3 : 2$.

7. Solder for lead consists of 1 part tin to $1\frac{1}{2}$ parts lead; how much of each of these substances is necessary to make 5 lb. of solder?

8. A, B, and C are charged $36 freight on a shipment of potatoes, of which A shipped 200 bu., B 300 bu., and C 500 bu.; how much should each pay?

9. Three men received $62.50 for husking a field of corn; the first husked 300 shocks, the second 450 shocks, and the third 500 shocks; how much should each receive?

10. After ducklings are 10 days old, a good feed for them consists of the following parts by weight:

3 parts bean	$1\frac{1}{2}$ parts beef scrap
1 part corn meal	$2\frac{1}{2}$ parts green food
1 part flour	

How many pounds of each ingredient are there in 360 lb. of this mixture?

PARTNERSHIP

368. A **partnership** is an association of individuals, called **partners**, for the transaction of business.

NOTE. A partnership is sometimes called a **firm**.

369. The **capital stock** is the amount which the partners invest in the business.

NOTE. The **assets** of a firm are all the property belonging to the firm. The **liabilities** are all the debts of the firm.

370. When the shares of the partners are invested for equal times, gains and losses are distributed among the partners in proportion to their respective shares.

371. When the shares of the partners are invested for unequal times, gains and losses are distributed among the partners in proportion to their respective shares and times, except when by special agreement some other plan is adopted.

Illustrative Examples

1. A, B, and C form a partnership, A investing \$ 600, B \$ 800, and C \$ 1000. Divide a gain of \$ 300 among them.

Explanation:

\$ 600 + \$ 800 + \$ 1000, or \$ 2400 = the capital stock.
$\frac{1}{4}$ of the capital stock = A's investment.
$\frac{1}{3}$ of the capital stock = B's investment.
$\frac{5}{12}$ of the capital stock = C's investment.
\$ 300 = the gain to be divided.

The gain is distributed in proportion to the investments.

$\therefore$ $\frac{1}{4}$ of \$ 300, or \$ 75 = A's gain.
$\frac{1}{3}$ of \$ 300, or \$ 100 = B's gain.
$\frac{5}{12}$ of \$ 300, or \$ 125 = C's gain.

2. Three boys invested their savings in a news stand; A invested \$50 Jan. 1, B \$30 Mar. 1, and C \$20 Aug. 1. At the end of the year they had a profit of \$80 to divide among them; find each one's share.

Explanation:

A's capital was invested 12 mo., B's 10 mo., and C's 5 mo.
A's gain = the gain on \$50 for 12 mo., or the gain on \$600 for 1 mo.
B's gain = the gain on \$30 for 10 mo., or the gain on \$300 for 1 mo.
C's gain = the gain on \$20 for 5 mo., or the gain on \$100 for 1 mo.
The total gain = the gain on \$1000 for 1 mo.
The total gain = \$80.
$\frac{6}{10}$ of \$80, or \$48 = A's gain.
$\frac{3}{10}$ of \$80, or \$24 = B's gain.
$\frac{1}{10}$ of \$80, or \$8 = C's gain.

Exercise 93

1. Three men invested jointly \$3000 in a creamery. At the end of the first year their gains were \$1500. The first invested \$800, the second \$1000, and the third \$1200; find each one's share of the profit.

2. A and B in partnership gain in 1 year \$8560. A contributes \$8000 and B \$7000. A is to retain 25% of the profits for managing the business; how much of the balance should each receive?

3. A merchant failing in business has property to the value of \$3200. He owes A \$4500, B \$2500, and C \$2000; how much should each receive?

4. A, B, and C established a coal yard, A furnishing \$5000, and B \$4000. C's gain for the first year was

$198. Find C's capital and A's and B's gain, if the total capital was $18,900.

5. A and B formed a partnership, A putting in $600 for 18 months, and B $800 for a year. They gained $510; what was each partner's share of the gain?

6. A, B, and C formed a partnership. A invested $500 on Jan. 1, B $400 on Mar. 1, and C $300 on July 1. At the end of the year they had gained $555; what was each partner's share of the gain?

7. A, B, and C take the contract for building a public road. A furnishes 12 men for 60 days, B 20 men for 40 days, and C 10 men for 8 days. They receive for the work $4200, of which A is paid $200 for supervising the work; how much of the balance should each of the men receive?

SUGGESTION. The work of 12 men for 60 days equals the work of 720 men for 1 day.

8. A, B, and C contracted to do a piece of work. A sent 24 men for 45 da., B 40 men for 25 da., and C 35 men for 42 da. If the sum paid for doing the work was $10,650, find each partner's share.

9. A, B, and C invested jointly $3000. A's capital was invested 12 mo., and his gain was $144; B's capital was invested 8 mo., and his gain was $144; C's capital was invested 6 mo., and his gain was $180. Find the capital of each.

POWERS, SQUARE ROOT, AND MENSURATION

POWERS

372. The **square** of a number is the result of using the number twice as a factor.

Thus, the square of 2 equals 2×2, or 4; the square of $\frac{1}{2}$ equals $\frac{1}{2} \times \frac{1}{2}$, or $\frac{1}{4}$; the square of .3 equals $.3 \times .3$, or .09.

373. The **cube** of a number is the result of using the number three times as a factor.

Thus, the cube of .5 equals $.5 \times .5 \times .5$, or .125.

374. A small figure called an **exponent**, written to the right and above a number, denotes how many times the number is to be used as a factor.

Thus, 7^2, read *7 squared*, equals 7×7, or 49; and 7^3, read *7 cubed*, equals $7 \times 7 \times 7$, or 343.

Exercise 94

1. Name the square of each number from 1 to 12.
2. Name the square of each of the nine tens.
3. Name the square of each of the nine hundreds.
4. Name the cube of each number from 1 to 5 inclusive.
5. Name the square and also the cube of $\frac{1}{2}$; $\frac{1}{3}$; $\frac{1}{4}$; $\frac{1}{5}$; .2; .3; .5; $\frac{2}{3}$; $\frac{3}{2}$; .1; .01; .001.

Find the square and also the cube of each of the following:

6. 14	7. 37	8. 45	9. 123	10. 417
11. 841	12. $\frac{3}{4}$	13. $6\frac{1}{4}$	14. $\frac{7}{8}$	15. $87\frac{1}{2}$
16. .75	17. .461	18. .079	19. 4.97	20. .0025

SQUARE ROOT

375. The **square root** of a number is the number whose square is the given number.

Thus, the square root of 49 is 7, since $7^2 = 49$; the square root of $\frac{16}{25}$ is $\frac{4}{5}$, since $(\frac{4}{5})^2 = \frac{16}{25}$.

376. The symbol $\sqrt{\ }$, called the **radical sign,** written before a number, indicates that the square root of the number is to be taken.

Thus, $\sqrt{9}$, read *the square root of 9,* equals 3.

Exercise 95

1. Name the square root of: 25; 9; 16; 121; 36; 81; 49; 100; 144.

2. Name the square root of: $\frac{1}{4}$; $\frac{4}{9}$; $\frac{25}{16}$; $\frac{36}{25}$; .01; 25; .81; .64.

3. Find the square root of: .0004; .0009; .0016; .0025; .0036; .0049; .0064; .0081.

4. Find the square root of 441.

Thus, $441 = 3 \times 3 \times 7 \times 7,$
$= (3 \times 7) \times (3 \times 7),$
$= 21 \times 21;$
therefore, $\sqrt{441} = 21.$

Find the square root of:

5. 196 6. 225 7. 484 8. 1089

9. 1225 10. 1296 11. 2601 12. 3025

13. Extract the square root of 576.

SUGGESTION. $576 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3$.

Now take one of each two equal factors and multiply them together.

Extract the square root of:

14. 2025 15. 1521 16. 1764 17. 11025

18. Find the length of a square that contains 64 sq. ft.

SUGGESTION. Since the area of a square is the square of the number of units in its length, the number of units in its length is equal to the square root of its area.

Find the lengths of squares that contain:

19. 100 sq. rd. 20. 625 sq. yd. 21. .0025 sq. mi.

General Rule for extracting Square Root

377. The square root of numbers may be found by the following rule:

1. **Point off the number into periods of two figures each, beginning with the units.**

2. **Take the square root of the nearest square number below that expressed by the first period on the left for the first figure of the root.**

3. **Subtract the square of the first figure of the root from the first period of the number and annex the next period to the remainder for a dividend.**

4. **Take twice the root found, as a trial divisor, and divide the dividend, exclusive of the right-hand figure, by it for the next term of the root.**

5. **Annex the last figure of the root now found to the trial divisor for the complete divisor.**

6. **Multiply the complete divisor by the last figure of the root now found and subtract the product from the remainder.**

7. **If the root is not fully found, repeat the above processes until all the periods of the number have been used.**

8. **Should a cipher occur in the root, annex a cipher to the trial divisor to form the next trial divisor, and bring down the next period.**

Illustrative Examples

1. Extract the square root of 217156.

Explanation:

```
 21'71'56 | 466
   16
86 | 571
     516
926 | 5556
      5556
```

Point off the number into periods of two figures each, beginning at units. The greatest number whose square is contained in 21 is 4, which is the first figure of the root. Square 4; subtract the result from 21, and annex 71, the next period, to the remainder, making the dividend 571. Double 4, the root found, for a trial divisor. Divide 571, exclusive of the right-hand figure, by 8 for the next figure of the root, which is found to be 6. Annex 6 to 8, making the complete divisor 86. Multiply 86 by 6, the last figure of the root found; subtract the product 516 from 571, and annex 56, the

last period, to the remainder, making the next dividend 5556. Double 46, the root already found, for a trial divisor. Divide 5556, exclusive of the right-hand figure, by 92 for the next figure of the root, which is found to be 6. Annex 6 to 92, making the complete divisor 926. Multiply 926 by 6, the last figure of the root found, and subtract the product from 5556; nothing remains. Therefore, the square root of 217156 is 466.

2. Extract the square root, correct to hundredths, of 142.3.

$$
\begin{array}{r|l}
1\,|42.\,|30\,|00\,|00 & 11.928 \\
1 & \\
\hline
21 \;|\; 42 & \\
\quad\;\; 21 & \\
\hline
229 \;|\; 2130 & \\
\quad\;\; 2061 & \\
\hline
2382 \;|\; 6900 & \\
\quad\;\; 4764 & \\
\hline
23848 \;|\; 213600 &
\end{array}
$$

Suggestion. Point off the number into periods of two figures each, rightward and leftward from the decimal point. The square root is found to be nearer 11.93 than 11.92; hence, the result is taken to be 11.93.

Exercise 96

Extract the square root of:

1. 1024	2. 7396	3. 524176	4. 498436
5. 2116	6. 9025	7. 258064	8. 577600
9. 3481	10. 14161	11. 318096	12. 643204

Extract the square root of:

13. 53.29	14. 7.7284	15. 5852.25	16. 65.2864

Extract the square root, to the nearest hundredth, of:

17. 32	18. 321	19. 3.42	20. .023

21. Extract the square root of $\frac{16}{25}$.

Suggestion. Since $\frac{4}{5} \times \frac{4}{5} = \frac{16}{25}$, the square root of $\frac{16}{25} = \frac{4}{5}$.

NOTE. If a fraction in its lowest terms is a perfect square, its terms are perfect squares. When the terms of a fraction in its lowest terms are not perfect squares, it is best to reduce it to a decimal before extracting the square root.

Extract the square root of:

22. $\frac{121}{144}$ 23. $\frac{256}{729}$ 24. $7\frac{1}{9}$ 25. $\frac{7}{8}$ to hundredths

MENSURATION OF PLANE SURFACES

(Lines, angles, the square, and the rectangle have been treated; see Sections 128–143.)

General Definitions

378. Anything that has length and breadth without thickness is called **surface.**

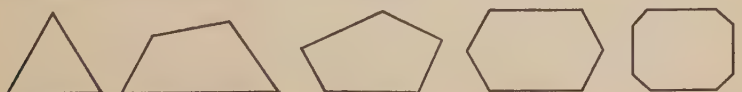
379. A surface on which any number of straight lines can be drawn in different directions is called a **plane surface,** or a **plane.**

380. Any portion of a plane surface bounded by straight lines is called a **polygon.**

Polygon

381. The points in which the sides of a polygon meet are the **vertices,** the sum of the sides is the **perimeter,** and a line joining two vertices, but not forming a side, is a **diagonal.**

382. Polygons are classified according to the number of their sides; some of the classes are *triangles*, *quadrilaterals*, *pentagons*, *hexagons*, and *octagons*.


Triangle Quadrilateral Pentagon Hexagon Octagon

383. (1) A **triangle** is a polygon of three sides.
(2) A **quadrilateral** is a polygon of four sides.
(3) A **pentagon** is a polygon of five sides.
(4) A **hexagon** is a polygon of six sides.
(5) An **octagon** is a polygon of eight sides.

Triangles

384. Triangles are classified according to angles, into *right triangles*, *acute triangles*, and *obtuse triangles*.

385. (1) A **right triangle** is a triangle which has a right angle. The side opposite the right angle is called the **hypotenuse**.
(2) An **acute triangle** is a triangle whose angles are all acute.
(3) An **obtuse triangle** is a triangle which has an obtuse angle.

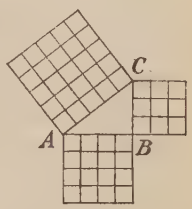
386. Triangles are classified according to sides, into *equilateral triangles*, *isosceles triangles*, and *scalene triangles*.

387. (1) An **equilateral triangle** is a triangle all of whose sides are equal.
(2) An **isosceles triangle** is a triangle two of whose sides are equal.
(3) A **scalene triangle** is a triangle no two of whose sides are equal.

388. The **base** of a triangle is the side upon which it is supposed to stand, and the **altitude** is the perpendicular drawn from the vertex opposite the base to the base, or the base extended.

Important Facts

1. *In a right triangle the square on the hypotenuse equals the sum of the squares on the other two sides.*



Thus, the square whose side is AC contains as many square units as the two squares whose sides are AB and BC.

2. *The sum of the angles of a triangle is 180°.*

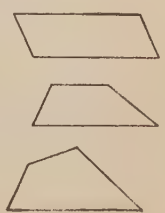
3. *The angles opposite the equal sides of an isosceles triangle are equal.*

4. *Each angle of an equilateral triangle is 60°.*

5. *If a right triangle has an angle of 30°, the side opposite this angle is one half the hypotenuse.*

Quadrilaterals

389. Quadrilaterals are divided into *parallelograms*, *trapezoids*, and *trapeziums*.



390. (1) A **parallelogram** is a quadrilateral whose opposite sides are parallel.

(2) A **trapezoid** is a quadrilateral two, and only two, of whose sides are parallel.

(3) A **trapezium** is a quadrilateral no two of whose sides are parallel.

391. The side on which a parallelogram is supposed to rest is called the **base**, and the perpendicular drawn from the side opposite the base to the base is called the **altitude.**

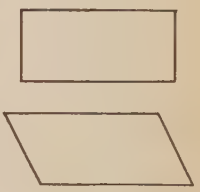
392. The parallel sides of a trapezoid are called the **bases**, and the perpendicular joining the bases is called the **altitude.**

Parallelograms

393. Parallelograms are divided into *rectangles* and *rhomboids.*

394. (1) A **rectangle** is a parallelogram whose angles are right angles. A rectangle whose sides are all equal is **a square.**

(2) A **rhomboid** is a parallelogram whose angles are not right angles. A rhomboid whose sides are all equal is a **rhombus.**



Important Facts

1. *The opposite sides of a parallelogram are equal.*
2. *The sum of any two adjacent angles of a parallelogram is 180°.*
3. *The diagonals of a parallelogram bisect each other.*
4. *A diagonal of a parallelogram divides the parallelogram into two equal triangles.*
5. *The diagonals of a rectangle are equal.*

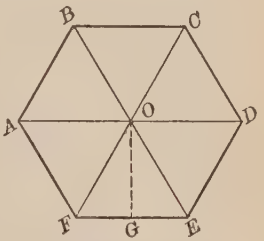
Regular Polygons

395. A **regular polygon** is a polygon whose sides and whose angles are equal.

396. The **center** of a regular polygon is a point within equally distant from the vertices; as the point *O.*

397. A **radius** of a regular polygon is a straight line drawn from the center to a vertex; as OA.

398. An **apothem** of a regular polygon is a perpendicular drawn from the center to a side; as OG.



A Regular Polygon

Important Facts

1. *The radii of a regular polygon are equal.*

2. *The apothems of a regular polygon are equal.*

3. *The radii of a regular polygon divide it into equal isosceles triangles.*

4. *The side of a regular hexagon equals the radius.*

5. *The apothem of a regular hexagon equals* $\frac{1}{2}\sqrt{3}$ *times the radius.*

Exercise 97

1. Cut from paper or cardboard figures like those shown in Section 382, and learn their names.

2. Cut from paper or cardboard triangles like those shown in Section 384, and learn their names.

3. Cut from cardboard triangles like those shown in Section 386, and learn their names.

4. If the base of a right triangle is 2 and its altitude is 1, what is the square of its hypotenuse?

5. Two angles of a triangle are 60° and 70°; what is the third angle?

6. The angle opposite one of the equal sides of an isosceles triangle is 50°; how many degrees are there in each of the other angles of the triangle?

7. One angle of an equilateral triangle is 60°; how many degrees are there in each of the other angles?

8. If one angle of a right triangle is 20°, how many degrees are there in each of the other angles?

9. One angle of a triangle is 100° and another is 30°; is the third angle acute, right, or obtuse?

10. Two angles of a triangle are 20° and 50°; is the third angle acute, right, or obtuse?

11. Two angles of a triangle are 30° and 60°; is the third angle acute, right, or obtuse?

12. Cut from paper or cardboard quadrilaterals like those shown in Section 390, and learn their names.

13. Cut from paper or cardboard parallelograms like those shown in Section 394, and learn their names.

14. One side of a parallelogram is 10 ft.; how long is the opposite side?

15. One diagonal of a rectangle is 6 ft.; how long is the other diagonal?

16. If one angle at the base of a right triangle is 30° and the hypotenuse is 12 ft., what is the altitude?

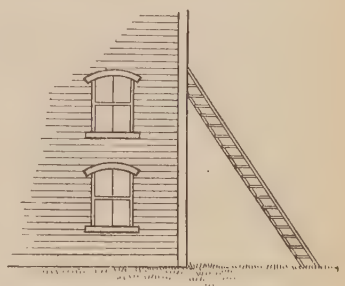
17. The base of a right triangle is 8 ft. and the altitude is 6 ft.; what is the hypotenuse?

SUGGESTION. $h = \sqrt{8^2 + 6^2}$, in which h stands for the number of units in the hypotenuse.

18. The hypotenuse of a right triangle is 29 ft. and the base is 21 ft.; what is the altitude?

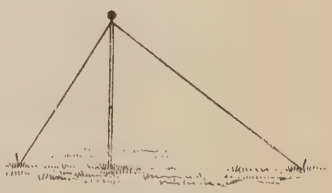
SUGGESTION. $a^2 + b^2 = h^2$, in which a, b, and h stand for the number of units in altitude, base, and hypotenuse, respectively; therefore, $a^2 = h^2 - b^2$ (§ 206).

19. The hypotenuse of a right triangle is 15 ft. and its altitude is 12 ft.; what is the base?



20. A ladder 20 ft. long is placed against a house so that the foot is 12 ft. from the house and on the same level with it; what is the distance from the top of the ladder to the ground?

21. A pole 52 ft. high stands on level ground; how long a rope will it take to reach from the top to a stake 39 ft. from the foot of the pole?



22. Two ropes, one 36 ft. long and the other 50 ft. long, are fastened 30 ft. above the ground to a pole, and are stretched to reach two stakes on opposite sides of the pole; how far apart are the two stakes, the surrounding ground being level?

23. Find the length of the longest line that can be drawn on a floor 24 ft. long and 18 ft. wide.

Areas

Finding the Area of a Parallelogram

399. If the shaded part of the parallelogram (Fig. 1) be cut off and placed as shown in Fig. 2, the parallelogram will be changed into a rectangle of the same base and altitude. The area of a rectangle is equal to the product of the number of units in the base and the number of units in the altitude; therefore:

Fig. 1.

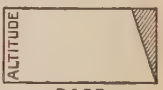


Fig 2.

400. The area of a parallelogram is equal to the product of the number of units in the base and the number of units in the altitude.

NOTE. The base and altitude must be stated in terms of the same unit.

401. If b represents the number of units in the base, a the number of units in the altitude, and S the area of a parallelogram,

$$S = a \times b.$$

Finding the Area of a Triangle

402. Each triangle in black is one half of a parallelogram of the same base and altitude, as may be seen by making figures like these, and by placing the shaded part of each upon the other part.

The area of a parallelogram is equal to the product of the number of units in the base and the number of units in the altitude; therefore:

403. The area of a triangle is equal to one half of the product of the number of units in the base and the number of units in the altitude.

404. If b represents the number of units in the base, a the number of units in the altitude, and S the area of a triangle,

$$S = \frac{a \times b}{2}.$$

405. To find the area of a triangle when its three sides are given, apply the following:

From half the sum of the three sides, subtract each side separately; multiply together the number of units in the half sum and in each of the three remainders, and extract the square root.

Illustrative Example

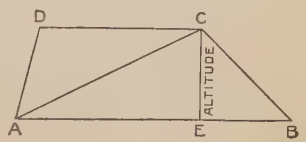
Let it be required to find the area of a triangle whose sides are 5 ft., 7 ft., and 8 ft. long.

$\frac{1}{2}$ of $(5+7+8)$, or $10 =$ the number of units in $\frac{1}{2}$ the perimeter.

$\therefore \sqrt{10 \times (10-5) \times (10-7) \times (10-8)}$, or $\sqrt{300}$, or $17.32 =$ the number of square feet in the triangle.

Finding the Area of a Trapezoid

406. Trapezoid $ABCD$ is divided into two triangles by the diagonal AC. The altitude of the trapezoid is the altitude of each triangle.



Area of triangle $CAB = \frac{1}{2}\ CE \times AB$.
Area of triangle $ADC = \frac{1}{2}\ CE \times DC$.
Area of trapezoid $ABCD = \frac{1}{2}\ CE \times (AB + DC)$.

407. The area of a trapezoid is equal to the product of half the number of units in the altitude and the number of units in the sum of the bases.

408. If B represents the number of units in the lower base, b the number of units in the upper base, a the number of units in the altitude, and S the area of a trapezoid,

$$S = \tfrac{1}{2}\, a(B + b).$$

Finding the Area of a Regular Polygon

409. A regular polygon may be divided into triangles, as shown here, each triangle having for its base a side of the polygon, and for its altitude the apothem of the polygon.

The sum of the bases of the triangles is the perimeter of the polygon.

The area of each triangle is equal to half the product of the apothem and a side of the polygon [§ 403]. The sum of the areas of the triangles is equal to half the product of the number of units in the apothem and the number of units in the sum of the sides; or:

410. The area of a regular polygon is equal to half the product of the number of units in the apothem and the number of units in the perimeter.

411. If a represents the number of units in the apothem, P the number of units in the perimeter, and S the area of a regular polygon,

$$S = \frac{a \times P}{2}.$$

Illustrative Examples

1. Find the area of a parallelogram whose base is 12 ft. and altitude 4 ft. 6 in.

Explanation:

12 ft. = the base.
4 ft. 6 in., or $4\frac{1}{2}$ ft. = the altitude.
$\therefore 4\frac{1}{2} \times 12$, or 54 = the number of square feet required.

Those who prefer to use formulæ may solve thus:
In a parallelogram, $S = a \times b$.
In the given parallelogram, $a = 4\frac{1}{2}$ and $b = 12$.
$\therefore S = 4\frac{1}{2} \times 12$, or 54.

2. Find the area of a triangle whose base is 5 rd. and altitude 20 yd.

Explanation:

5 rd., or $27\frac{1}{2}$ yd. = the base.
20 yd. = the altitude.
$\therefore \frac{20 \times 27\frac{1}{2}}{2}$, or 275 = the number of square yards required.

Remark. Those who desire to do so may use formula of Section 404.

3. Find the area of a trapezoid whose bases are 15 ft. and 7 ft., and whose altitude is 5 ft.

Explanation:

15 ft. and 7 ft. = the bases.
5 ft. = the altitude.
$\therefore \frac{5}{2} \times (15 + 7)$, or 55 = the number of square feet required.

Remark. Those who desire to do so may use formula of Section 408.

4. Find the area of a regular hexagon one of whose sides is 4 ft. and whose apothem is 3.46 ft.

Explanation:

6×4 ft., or 24 ft. = the perimeter.

3.46 ft. = the apothem.

$\therefore \frac{3.46 \times 24}{2}$, or 41.52 = the number of square feet required.

REMARK. Those who desire to do so may use formula of Section 411.

Exercise 98

Find the area of each of these parallelograms:

	Base	Altitude		Base	Altitude
1.	10 ft.	8 ft.	2.	9 ft.	5 ft.
3.	8 yd.	7 ft.	4.	3 rd.	4 yd.
5.	6 ft. 4 in.	9 ft.	6.	4 yd. 2 ft.	7 ft.

Find the area of each of these triangles:

	Base	Altitude		Base	Altitude
7.	12 ft.	7 ft.	8.	10 ft. 6 in.	20 ft.
9.	8 yd.	21 ft.	10.	4 rd. 2 ft.	12 ft.
11.	6 ch.	8 rd.	12.	4 ft. 9 in.	6 ft. 6 in.

Find the area of each of these trapezoids:

	Bases	Altitude		Bases	Altitude
13.	6 ft. and 4 ft.	10 ft.	14.	12 ft. and 3 yd.	10 ft.
15.	7 yd. and 6 ft.	9 ft.	16.	6 ft. and $4\frac{1}{2}$ ft.	2 ft.
17.	8 ch. and 3 ch.	6 ch.	18.	5 ft. and $6\frac{1}{2}$ ft.	2 yd.

In the following find the missing number:

Class of Polygon	Base	Altitude	Area
19. Parallelogram	8 yd.	6 ft.	( ) sq. yd.
20. Parallelogram	12 ft.	( ) ft.	72 sq. ft.
21. Parallelogram	( ) yd.	6 ft.	144 sq. ft.
22. Triangle	40 ft.	8 ft.	( ) sq. ft.
23. Triangle	10 yd.	( ) yd.	75 sq. yd.
24. Triangle	( ) rd.	30 rd.	150 sq. rd.
25. Parallelogram	8 ch.	5 ch.	( ) acres
26. Parallelogram	12 ch.	( ) ch.	24 acres
27. Triangle	( ) ch.	20 rd.	5 acres

28. Find the area of a regular hexagon one of whose sides is 8 ft., and whose apothem is 6.93 ft.

29. Find the area of a regular octagon one of whose sides is 10 ft., and whose apothem is 12.07 ft.

30. Find the area of a regular pentagon one of whose sides is 4 ft., and whose apothem is 2.75 ft.

31. Find the area of a triangle whose sides are 13 ft., 14 ft., and 15 ft.

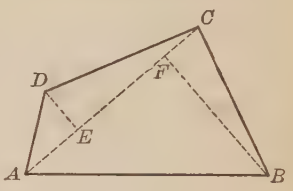
32. The base of a rectangle is 48 ft. and the altitude is 36 ft. What is the diagonal?

33. A field in the form of a rhomboid contains 9 A. 96 sq. rd., and the altitude is 32 rd. What is the base?

34. The perimeter of a rectangle is 112 ft. and the width is 24 ft. What is the area?

35. A square lot of land that contains 1 acre is how many feet long? Answer correct to the nearest tenth of a foot.

36. To find the area of trapezium $ABCD$, find the areas of the triangles of which it is composed. Diagonal AC is 34 ft. long and the perpendiculars DE and BF upon the diagonal are 4 ft. and 13 ft., respectively.



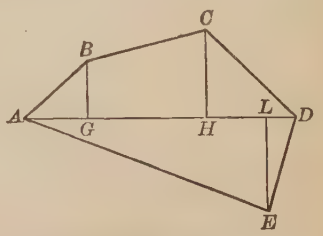
37. How many board feet are there in a board 16 ft. long, 18 in. wide at one end, and 12 in. wide at the other end?

38. The perimeter of a rectangle that is 80 rd. long is 200 rd.; how many acres does it contain?

39. The length of a rectangle is to its width as 5 is to 4, and it contains 8 A. Find its length and its width in chains.

40. One side of a field in the form of a trapezoid measures 24 ch. 80 li., another side parallel to it measures 18 ch. 40 li., and the distance between these sides is 12 ch. 60 li. How many acres does the field contain?

41. Find the area of the polygon $ABCDE$ in which $AG=2$ ch., $GH=4$ ch., $HL=2$ ch., $LD=1$ ch., and the perpendiculars $BG=2$ ch., $CH=3$ ch., and $EL=3$ ch.



NOTE. The area of a polygon may be found by dividing it into right triangles and trapezoids, and taking the sum of their areas.

The Circle

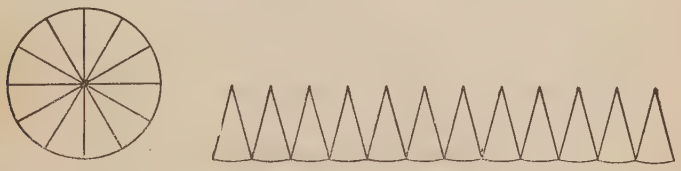
(For definition of terms and important facts see page 124.)

412. It is proved in geometry that the ratio of the circumference of a circle to the diameter is 3.1416, nearly. The Greek letter π (pi) stands for this ratio; then if the circumference is represented by C, the diameter by D, and the radius by R,

$$(1)\ \frac{C}{D}=\pi. \quad (2)\ C=\pi D. \quad (3)\ C=2\pi R.$$

NOTE. Sometimes $\frac{22}{7}$ instead of 3.1416 is used to express the ratio of the circumference to the diameter.

413. A circle may be cut and separated, as here shown, into a series of figures that resemble triangles.



If we should call them triangles, each figure might then be regarded as having the curved side for the base and the radius of the circle for the altitude. The sum of the areas of the figures, or the area of the circle, would be the product of half the number of units in the radius and the number of units in the sum of the bases. It is shown in geometry that the area of the circle is equal to this product; therefore:

414. The area of a circle is equal to the product of half the number of units in the radius and the number of units in the circumference.

415. If C represents the number of units in the circumference, R the number of units in the radius, and S the area of the circle,

$$S = \tfrac{1}{2} R \times C.$$

Since $C = 2\pi R$,

$$S = \tfrac{1}{2} R \times 2\pi R.$$

Or, $S = \pi R^2$ (the area in terms of the radius).

Illustrative Examples

1. Find the area of a circle whose radius is 4 ft.

Explanation:

4 ft. = the radius.

In a circle, $S = \pi R^2$.

$S = 3.1416 \times 4^2$, or 50.2656.

That is, the circle contains 50.2656 sq. ft.

2. Find the area of a circle whose circumference is 31.416 ft.

Explanation:

31.416 ft. = the circumference.

$C = 2\pi R$.

$R = 31.416 \div 6.2832$, or 5.

$S = \pi R^2$.

$\therefore S = 3.1416 \times 5^2$, or 78.54.

That is, the circle contains 78.54 sq. ft.

Exercise 99

Find the areas of circles whose radii are as follows:

1. 6 in. 2. 4 yd. 3. 12 rd. 4. 2 ch. 5. 2 ft. 6 in.

Find the areas of circles whose diameters are as follows:

6. 10 yd. 7. 5 ch. 8. 6.4 ft. 9. 2.75 rd. 10. 5.9 ch.

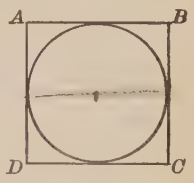
Find the areas of circles whose circumferences are as follows:

11. 18.8496 ft. 12. 31.416 rd. 13. 3.1416 ch.

14. 12.5664 in. 15. 628.32 yd. 16. 157.08 ft.

17. Find the perimeter and area of a square circumscribed about a circle whose diameter is 8 ft.

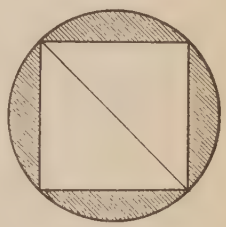
18. Find the difference between the circumference of a circle and the perimeter of a circumscribed square whose side is 12 ft.



A Circumscribed Square

19. Find the difference between the area of a circle and the area of a circumscribed square whose length is 10 ft.

20. A square courtyard, as large as possible, is made within a circle whose diameter is 66 ft., and the remainder of the circle is sodded. How many square feet are sodded? Answer correct to the nearest .1 ft.



SUGGESTION. The diameter of the circle is the diagonal of the square.

21. A cow is fastened by a rope 20 yards long to a stake in the center of a square lot containing 1600 sq. yd. Over how much land can she graze?

22. Find the area of the largest square that can be cut from a circular board whose diameter is 20 in.

MENSURATION OF SOLIDS

General Definitions

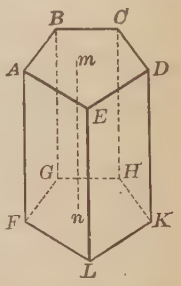
416. A **solid** is any bounded portion of space; as, a cube.

417. The **volume** of a solid is the number of cubic units it contains.

Right Prisms and Cylinders of Revolution

Definitions

418. A **right prism** is a solid bounded by two equal and parallel polygons, called **bases**, and by rectangles called **lateral faces**.



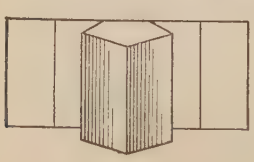
419. The sum of the areas of the lateral faces is called the **lateral area**.

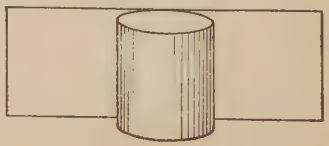
The perpendicular line joining the bases is the **altitude**; as, *mn*.

REMARK. For treatment of the cube and rectangular solid, which are classes of right prisms, see pp. 118, 119. For partial treatment of the cylinder of revolution, see pp. 128, 129. In this book right prisms and cylinders of revolution only are treated; they will be called merely prisms and cylinders.

Areas

420. The lateral surface of a prism or of a cylinder may be thought of as unrolled like a rectangular sheet of





paper, the perimeter of the base of the prism or the circumference of the base of the cylinder being the base of the rectangle, and the altitude of the prism or the cylinder, the altitude of the rectangle.

The area of a rectangle being the product of the number of units in the altitude and the number of units in the base, it follows that:

421. 1. **The lateral area of a prism is equal to the product of the number of units in the altitude and the number of units in the perimeter of the base.**

2. **The lateral area of a cylinder is equal to the product of the number of units in the altitude and the number of units in the circumference of the base.**

Volumes

422. The rectangular solid (Fig. 1), the prism (Fig. 2), and the cylinder (Fig. 3) have equal altitudes and the same number of square units in their bases.

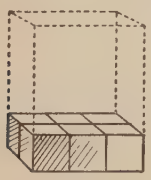


Fig. 1.

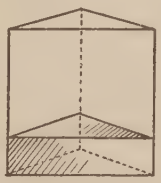


Fig. 2.

Fig. 3.

In Fig. 1 it is seen that a layer 1 unit high contains as many cubic units as there are square units in the base; hence for each layer one unit high in Figs. 2 and 3 there are as many cubic units as there are square units in the base. In each figure the number of cubic units is equal to the number of cubic units in a layer multiplied by the number of layers; it follows that:

423. The volume of a prism or of a cylinder is equal to the product of the number of units in the altitude and the number of square units in the base.

424. If S denotes the lateral area, V the volume, H the number of units in the altitude, P the number of units in the perimeter of the base, and B the number of square units in the base of a prism,

(1) $S = P \times H$, (2) $V = B \times H$.

If S denotes the lateral area, V the volume, H the number of units in the altitude, R the number of units in the radius of the base of a cylinder,

(3) $S = 2\pi RH$, (4) $V = \pi R^2 H$.

Exercise 100

1. Find the lateral area of a prism each side of whose square base is 6 ft. and whose altitude is 3 ft.

SUGGESTION. In a prism, $S = P \times H$. [§ 424, 1]
$\therefore S = 4 \times 6 \times 3$.

2. Find the lateral area of a cylinder whose altitude is 6 in. and whose radius is 4 in.

SUGGESTION. In a cylinder, $S = 2\pi RH$. [§ 424, 3]
$\therefore S = 2 \times 3.1416 \times 4 \times 6$.

3. Find the volume of a prism the area of whose base is 64.96 sq. ft. and whose altitude is 8 ft.

SUGGESTION. In a prism, $V = B \times H$. [§ 424]
$\therefore V = 64.96 \times 8$.

4. Find the volume of a cylinder whose altitude is 5 in. and whose radius is 6 in.

SUGGESTION. In a cylinder, $V = \pi R^2 H$. [§ 424]
$\therefore V = 3.1416 \times 6^2 \times 5$.

5. Find the lateral area of a prism whose base is a square 4 ft. long, and whose altitude is 6 ft.

6. Find the lateral area of a prism whose base is a regular hexagon 2 ft. on a side, if the altitude of the prism is 8 ft.

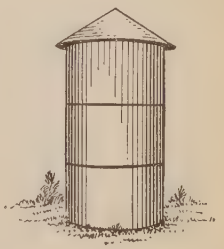
7. Find the lateral area of a cylinder whose radius is 4 in. and whose altitude is 1 ft.

8. Find the volume of a prism whose base is a square 28 ft. on a side, and whose altitude is 18 in.

9. What is the volume of a cylinder whose radius is 6 in., and whose altitude is 1 ft. 6 in.?

10. Find the number of square feet in the entire surface of a cube whose edge is 3 in.

11. Find, correct to the nearest gallon, the capacity of a tank in the form of a cylinder, the radius of whose base is 3 ft. and whose altitude is 6 ft.



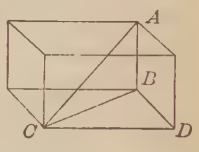
12. This figure represents a cylindrical silo 20 ft. deep and 10 ft. in diameter. How many cubic feet will it hold?

13. What is one side of the base of a square prism whose volume is 150 cu. ft. and whose altitude is 6 ft.?

14. Find the weight of a granite pillar of uniform size, whose base is a circle with a radius of 12 in. and whose height is 14 ft., if 1 cu. ft. of granite weighs $165\frac{5}{8}$ lb.

15. What is the distance from the upper corner of a room to the opposite lower corner, if the length of the room is 16 ft., the width 14 ft., and the height 12 ft.?

SUGGESTION. Let AC be the diagonal of the room; then $\overline{AC}^2 = \overline{AB}^2 + \overline{BC}^2$ and $\overline{BC}^2 = \overline{BD}^2 + \overline{CD}^2$; whence, $\overline{AC}^2 = \overline{AB}^2 + \overline{BD}^2 + \overline{CD}^2$.



16. What must be the depth of a bin 10 ft. long and 5 ft. wide to hold 12 tons of wheat, regarding 1 bu. as equal to $\frac{5}{4}$ of a cu. ft.?

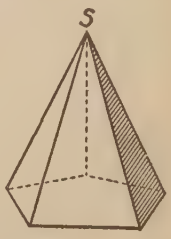
17. This cylinder represents a tank 6 ft. in diameter and 8 ft. high. Find, correct to the nearest gallon, its capacity.

18. Find the altitude of a cylinder that contains 125.664 cu. ft., if the radius of its base is 2 ft.

Regular Pyramids and Cones of Revolution

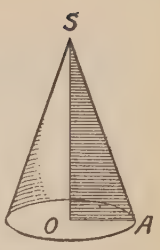
Definitions

425. A **regular pyramid** is a solid bounded by a regular polygon, called the **base**, and a number of equal isosceles triangles, called **lateral faces**, which meet in a point called the **vertex** of the pyramid.



426. The sum of the lateral faces is the **lateral area**, the perpendicular from the vertex to the base is the **altitude**, and the line drawn from the vertex to the middle point of a side of the base is the **slant height.**

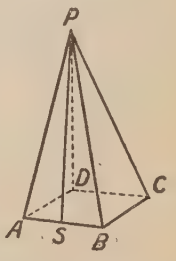
427. A **cone of revolution** is a solid formed by revolving a right triangle about one of its perpendicular sides as an axis.



428. The side about which the right triangle revolves is the **altitude**, the side perpendicular to it is the **radius of the base**, and the hypotenuse is the **slant height** of the cone.

REMARK. In this book regular pyramids and cones of revolution only are treated; they will be called merely pyramids and cones.

Areas



429. In the pyramid P–$ABCD$,

$$\text{Area } PAB = \tfrac{1}{2}\, PS \times AB.$$
$$\text{Area } PBC = \tfrac{1}{2}\, PS \times BC.$$
$$\text{Area } PCD = \tfrac{1}{2}\, PS \times CD.$$
$$\text{Area } PDA = \tfrac{1}{2}\, PS \times DA.$$
$$\text{Lateral area} = \tfrac{1}{2}\, PS \times (AB + BC + CD + DA).$$

430. The lateral area of a pyramid is equal to the product of one half the number of units in the slant height and the number of units in the perimeter of the base.

431. The lateral area of a cone is nearly equal to the lateral area of a pyramid whose base is a regular polygon of a great number of sides, whose slant height is the same as the slant height of the cone and the perimeter of whose base nearly equals the circumference of the base of the cone. Just as the lateral area of a pyramid is the product of one half of the number of units in the slant height and the number of units in the perimeter of the base, so it is proved in geometry that:

432. The lateral area of a cone is equal to the product of one half the number of units in the slant height and the number of units in the circumference of the base.

Volumes

433. By taking a hollow pyramid and a hollow prism of equal base and equal altitude, it may be shown, by measuring, that the pyramid holds one third as much as the prism. In the same way it may be shown that a cone holds one third as much as a cylinder of equal base and equal altitude; hence, from Section 423 it follows that:

434. The volume of a pyramid or of a cone is equal to the product of one third of the number of units in the altitude and the number of square units in the base.

435. If S denotes the lateral area, V the volume, H the number of units in the altitude, L the number of units in the slant height, P the number of units in the perimeter of the base, and B the number of square units in the base of a pyramid,

$$(1)\ S=\frac{P\times L}{2}, \qquad (2)\ V=\frac{B\times H}{3}.$$

436. If S denotes the lateral area, V the volume, H the number of units in the altitude, L the number of units in the slant height, R the number of units in the radius of the base of a cone,

$$(1)\ S=\pi RL, \qquad (2)\ V=\frac{\pi R^2 H}{3}.$$

Exercise 101

1. Find the lateral area of a pyramid whose base is a regular hexagon 8 ft. on a side, if the slant height of the pyramid is 10 ft.

SUGGESTION. $S = \frac{P \times L}{2}$. [§ 435]

$$\therefore S = \frac{6 \times 8 \times 10}{2}.$$

2. Find the lateral area of a cone the radius of whose base is 6 in. and whose slant height is 8 in.

SUGGESTION. $S = \pi RL$. [§ 436]

$$S = 3.1416 \times 6 \times 8.$$

3. Find the volume of a pyramid the area of whose base is 93.53 sq. yd. and whose altitude is 6 yd.

SUGGESTION. $V = \frac{B \times H}{3}$. [§ 435]

$$\therefore V = \frac{93.53 \times 6}{3}.$$

4. Find the volume of a cone the radius of whose base is 3 in. and whose altitude is 6 in.

SUGGESTION. $V = \frac{\pi R^2 H}{3}$. [§ 436]

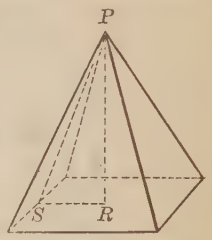
$$\therefore V = \frac{3.1416 \times 3^2 \times 6}{3}.$$

5. Find the lateral area of a pyramid whose base is a regular pentagon 10 in. on a side and whose slant height is 12 in.

6. Find the lateral area of a cone whose radius is 1 ft. and whose slant height is 18 in.

7. Find the volume of a pyramid whose base is a square 6 in. on a side and whose altitude is 9 in.

8. How many cubic inches will a conical vessel hold that is 9 in. high and the radius of whose base is 3 in.?



9. A side of a square pyramid is 6 ft. and the altitude (PR) is 4 ft. Find the length of the slant height PS.

SUGGESTION. PRS is a right triangle.

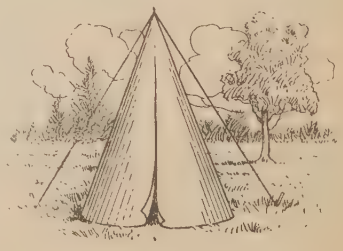
10. Find the lateral area of a pyramid whose slant height is 5 ft. and whose altitude is 3 ft., if its base is a square.

11. Find the volume of a pyramid whose base is a square 6 ft. on a side, if the slant height of the pyramid is 5 ft.

12. The slant height of a cone, the radius of whose base is 6 in., is 10 in. Find the altitude.

13. Find the lateral area and volume of a cone the radius of whose base is 8 in. and whose slant height is 10 in.

14. Find the volume of a cone the radius of whose base is 12 in. and whose slant height is 15 in.



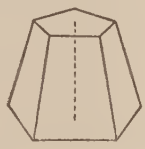
15. How many square yards of canvas are required to make a conical tent 18 ft. in diameter at the base and 12 ft. high?

16. Find the lateral area and volume of a pyramid whose base is a square 24 ft. on a side, the altitude of the pyramid being 16 ft.

17. Find the lateral area and volume of a cone whose altitude is 15 ft. and the diameter of whose base is 40 ft.

Frustums

437. A **frustum** of a pyramid or of a cone is the part between the base and a plane parallel to the base.



REMARK. In this book frustums of regular pyramids and cones of revolution only are treated. They are called merely frustums.

438. The **lateral surface** of a frustum is the entire surface less the bases, and the area of the lateral surface is the **lateral area**. The line joining the centers of the bases is the **altitude**, and that part of the slant height of the pyramid or of the cone, included between the bases of the frustum, is the **slant height** of the frustum.

439. To find the lateral area of a frustum:

Multiply the number of units in the sum of the perimeters or circumferences of the bases by half the number of units in the slant height.

440. To find the volume of a frustum:

Multiply the number of square units in the lower base, plus the number of square units in the upper base, plus the square root of the product of these numbers, by one third of the number of units in the altitude.

Exercise 102

1. Find the lateral area of a frustum of a pyramid whose bases are squares, if each side of the upper base is 4 ft., each side of the lower base 12 ft., and the slant height of the frustum 5 ft.

2. Find the lateral area of a frustum of a cone the radii of whose bases are 3 ft. and 1 ft. and whose slant height is 6 ft.

3. Find the volume of the frustum of a pyramid whose bases are squares, if each side of the lower base is 3 in., each side of the upper base 2 in., and the altitude of the frustum 6 in.

4. Find the lateral area of the frustum of a pyramid whose bases are regular hexagons, if each side of the lower base is 10 in., each side of the upper base 5 in., and the slant height 8 in.

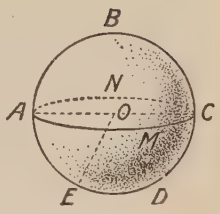
5. Find the volume of the frustum of a cone the radii of whose bases are 6 in. and 4 in. and whose altitude is 10 in.

6. How many cubic inches of water will a vessel hold which is 6 in. in diameter at the top, 4 in. in diameter at the bottom, and 9 in. deep?

7. How many square feet of zinc will be required to line the sides and bottom of an ash hopper 5 ft. square at the top and 3 ft. square at the bottom, if the slant height is 6 ft.?

8. How many quarts of milk, correct to the nearest hundredth, will a tin bucket hold whose top diameter is 14 in., bottom diameter 10 in., and depth 8 in.?

Spheres



441. A **sphere** is a solid bounded by a curved surface every point of which is equally distant from a point within called the center.

A **diameter** of a sphere is a straight line drawn from surface to surface through the center; as AC.

A **radius** of a sphere is a line drawn from the center to the surface. It is half a diameter; as OC.

A **circumference** of a sphere is a circumference of a circle whose radius is the radius of the sphere; as $AMCN$.

442. If S denotes the area of the surface, V the volume, and R the number of units in the radius of a sphere, then,

$$(1)\ S = 4\pi R^2, \qquad (2)\ V = \tfrac{4}{3}\pi R^3.$$

Exercise 103

1. Find the area of the surface of a sphere whose radius is 2 in.

SUGGESTION. $S = 4\pi R^2$. [§ 442]
$\therefore S = 4 \times 3.1416 \times 4.$

2. Find the volume of a sphere whose radius is 3 in.

SUGGESTION. $V = \tfrac{4}{3}\pi R^3$. [§ 442]
$\therefore V = \tfrac{4}{3} \times 3.1416 \times 27.$

Find the area of surfaces of spheres with the following radii:

3. 6 in. 4. 4 ft. 5. 2 rd. 6. 5 ch.
7. 3.2 in. 8. 4.5 ft. 9. 6.75 yd. 10. 3.4 ch.

Find the volumes of spheres with the following radii:

11. 1 in. 12. 5 ft. 13. 12 rd. 14. 7.5 ch.

15. 1.5 in. 16. 2.75 ch. 17. 12.5 rd. 18. .5 ft.

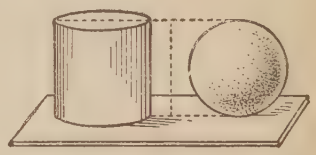
19. Find the area of the surface of a sphere whose circumference is 12.5664 ft.

20. Find the volume of a sphere whose circumference is 18.8496 yd.

21. What is the radius of a sphere the area of whose surface is 113.0976 sq. ft.?

22. Considering the earth to be a sphere whose radius is 4000 miles, find the area of the earth's surface.

23. The diameter of this sphere, the diameter of this cylinder, and the altitude of the cylinder are each 3 in. Find the volume of each, the area of the surface of the sphere, and the lateral surface of the cylinder. Compare the volumes and also the areas.



Similar Figures

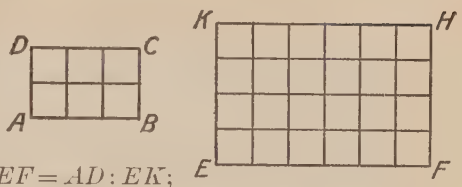
443. Similar figures are such as have the same shape.

Thus, all squares are similar, all circles are similar, all cubes are similar, and all spheres are similar.

444. In similar figures lines that are similarly placed are said to be **corresponding**, or **homologous**.

Thus, the radii of two circles are homologous lines, also the radii of two spheres.

445. In similar rectangles $ABCD$ and $EFHK$, AB and EF are homologous; also AD and EK.



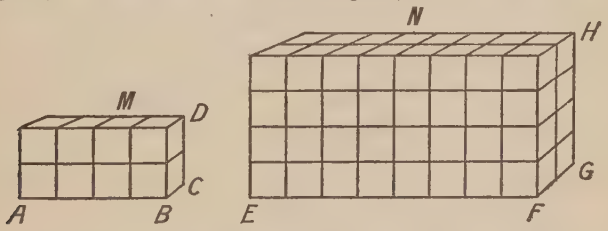
Observe that: (1) $AB : EF = AD : EK$;

(2) Area $ABCD$: Area $EFHK = \overline{AB}^2 : \overline{EF}^2 = \overline{BC}^2 : \overline{FH}^2$.

446. 1. **In similar surfaces homologous lines are proportional.**

2. **The areas of two similar surfaces are to each other as the squares of any two homologous lines.**

447. In similar rectangular solids M and N, AB and EF are homologous, BC and FG are homologous, CD and GH are homologous.



Observe that: (1) $AB : EF = BC : FG = CD : GH$;

(2) $M : N = \overline{AB}^3 : \overline{EF}^3 = \overline{BC}^3 : \overline{FG}^3 = \overline{CD}^3 : \overline{GH}^3$.

448. 1. **In similar solids homologous lines are proportional.**

2. **The volumes of two similar solids are to each other as the cubes of any two homologous lines.**

REMARK. $\overline{AB}^2 : \overline{EF}^2$ is understood to mean the ratio of the square of the *number* of units in AB to the square of the *number* of units in EF. Similarly, $\overline{AB}^3 : \overline{EF}^3$ denotes the ratio of the cube of the *number* of units in AB to the cube of the *number* of units in EF

Exercise 104

1. The bases of two similar triangles are 6 ft. and 10 ft. and the altitude of the first is 2 ft. What is the homologous altitude of the second.

SUGGESTION. Let x be the number of feet in the required altitude.

2. The bases of two similar triangles are 3 ft. and 6 ft. If the area of the first is 24 sq. ft., what is the area of the second?

3. The radii of two circles are 4 ft. and 5 ft. and the circumference of the first is 12.5664 ft. What is the circumference of the second?

4. The area of an equilateral triangle one of whose sides is 5 ft. is 10.826 sq. ft. Find the area of an equilateral triangle one of whose sides is 10 ft.

5. What is the ratio between the area of two circles whose radii are 1 ft. and 2 ft.?

6. The altitudes of two similar cones are 4 ft. and 5 ft. If the volume of the first is 800 cu. ft., what is the volume of the second?

7. A farmer has two fields of the same shape, their lengths being 30 rd. and 20 rd. If the width of the second is 8 rd., what is the width of the first?

8. If a cone 3 in. high weighs 54 lb., what is the weight of a similar cone 4 in. high if made of the same material?

9. If a sphere whose radius is 1 in. weighs 10 lb., what is the weight of a sphere of the same material whose radius is 2 in.?

10. If 240 gallons of water will flow through a certain orifice 3 in. in diameter in a certain time, how much water will flow through an orifice 6 in. in diameter in the same time?

SUGGESTION. The amounts of water or other liquid that flow through two similar orifices are to each other as the squares of the homologous lines of the orifices.

11. If it requires 18 hours for a pipe 2 in. in diameter to empty a certain cistern, how long will it require a pipe 1 in. in diameter to empty the same cistern?

SUGGESTION. The time is inversely proportional to the squares of the diameters of the pipes.

REVIEW

Exercise 105

1. How many acres does a rectangular field contain that is 20 ch. long and 8 ch. wide?

2. How many acres does a field contain if it has the form of a right triangle whose base is 16 ch. and hypotenuse 20 ch.?

3. A square lawn is surrounded by a walk 8 ft. 3 in. wide; the walk and the lawn together contain 6.4 acres. Find the cost of paving the walk at $0.50 per square yard.

4. Find the volume of a prism whose base is a right triangle, the perpendicular sides of which are 20 ft. and 21 ft., if the altitude of the prism is 11 ft.

5. Find the lateral area of a prism whose base is a right triangle, the perpendicular sides of which are 3 ft. and 4 ft., if the altitude of the prism is 10 ft.

6. The length of a rectangle is to its width as 5 is to 4 and its area is 2 acres. Find its length and its width.

7. A has a rectangular lot whose frontage is 60 ft. and depth 120 ft.; on it he builds a house 48 ft. long and 42 ft. wide. The remainder of the lot is made into a yard. Find the area of the yard.

8. A rectangular glass 36 in. long and 20 in. wide is surrounded by a frame 4 in. wide. Find the area of the frame.

9. A rug 24 ft. long and 18 ft. wide is placed in the center of a floor 28 ft. long and 22 ft. wide. How many square feet of the floor remain uncovered?

10. If a rectangular piece of land 88 yd. long and 40 yd. wide produced 960 lb. of wheat, what was the yield per acre, wheat weighing 60 lb. to the bushel?

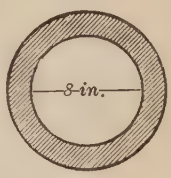
11. The area of the entire surface of a cube is 54 sq. in. What is its volume?

12. The bases of a trapezoid are 12 ft. and 10 ft. and the altitude is $8\frac{1}{2}$ ft. Find the area.

13. Find, correct to the nearest hundredth of a square foot, the area of a regular hexagon each side of which is 10 ft.

14. Find the total area of a cylinder if the altitude is 8 ft. and the radius of its base 4 ft. (Use $\pi = \frac{22}{7}$.)

15. How many square inches are there in the surface of a frame 1 in. wide around a clock dial 8 in. in diameter?



16. Trees are planted 40 ft. apart on both sides of a certain street for the distance of a mile. Find the number of trees.

17. For one inch of rainfall what weight of water falls on one square yard of ground, 1 cu. ft. of water weighing 62.5 lb.?

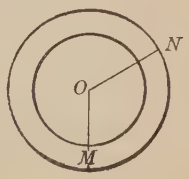
18. The diameters of the earth and the moon are as 1 to $\frac{3}{11}$. How many times as large as the moon is the earth?

19. Find the weight of a granite block 4 ft. long, 3 ft. wide, and 18 in. high, granite weighing 2.65 times as much as an equal bulk of water, which weighs 62.5 lb. per cubic foot.

20. A cubic foot of iron was formed into a bar 3 in. square. Find its length.

21. Find the cost, at 15 cents per square foot, of lining the sides and bottom of a cubical tank which is 6 ft. long.

22. Find the number of square feet in the walk surrounding the circle whose radius OM is 6 yd., if the distance ON from the center of the circle to the outer edge of the walk is 8 yd.



23. The length of the minute hand of the clock on City Hall, Philadelphia, is 10 ft. 8 in. How many feet does the point of this hand move in an hour?

24. A line 50 ft. long extends from the top of a center pole 40 ft. high to the edge of a circus ring. Find the area of the ring.

25. How many square feet of boards are required for the sides and ends of a barn 50 ft. long, 36 ft. wide, and 24 ft. high to the square, the ridgepole being 16 ft. higher than the square?

26. How many revolutions per minute are made by a locomotive wheel 16 ft. in circumference when the locomotive is running 45 miles per hour?

27. Regarding 1 cu. ft. as equal to $\frac{16}{25}$ of a heaped bushel, how many bushels of ashes, heaped measure, will a hopper hold whose upper base is a square 4 ft. long, and lower base a square 9 ft. long, the depth of the hopper being 6 ft.?

28. How many gallons will a cubical tank hold that is 4 ft. long?

29. Find the number of square feet in an equilateral triangle each of whose sides is 6 ft.

30. Estimating that a bushel is equal to $\frac{5}{4}$ of a cubic foot, how many bushels of wheat will a box hold that is 6 ft. long, 4 ft. wide, and 5 ft. deep?

31. Find, correct to the nearest square yard, the amount of rubber cloth required to make 100 footballs, spherical in form, the diameter of each to be 1 ft.

32. A tract of land 10 ch. long and 3.75 ch. wide was bought at \$75 per acre. The purchaser had it inclosed by a fence which cost \$0.60 per rod. Find the total cost.

33. If a pipe 2 in. in diameter will exhaust the water in a certain cistern in $1\frac{1}{2}$ hr., in what time will a pipe $\frac{1}{2}$ in. in diameter exhaust the water?

34. A farmer has a field in the form of a right triangle containing 2 A. 142 sq. rd. If one of the perpendicular sides is 242 yd., find the other.

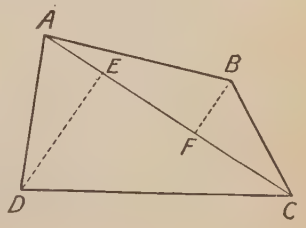
35. 1 cu. ft. of water equals how many quarts? Answer correct to the nearest .01 qt.

36. A cube whose volume is 1000 cu. in. was plated uniformly with 331 cu. in. of copper. Find the thickness of the plating.

37. Find the cost of 86 planks, each 16 ft. by 6 in. by 3 in., at \$24 per M board feet.

38. Find in feet per minute the flow of water through a pipe 8 in. in diameter, if the discharge is 50 gal. per minute. Answer correct to the nearest .01 ft. (Use $\pi = \frac{22}{7}$.)

39. $ABCD$ is a field whose diagonal AC is 18 ch., and the perpendiculars DE and BF are 12 ch. and 6 ch. Find the number of acres in the field.



GENERAL REVIEW

1. A man has 150 quarter-acre lots. How many half-acre lots can he make of these?

2. 100,000 five-cent pieces are worth how many dollars? how many quarter dollars?

3. Find the selling price of 240,000 eggs at 25¢ per dozen.

4. Estimating a step to be a yard, how many steps wide must a patch of ground be made in order that it may contain an acre, if it is 88 steps long?

5. Tell which of the following numbers are (1) prime, (2) composite, (3) even, (4) odd; and give the reason for your answer: 18, 15, 23, 2, 1, 49, 81, 121, 93, 69.

6. Tell at sight which of the numbers 2, 3, 4, 5, and 9 will divide: 109; 111; 124; 819; 478; 215; 2340.

7. Sound travels 1090 ft. per second. Express this rate in miles per hour.

8. When a locomotive is running at the rate of 40 mi. per hour, how many revolutions does the driving wheel make per minute, if its circumference is 20 ft.?

9. Divide the difference between .5 and $.33\frac{1}{3}$ by their sum, and express the result as a fraction.

10. The quotient resulting from solving a problem in division is 48. What would be the quotient if:

(1) the dividend were multiplied by 2?
(2) the divisor were multiplied by 3?
(3) the divisor were divided by 3?
(4) the dividend were multiplied by 12?

11. Onions should yield about 400 bu. per acre. At this estimate, how much should a boy receive from a plat of land 110 yd. long and 11 yd. wide, planted with onions, if he sells the crop at $0.50 per bushel?

12. Find the area of the entire surface of a cube whose volume is 1000 cu. in.

13. Show that a vessel 11 ft. long, 7 ft. wide, and 3 ft. deep will hold 1728 gal.

14. A street 33 ft. wide and $\frac{3}{4}$ mi. long is paved and curbed. The paving costs $1.75 per square yard and each line of curbing 25¢ per linear foot. Find the total cost.

15. A bushel of oats contains 10 oz. of nitrogen, 3 oz. of phosphoric acid, and 2 oz. of potash. How many pounds of each of these essential fertilizing substances were taken from the soil by a crop of 6 A. of oats that yielded 40 bu. per acre?

16. A man agreed to work for $20 a month and house rent free; he worked only 9 mo., but continued to use the house. The house rent was estimated at $100 per year. How much was due him at the end of the year?

17. How many grains of pure gold are there in a piece of jewelry 18 carats fine, if it weighs $\frac{3}{4}$ of an ounce?

18. My bill for 756 rails was $68.04. Find the price per hundred.

19. When lots 300 ft. deep are selling at $10 per front foot, what would be the selling price per acre at the same rate?

20. A bushel of shelled corn contains 14 oz. of nitrogen, 5 oz. of phosphoric acid, and 3 oz. of potash. How many pounds of each of these fertilizing substances are taken from each acre of the soil by a corn crop of 50 bu. per acre?

21. A farmer paid $70.10 for 5608 lb. of bran. Find the price per ton.

22. In a recent year there were shipped from Portland, Ore., to foreign countries 5,201,367 bu. of wheat, valued at $4,531,865. Find, correct to the nearest tenth of a cent, the average value per bushel.

23. Find the number of tons of rails, and the number of ties required for each mile of a single-track railroad, if each rail used is 30 ft. long and weighs 3000 lb., and there are 16 ties to each 30 ft. of track.

24. A good ration for cows consists of 10 bu. of corn and 10 bu. of oats ground together, and 1 T. of clover hay. This should last a cow 120 da. How much ground feed and how much hay does this allow the cow per day?

25. The perimeter of a rectangular field 4 ch. wide is 84 ch. How many acres does it contain?

26. In one year recently the farmer in the United States received $6,000,000,000 for his produce, while the consumer finally paid $13,000,000,000 for the same produce. The amount the consumer paid was how many times the amount the farmer received?

27. Count by $7\frac{1}{2}$'s to 120.

28. Simplify: $\dfrac{.875 \times \frac{3}{8} \times .16\frac{2}{3} \times .625 \times 2.5}{.375 \times \frac{7}{8} \times .33\frac{1}{3} \times .125 \times 5}$. (Cancel.)

29. What average price per pound does a farmer receive for his tobacco crop consisting of 2000 lb. of wrappers which he sells for $10\frac{1}{2}¢$ per pound and 500 lb. of fillers which he sells for $3¢$ per pound?

30. The distance between two places on a map is $1\frac{3}{8}$ in. If the actual distance between these places is 13.75 mi., the map is drawn on a scale of how many miles to the inch?

31. Assuming that one geographical mile equals 6086 ft., find, correct to the nearest thousandth, the number of statute miles in a league.

32. How many acres must be planted with corn to produce silage sufficient for a herd of 20 cows for 180 da., if each cow is to be fed 40 lb. of silage per day, and each acre produces 15 T. of silage?

33. A lot in one of our largest cities was recently sold for $1000 per front foot. The lot has a frontage of 20 ft. and a depth of 94 ft. What would be the price per acre at the same rate?

34. How many boards 12 ft. long are required to build a fence 40 rd. long and 4 boards high? If the boards are 6 in. wide and 1 in. thick, find their cost at $25 per 1000 board feet.

35. A herd of cows is to be fed 300 lb. of silage per day. How long will the silage produced from 2 A. keep this herd, if each acre planted with corn produced 15 T. of silage?

36. At what rate per cent is a man taxed who pays $2\frac{1}{2}$ mills on each $1 of the valuation of his property?

37. What per cent is gained on pencils bought at $1.80 per gross and retailed at 3¢ each?

38. An article was sold for $10 at a gain of $33\frac{1}{3}$ %. Had the cost been $0.50 more than it was, what would have been the gain per cent?

39. The enrollment of a certain school is 225 boys and 310 girls. What per cent of the pupils enrolled are boys and what per cent are girls? Answer correct to the nearest .01 %.

40. When lard is advanced from 12¢ per pound to 17¢, what is the per cent of the advance?

41. An attorney collected a debt, retained a commission of $7\frac{1}{2}$ %, and remitted to his client $370. What commission did he charge?

42. When a laborer's wages are increased from $23\frac{1}{2}$¢ per hour to 25¢, what is the per cent of increase, correct to the nearest .1 %?

43. A merchant gains 20 % when he sells goods at wholesale. If his retail price is 20 % above his wholesale price, find his gain per cent when he sells at retail.

44. A merchant sold a couch for $15, which was at a discount of 25 % from the marked price. If his gain was 20 %, find what per cent the marked price was above cost.

45. A dealer gained 200 % by selling matches at the rate of 2 boxes for 3¢. What per cent would he have gained, if he had sold them at the rate of 3 boxes for 5¢?

46. A dealer gained 25 % when he sold lemons at 25¢ per dozen. What per cent did he gain on those that he sold at the rate of 3 for 10¢?

47. A dealer bought 50 bu. of white lime at 20¢ per bushel and paid $5 for having it hauled. At what price per peck must he retail it to make a profit of 60 %?

48. Two successive discounts of 20 % and $12\frac{1}{2}$ % on a bill of goods amount to $21. What is the bill?

49. If 35¢ interest is charged for $100 for 1 month, what rate per cent per annum is charged?

50. At what price must an article costing $0.80 be marked so that after deducting $16\frac{2}{3}$ % from the marked price a dealer may make a profit of $12\frac{1}{2}$ %?

51. Goods are marked to sell at a profit of 60 %, if sold at retail; the retail price is fixed at $33\frac{1}{3}$ % above the wholesale price. Find the gain per cent on the goods when sold at wholesale.

52. A invests 40 % of his money at 3.5 %, 35 % of it at 4 %, 10 % of it at 6 %, and the remainder at 5.5 %. What average rate of interest does he make on his money?

53. An article was marked at $4, but was sold at a discount of 20 % and $12\frac{1}{2}$ %. Find the selling price.

54. A man charges $30 a month rent for property that costs him $5000. The outlay on the property for a certain year was: taxes of $1.50 per $100 on an assessed valuation of $3500; fire insurance 90¢ per $100 on a policy of $4000; repairs $31.50. What was the per cent of net income for that year?

55. The assessed valuation of the property of a certain town is $1,250,000. How many mills tax must be levied on each dollar to raise $5000?

56. At what price must an article costing $3.20 be marked so that after deducting 20 % from the marked price a profit of 25 % may be made?

57. Find, correct to the nearest cent, the interest on $1100 from Nov. 22 to Feb. 28 of the following year, if the rate is 5 %.

58. Find the gain on an article that sold for 30¢ at a profit of 500 %.

59. A dealer bought lemons at 20 ct. a dozen and twice as many at 30 ct. a dozen. If he mixed them and retailed them all at 40 ct. a dozen, find his gain per cent.

60. The publisher's price for a book is \$0.90, but he sells to retailers at a discount of $16\frac{2}{3}\%$. If the retailer sells the book at the publisher's price, what per cent does the retailer gain?

61. What should be the selling price per pound of a mixed spice formed of 8 lb. costing 50 ct. per pound, 8 lb. costing 40 ct. per pound, and 4 lb. costing 60 ct. per pound, in order that the profit may be 25%?

62. A dealer sold 2 lots for \$1000 each, gaining 25% on the one and losing 20% on the other. Find his gain or loss.

63. A boy buys apples at the rate of 4 for 1¢ and twice as many at the rate of 2 for 1¢. If he sells them all at 1¢ each, find his gain per cent.

64. A piece of property was taxed \$33.60 at a tax rate of $\frac{3}{4}$ of one per cent. Find the assessed value of the property.

65. Green coffee loses 10% of its weight in roasting. At what price must the grocer sell the roasted coffee to gain 20%, if he paid 27¢ per pound for the green coffee?

66. If envelopes are bought at the rate of \$1 per 1000 and retailed at the rate of 2 for 1¢, what is the gain per cent?

67. If a manufacturer sells goods to a retailer at a profit of 20% and the retailer sells them to the consumer at a profit of 25%, what fractional part of the cost of the goods to the consumer is the cost of manufacturing them?

68. I sold coal at $6.25 per ton, thereby making a profit of 25%, and a total profit of $250. How many tons did I sell?

69. My agent purchased for me 154,014 lb. of corn at $0.60 per bushel (56 lb.) He prepaid freight amounting to 5¢ per hundredweight and charged me a commission of 2%. Find the total cost of the corn.

70. A farmer's herd of 20 cows averaged 8.75 lb. of milk daily for the month of November. The milk tested 3.6% butter fat which he sold at 32¢ per pound. What was his income for the month?

71. A dealer bought a bunch of 72 bananas for $0.90. He retailed half of them at the rate of 2 for 5¢, a dozen of them at the rate of 3 for 5¢, and the remainder at 1¢ each. Find his gain per cent.

72. A tax of $22,315 is to be raised in a town whose property is assessed at $4,275,800 and 624 polls. If the polls are assessed at $1.50 each, what must be the rate of taxation?

73. Find the duty on a rug 16 ft. by 12 ft., whose dutiable value is $120 and on which there is charged a specific duty of 10¢ per square foot and 40% ad valorem.

74. A house and lot cost $2500 and $1850 respectively. The house was insured for $\frac{4}{5}$ of its value at $\frac{3}{5}$%. If the house was destroyed by fire within a year, and the lot was sold for $2350, find the gain or loss.

75. If a building is insured for \$ 6000 and the annual premium is \$ 19.20, find the annual rate per \$ 100. What is the rate per \$ 100 for 2 years? What is the rate per \$ 100 for 3 years?

76. Find the annual premium that a man must pay on a 20-year endowment policy for \$ 10,000 taken out April 1, 1913, if the insured was born December 20, 1887.

77. July 1, A borrowed \$ 200 at $4\frac{1}{2}\%$, and when the principal had amounted to \$ 203.75, he paid the debt. On what date did he make payment?

78. My agent purchased for me 1000 bu. of wheat at \$ 0.80 per bushel. I sent him a check for \$ 856, which paid the cost of the wheat, freight charges amounting to \$ 40, and his commission. What rate of commission did he charge me?

79. What is the interest for 1 year on a \$ 10 postal savings certificate, the rate of interest being 2%?

80. If the income from \$ 100 invested in a certain kind of stock was \$ 4.3846, estimated correct to the nearest tenth of a mill, find the investor's rate of income, estimated correct to:

(1) The nearest one per cent.
(2) The nearest tenth per cent.
(3) The nearest hundredth per cent.
(4) The nearest half per cent.
(5) The nearest quarter per cent.

81. When 6% stock pays 4% on the investment, what is its market value, if the rate of brokerage is $\frac{1}{8}\%$?

82. What rate of dividend does stock pay, if an investor realizes $4\frac{1}{2}\%$ on his investment, the stock costing him $\$120$ per share?

83. A sent his broker $\$2445$ to pay for 40 shares of P. R. R. stock (par value $\$50$), brokerage $\frac{1}{8}\%$. Find the market value of each share.

84. Find the profit on 20 shares of stock (par value $\$50$), bought at $\$58\frac{1}{4}$ per share and sold at $\$60\frac{1}{4}$, if two semiannual dividends of $2\frac{1}{2}\%$ each were received between the transactions, brokerage $\frac{1}{4}\%$ on the sale and the purchase.

85. What sum must be invested in stock at 199%, paying quarterly dividends of 2%, in order that the annual income may be $\$200$, brokerage $\frac{1}{8}\%$?

86. A laborer deposited $\$100$ in a savings bank at the beginning of a year, and withdrew principal and interest at the beginning of the next year. How much was he entitled to, if the savings bank paid 3% interest compounded quarterly?

87. What must be paid for sufficient P. R. R. stock (par value $\$50$) selling at $\$61$ per share, brokerage $\frac{1}{8}\%$, to yield the investor an annual income of $\$375$, if this stock pays a semiannual dividend of 3%?

88. When 6% stock (par value $\$50$) is bought at $\$71\frac{1}{8}$ per share, brokerage $\frac{1}{4}\%$, what rate of income, correct to the nearest .01%, does it pay the investor?

89. When 6% stock pays 5% on the investment, what is the market value, if the rate of brokerage is $\frac{1}{4}\%$?

90. Type metal is made from antimony and lead mixed in the ratio of 1 part of the former to $4\frac{1}{2}$ parts of the latter. Find, correct to the nearest .01 of a pound, the amount of each substance in a ton of type metal.

91. How many posts set 12 ft. apart are necessary to fence a rectangular tract of land 8 rd. wide, containing 2 A.?

92. A rectangular piece of land containing $1\frac{1}{4}$ A. is 121 yd. long. Find its width.

93. A piece of land in the form of a trapezoid contains 4 A. If the length of one of the parallel sides is 118 rd. and of the other 10 rd., how far are they apart?

94. What must be the market value of 6% stock (par value \$50) in order that it may pay an investor 4% interest on his money, the rate of brokerage being $\frac{1}{4}$%?

95. The diameters of two cylindrical vessels of the same shape are 3 ft. and 4 ft. If the first holds 27 gal., how much does the second hold?

96. A and B entered into partnership. A invested \$500 for 10 mo. and gained \$50; B invested his capital for 6 mo. and gained \$60. What was B's capital?

97. A lady found that if she paid \$1.10 per yard for silk, she would lack \$1.20 of having enough to pay for the quantity desired, but that if she paid \$0.85 per yard, she would have \$1.80 remaining. How much money had she?

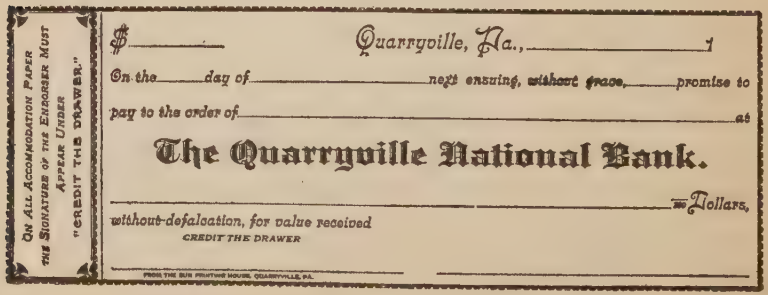
98. Through my broker I sold \$4000 U. S. 2s at $101\frac{1}{2}$% and invested the proceeds in railroad stock at

$126\frac{1}{4}\%$. The stock bought pays a quarterly dividend of $1\frac{1}{4}\%$. Find the change in my annual income and the surplus from the sale of the bonds, brokerage $\frac{1}{8}\%$ on the sale and the purchase.

99. The altitude of a prism is 12 ft. Its base is a right triangle with an hypotenuse 15 ft. long, and one side 12 ft. long. Find the lateral area and the volume of the prism.

100. A man at the age of 25 years took out an ordinary life policy for $1000; at the age of 30 years a twenty-payment life policy for another $1000; at the age of 35 a twenty-year endowment policy for another $1000; he died at the age of between 39 and 40 years. By how much did the amount of insurance exceed the sum of the premiums paid?

101. Copy and fill out the following note, dating it July 1 of the current year, and making it payable at the expiration of 60 days. Use appropriate names and explain the transaction that this note would require. Write such indorsements as are necessary before the Quarryville National Bank will cash the note:


On All Accommodation Paper the Signature of the Endorser Must Appear Under "Credit the Drawer."
$
Quarryville, Pa.,
On the day of next ensuing, without grace, promise to
pay to the order of at
The Quarryville National Bank.
Dollars,
without defalcation, for value received
CREDIT THE DRAWER

102. Find the amount due on the following note Dec. 31, 1913. When is the note due?

$150.00 Chicago, Ill. Feb 4, 1913.

Nine months after date for value received I promise to pay to the order of F. L. Fields

One hundred fifty Dollars,

with interest at the rate of 6 per cent. per annum after April 1, 1913.

Payable at Continental and Commercial National Bank

No. Due A. J. Garrett

103. Find the area of the regular hexagon whose side is 4 rd.

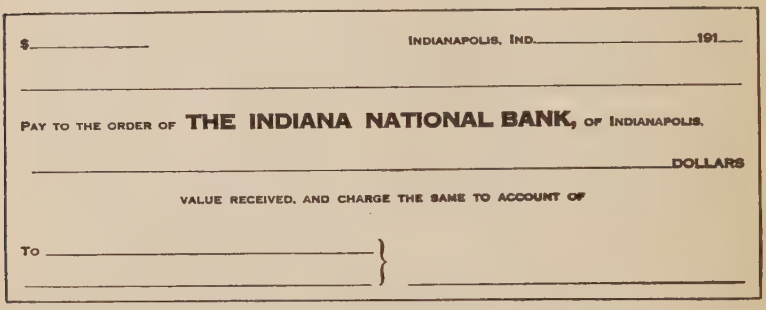
104. A tract of land in the form of a square containing 10 A. was divided into 4 equal square lots and fenced. At $1.50 per rod, find the cost of the fence necessary to inclose the 4 lots.

105. A and B are 180 yd. apart and walk toward each other, A walking 2 yd. while B walks 3 yd. How far will each walk before they meet?

106. A pole 10 ft. long casts a shadow 12 ft. long at the same time that a tree casts a shadow 60 ft. long. How high is the tree?

107. Write a promissory note dated Sept. 1 of the current year, payable in 60 da., with yourself as payee and H. P. Worth as maker. Properly indorse the note for discount and find the proceeds, if discounted at a bank in your state Sept. 1.

108. Copy and fill out the following draft, making yourself drawer, and W. C. Dunlap drawee. Date the draft May 15 of the current year and make it payable 60 days after sight, face $ 50 :



After filling out a form like the above, write the acceptance. Explain which of the three gets the $ 50, and from whom. When is the draft due? Who holds the draft after it has been paid?

109. A Pittsburg firm drew a sight draft for $ 1000 on a New Orleans firm and had the draft cashed at a Pittsburg bank. How much was received for it, exchange being at $\frac{1}{2}\%$ discount?

110. A lawn 60 yd. long and 50 yd. wide is surrounded by a walk 6 ft. wide. Find the cost of graveling this walk at $ 0.12 per square yard.

111. A sphere whose radius is 6 in. weighs 324 oz. Find the weight of a sphere of the same material whose radius is 10 in.

112. Find the diagonal of a cube whose edge is 10 inches.

113. One man can build a fence in 9 days, working 10 hours a day; another can build it in $4\frac{1}{2}$ days, working 8 hours a day. In how many days can they build it together, working 6 hours a day?

114. Two thirds of my money equals four fifths of yours. If we put our money together, what part of the whole will you own?

115. A cylindrical tank 40 ft. high and 15 ft. in diameter is represented in a drawing as 15 in. high. What is the length of the diameter as represented in the drawing?

116. A painter received \$ 3 per day for his labor and paid \$ 5 per week for his board, that being his only expense; at the expiration of 10 wk. he had saved \$ 100. How many days was he idle?

117. A and B are 180 yd. apart and travel in the same direction. A travels 3 yd. while B travels 5 yd. How far will each travel before B overtakes A?

118. Three men invested jointly \$ 3000, and after a certain time, when the gains were to be divided, it was found that A was entitled to \$ 3 as often as B to \$ 4 and C to \$ 5. Find the investment of each.

119. Find the volume of a triangular pyramid the sides of whose base are 52 ft., 56 ft., and 60 ft., and whose altitude is 21 ft.

120. Find, correct to the nearest .01 of an acre, the area of a circle whose radius is 5 chains.

121. How many $\frac{1}{2}$-in. cubes can be placed in a box $4\frac{1}{2}$ in. long, $2\frac{1}{2}$ in. wide, and 2 in. high?

122. A farmer drills his corn in rows $2\frac{3}{4}$ ft. apart. How many rods are there in the width of a field that contains 160 rows, the outside ones being 1 yd. from the fence?

123. A high wind that has a velocity of 30 mi. per hour exerts a force of 4.428 lb. per square foot. What force would such a wind exert on a tight board fence 80 yd. long and 6 ft. high?

124. If it requires 1 lb. of barbed wire of a certain kind for each wire in a rod of fence, how many pounds of wire would be required to build a fence of 3 wires around a field, the sides of which measure 90 rd., 75 rd., 60 rd., and 52 rd.?

125. A year consists of 365 da. 5 hr. 48 min. 49.7 sec. How much time is lost in 4 yr. by counting 365 da. to the year?

126. Find the missing term in each of the following:

$$x : \tfrac{2}{3} = \tfrac{3}{4} : 1 \qquad 1 : \tfrac{1}{2} = x : 4$$
$$\tfrac{3}{4} : x = \tfrac{1}{4} : 2 \qquad \tfrac{2}{3} : \tfrac{3}{4} = \tfrac{4}{9} : x$$

127. Find $\sqrt{21 \times 42 \times 200}$ by first reducing the numbers to their prime factors.

128. How many cubic feet of masonry are necessary to build the breast of a dam, if the wall is 8 ft. thick at the base, 4 ft. thick at the top, 40 ft. long, and 8 ft. high?

129. The quantity of air taken into the lungs at each ordinary inspiration of an adult is on an average 30 cu. in. Assuming that he takes 17 inspirations per minute, how many cubic feet of air does he breathe in 24 hr.?

130. If $\frac{3}{4}$ of A's money equals $\frac{3}{5}$ of B's, then $\frac{3}{4}$ of B's is what part of A's?

131. Find the lateral area of the frustum of a cone the radii of whose bases are 10 ft. and 4 ft. respectively, and whose altitude is 8 ft.

132. Two men are to receive \$ 48 for painting a barn. If they work together 6 days and then one of them finishes the work in 4 days more, how much should each receive?

133. What is the diameter of a wheel which makes 480 revolutions in going a mile?

134. One pipe can fill a tank in 20 hr. and another can fill it in 10 hr. The tank can be emptied by a spigot in 15 hr. If the tank is empty and the pipes and spigot set to flow at the same time, in how many hours will the tank be filled?

135. One geographic mile equals one minute of the earth's equatorial circumference, or about 6086 ft. Find the circumference of the earth in statute miles.

136. A woman deposited \$ 200 in a savings bank at the beginning of the year and withdrew principal and interest at the end of 18 mo. How much was she entitled to, if the savings bank paid 3 % interest compounded semi-annually?

137. If a ball 1 ft. in diameter weighs 24 lb., what is the weight of a ball $\frac{1}{2}$ ft. in diameter, made of the same material?

138. A barn valued at \$ 2400 was insured for $\frac{3}{4}$ of its value. Find the annual premium, if the rate charged was \$ 2.60 per \$ 1000 of insured value.

139. Six men agreed to do a piece of work in a certain time, but, two of them failing to come, the work was prolonged 5 days. How long would it have taken the 6 men to do the work?

140. Find the length of the diagonal of a rectangular solid 12 ft. long, 9 ft. wide, and 10 ft. high.

141. 1 cu. in. of steel weighs as much as 7.83 cu. in. of water. Find the weight of 1 cu. ft. of steel, water weighing 62.5 lb. per cubic foot.

142. Find the number of square feet in a square whose perimeter is 1 chain.

143. To hold silage sufficient for a herd of 10 cows for 6 mo., allowing each cow 40 lb. daily, a silo should have a diameter of 10 ft. and a height of 25 ft. How many cubic feet will this silo hold?

144. The area of a rectangle is 490 sq. ft., and its length is to its width as 5 is to 2. Find its perimeter.

145. Find the area of an equilateral triangle each of whose sides is 4 rods.

146. A field in the form of a rectangle contains 7 A. 80 sq. rd.; it is 40 rd. long. How many rods of fence are necessary to inclose it?

147. A dealer gained 15% when he sold bran at \$1.15 per hundredweight. What per cent would he gain by selling it at \$22.50 per ton?

148. A woman paid 45¢ for a dozen eggs that weighed 21 oz. At this rate what is a dozen eggs worth that weighs 28 oz.?

149. The four sides of a tract of land measure $108\frac{1}{3}$ rd., $116\frac{2}{3}$ rd., 75 rd., and 100 rd. The last two sides form a right angle. Find the number of acres in the tract.

150. Find the interest on \$60 for 1 day at 6%.

151. A class of 150 pupils was examined in grammar, 20% of them being boys and the rest girls. If $16\frac{2}{3}$% of the boys and 20% of the girls failed, what per cent of the class passed?

152. At what price must I purchase 3% stock (par value \$100) to secure the same rate of income that I obtain from 4% stock (par value \$50) purchased at $\$59\frac{7}{8}$ per share, brokerage $\frac{1}{4}$% in each case?

153. What is the cost of a city square, exclusive of streets, at \$1000 per acre, if the square is $\frac{1}{8}$ of a mile long and $\frac{1}{12}$ of a mile wide, measuring from the center of a 60 ft. street on each side?

154. A father willed \$6600 to his two sons, whose ages are 11 years and 16 years, respectively, so that they may have equal sums on coming of age, if the money is loaned at 5%; how much did each receive?

155. A whitewash adopted by the United States Government is composed of 10 parts of freshly slaked lime to 1 part of hydraulic potash, and the whole is mixed with salt water. How much potash must be mixed with 2 bu. 2 pk. of lump lime, if the lump lime in slaking doubles its volume?

156. A school was open 20 da. during a certain month; 6 pupils attended every day, 8 attended 19 da., 4 attended 18 da., 4 attended 17 da., and 1 attended 8 da. Find the average daily attendance for that month.

157. Find the average cost per pound of 10 lb. of Japan tea costing 75¢ per pound, 40 lb. of Formosa costing 60¢ per pound, 80 lb. of Salada costing 60¢ per pound, and 90 lb. of Ceylon costing 70¢ per pound.

158. A man owns the northwest corner lot, rectangular in form, fronting 100 ft. on New Street, which runs east and west, and 80 ft. on State Street, which runs north and south. Along each street a pavement 10 ft. wide is taken from the lot. Back 20 ft. from each street the ground is staked out for a house fronting 40 ft. on New Street, the house to be 30 ft. wide.

(*a*) Make a drawing, scale $\frac{1}{2}$ in. to 10 ft., representing the outline of the pavements, the lot, and the cellar of the house.

(*b*) Find the number of square yards of the lot that remains after the pavement is made and the house built.

SUPPLEMENT

LATITUDE AND LONGITUDE

1. To locate places on the surface of the earth, a circumference, called the meridian of Greenwich, is supposed to be drawn through Greenwich, England, and the north and south poles of the earth. Halfway, or 90°, from either pole another circumference, called the **equator,** is supposed to be drawn east and west around the earth. These two circumferences divide each other into two equal parts of 180° each.

2. The **meridian** of a place is the half circumference supposed to be drawn from pole to pole through the place. It is divided by the equator into two equal parts of 90° each.

3. The meridian of Greenwich is called the **prime meridian,** and its opposite meridian, or the other half of the circumference of which the prime meridian is one half, is the 180th meridian.

4. The **latitude** of a place is its distance from the equator measured in degrees upon the meridian of the place, and is north or south according as the place is north or south of the equator.

5. The **longitude** of a place is the distance in degrees on the equator from the prime meridian to the meridian of the place, and is east or west according as the place is east or west of the prime meridian.

6. Parallels of latitude are circumferences extending east and west around the earth parallel to the equator.

7. The following truths are evident:

1. *All places on the equator have no latitude.*
2. *All places on the same parallel of latitude have the same latitude.*
3. *All places on the prime meridian have no longitude.*
4. *All places on the same meridian have the same longitude.*
5. *The greatest latitude that a place may have is 90°.*
6. *The greatest longitude that a place may have is 180°, being the distance from the prime meridian to the 180th meridian.*

Exercise 1

How far and in what direction by the shorter arc is:

1. 75° N. from 30° N.?
2. 5° N. from 70° N.?
3. 50° S. from 90° S.?
4. 50° S. from 40° S.?
5. 40° N. from 45° S.?
6. 90° S. from 20° N.?
7. 25° E. from 10° E.?
8. 25° E. from 90° E.?
9. 110° W. from 60° W.?
10. 25° W. from 65° W.?
11. 140° E. from 140° W.?
12. 15° W. from 55° E.?

LONGITUDE AND TIME

8. The earth rotates on its axis from west to east in 24 hours. This makes the sun appear to revolve about the earth from east to west once in the same time.

Since the sun appears to revolve through 360° in 24 hours, its apparent motion is 15° per hour, 15′ per minute, and 15″ per second.

It is noon at any place when the sun is on the meridian of that place, and since the sun apparently moves 15° from east to west in 1 hour, it is then 1 P.M. 15° east of that place, 11 A.M. 15° west of it, 2 P.M. 30° east of it, 10 A.M. 30° west of it, and so on. From this we see that a difference of 15° in the longitude of two places makes a difference of 1 hour in their time. Similarly, a difference of 15′ in their longitude makes a difference of 1 minute in their time, and a difference of 15″ in their longitude makes a difference of 1 second in their time. Hence:

9. *$\frac{1}{15}$ of the number of degrees, minutes, and seconds in the difference of longitude of two places equals the number of hours, minutes, and seconds in the difference of their time.*

10. *15 times the number of hours, minutes, and seconds in the difference of time of two places equals the number of degrees, minutes, and seconds in the difference of their longitude.*

REMARK. It is evident from § 8 that places east of any given meridian have later time and those west of it earlier time than places on the meridian.

11. This figure shows the times on different given meridians when the time is noon on the meridian of Greenwich.

Exercise 2

(Refer to figure, Sec. 11, in answering the questions of this exercise; answer as many as possible orally.)

State how many degrees and in what direction by the shorter arc is:

1. 135° E. from 30° E.
2. 45° W. from 60° E.
3. 75° W. from 15° W.
4. 120° E. from 30° W.
5. 15° E. from 105° E.
6. 135° E. from 165° W.
7. 30° W. from 120° W.
8. 120° W. from 120° E.

State what meridian is:

9. 45° east of 75° E.
10. 30° east of 15° W.
11. 30° west of 60° W.
12. 60° west of 45° E.
13. 30° west of 120° E.
14. 90° west of 120° W.
15. 15° east of 90° W.
16. 60° east of 150° E.

Observe that:

If A and B stand for two cities, and A is west of B, A's time is 1 hour earlier (slower) than B's for every 15° that A is west of B.

If A and B stand for two cities, and B is east of A, B's time is 1 hour later (faster) than A's for every 15° that B is east of A.

What is the time at:

17. 90° E. when the time at 30° E. is 6 P.M.?
18. 150° W. when the time at 90° W. is 2 P.M.?
19. 30° E. when the time at 75° E. is 4 P.M.?
20. 15° W. when the time at 60° W. is 9 A.M.?
21. 45° W. when the time at 15° E. is 1 P.M.?
22. 60° E. when the time at 30° W. is noon?

On what meridian is the time:

23. 3 P.M. when the time at 60° W. is 6 P.M.?
24. 4 A.M. when the time at 90° W. is 2 A.M.?
25. 10 A.M. when the time at 60° E. is 8 A.M.?

Illustrative Examples

1. The longitude of Chicago is 87° 34′ 8″ W., and of Boston 71° 3′ 30″ W. When it is 10 A.M. at Boston, what is the time at Chicago?

87°	34′	8″	W.	is the longitude of Chicago.
71°	3′	30″	W.	is the longitude of Boston.
16°	30′	38″		is the difference of longitude.

$\frac{1}{15}$ of the number of degrees, minutes, and seconds in the difference of longitude of two places equals the number of hours, minutes, and seconds in the difference of their time.

15)16	30	38	
1	6	2.5	

∴ 1 hr. 6 min. 2.5 sec. is the difference of time, correct to the nearest tenth of a second.

hr.	min.	sec.	
10	0	0	A.M. is the time at Boston.
1	6	2.5	is the difference of time.
8	53	57.5	A.M. is the time at Chicago.

The difference of time is subtracted from the time of Boston, since Chicago is west from Boston, and, therefore, has earlier time than Boston.

2. When it is 3 hr. 45 min. A.M. Wednesday at Paris, longitude 2° 20′ 30″ E., it is 9 hr. 11 min. 57.5 sec. P.M. Tuesday at Omaha. Find the longitude of Omaha.

da.	hr.	min.	sec.	
4	3	45	0	is the time at Paris.
3	21	11	57.5	is the time at Omaha.
	6	33	2.5	is the difference of time.

6	33	2.5
		15
98	15	37.5

15 times the number of hours, minutes, and seconds in the difference of time of two places equals the number of degrees, minutes, and seconds in the difference of their time.

Since the time is later at Paris than at Omaha, Omaha must be west of Paris.

98°	15′	37.5″	is the distance of Paris from Omaha.
2°	20′	22 ″	is the distance of Paris east from the prime meridian.
95°	55′	15.5″	is the distance of Omaha west from the prime meridian.

∴ 95° 55′ 15.5″ W. is the longitude of Omaha.

Exercise 3

1. The longitude of St. Louis is 90° 15′ 16″ W., and of New Haven, 72° 55′ 34″ W. When it is 6 P.M. at New Haven, what is the time at St. Louis?

2. The longitude of Philadelphia is 75° 9′ 5″ W., and of San Francisco 122° 26′ 15″ W. What is the time in Philadelphia when it is noon at San Francisco?

3. The longitude of Rome is 12° 27′ 14″ E., and of New York 74° 3″ W. When it is 11 P.M. Tuesday at New York, what time is it at Rome?

4. The time at Galveston, Texas, is 6 hr. 9 min. 9.9 sec. earlier than the time on the prime meridian. What is the longitude of Galveston, Texas?

Standard Time

12. For the convenience of business in general and of railroads especially, a system of **standard time** has been adopted; it was put in force in the United States on Sunday, Nov. 18, 1883. This system practically divides the country into four **belts** and adopts the local time of a meridian in each belt as the common time for all places in that belt.

13. The standard belts are:

(1) *Eastern*, whose standard time is the local time of the 75th meridian.

(2) *Central*, whose standard time is the local time of the 90th meridian.

(3) *Mountain*, whose standard time is the local time of the 105th meridian.

(4) *Pacific*, whose standard time is the local time of the 120th meridian.

14. It is evident that within each of these belts the standard time is uniform, and that the time of each belt differs from that next to it by exactly one hour.

NOTE 1. It was intended that each standard meridian should govern the belt $7\frac{1}{2}$ degrees on each side of it, but there have been some slight variations from this, due to the fact that railroad companies often change time at important railroad centers near a boundary when such centers are not located on the boundary.

NOTE 2. The Eastern belt includes all territory between the Atlantic and an irregular line drawn from Buffalo to Central Junction, Ga., the latter city being its southernmost point. The Central belt includes all the territory between this eastern line and another irregular line drawn from Bismarck, N.D., to El Paso, Tex. The Mountain belt includes all the territory between the last named line and nearly all the western boundaries of Idaho, Utah, and Arizona. The Pacific belt includes all the territory of the United States between the western boundary of the Mountain belt and the Pacific coast.

NOTE 3. It is evident that in any belt the local time of places east of the standard meridian is faster, and that of places west slower, than the standard time.

15. Map of Standard Time Belts

Exercise 4

1. When it is noon in the Eastern belt, what is the time in each of the other belts?

2. When it is noon by Mountain time, what is the time in each of the other belts?

3. Find the difference between the standard time of New York and that of Chicago, longitude 87° 34′ 8″ W.

4. In traveling from Philadelphia to San Francisco would a traveler find his watch too slow or too fast, and by how many hours?

5. How much slower is local, or sun, time than standard, or railroad, time at longitude 77° 30′ W.?

6. When it is noon at the 75th meridian west, is the local time of places that are east of the 75th meridian and in the Eastern belt later or earlier than noon?

7. A traveler that left San Francisco at noon Sunday, Pacific time, arrived in New York at noon Friday, Eastern time. How long did it require to make the journey?

PUBLIC LANDS

16. Most of the public lands north of the Ohio River and west of the Mississippi are laid out by lines running north and south and east and west.

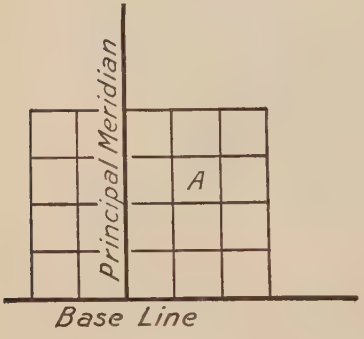


Fig. 1.

17. The method is to select a point in a tract and through it establish a north and south line called a **principal meridian**, and an east and west line called a **base line**. Lines are run parallel to the principal meridian and to the base line at intervals of 6 miles, thus dividing the tract into townships that are approximately 6 miles square.

18. A series of townships extending north and south is called a **range**; a series extending east and west is called a **tier**. Tiers are numbered north and south from the base line; ranges are numbered east and west from the principal meridian. A township is designated by the number of the tier and the number of the range in which it is located.

Thus, township marked A, Fig. 1, is designated T. 3 N., R. 2 E., read *township three north, range two east*, which means that A is

6	5	4	3	2	1
7	8	9	10	11	12
18	17	16	15	14	13
19	20	21	22	23	24
30	29	28	27	26	25
31	32	33	34	35	36

Fig. 2.

in the third tier north of the base line and in the second range east of the principal meridian.

19. A township is divided into **sections** approximately * 1 mile square and numbered as shown in Fig. 2, beginning at the northeast corner.

A section is divided into quarter-sections, and the township, section, and quarter-section corners are permanently marked.

Quarter-sections may be sub-divided, if desired, into half-quarter-sections and quarter-quarter-sections.

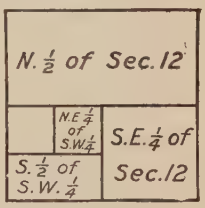


Fig. 3.

Exercise 5

1. How many square miles are there in a township?

2. How many acres are there in a section? in a half-section? in a quarter-section? in a quarter-quarter-section?

In Examples 3–8 make drawing as in section 17 and locate with respect to the principal meridian and the base line:

3. T. 2 N., R. 3 W.
4. T. 2 N., R. 2 E.
5. T. 4 N., R. 1 W.
6. T. 1 N., R. 3 E.
7. T. 2 S., R. 2 W.
8. T. 3 S., R. 2 E.
9. Read N.W. ¼, Sec. 12, T. 3 N., R. 1 W.
10. Read S. ½, Sec. 30, T. 2 N., R. 4 W.
11. Read W. ½, Sec. 10, T. 2 S., R. 3 W.

Make drawing as in section 19 and locate each of the following tracts of land; give the number of acres in the tract:

12. S. ½ of N.W. ¼.
13. W. ½ of S.E. ¼.
14. S.E. ¼ of N.E. ¼.
15. S.W. ¼ of N.W. ¼.

METRIC SYSTEM

20. The **metric system** of weights and measures has been adopted by nearly all the countries of Europe and South America and by

* Townships and sections would be exact squares if it were not for the fact that the meridians converge; that is, all north and south lines meet at the North Pole.

Mexico. Its use is permitted in the United States, and even required in certain work of the departments of the Government, such as the medical work of the Navy and the War Departments. It is the legal system of Porto Rico and the Philippines.

21. The fundamental unit of the metric system is the **meter** — the unit of *length*. From this the units of *capacity* (**liter**) and *weight* (**gram**) were derived. All other units are the decimal subdivisions or multiples of these. The three units named are simply related; thus:

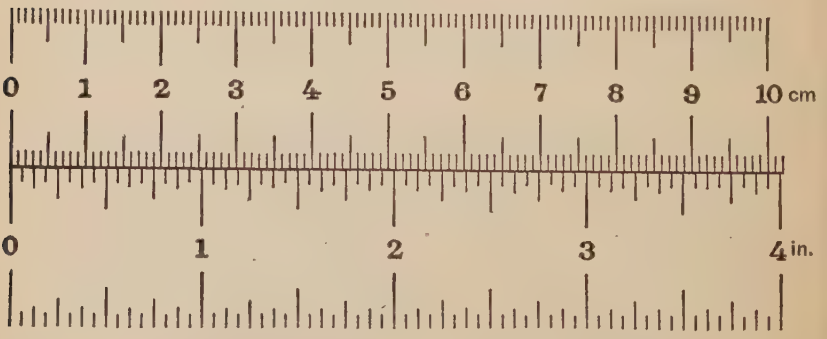
A liter is a cube .1 of a meter on an edge.

A gram is the weight of a cube of pure water .01 of a meter on an edge at the greatest density of water, 39.2° Fahrenheit.

22. The metric tables are formed by combining the words "meter," "gram," and "liter" with the six prefixes here shown:

Prefixes	Meaning		Units
milli-	.001		
centi-	.01		
deci-	.1	prefixed to	**meter** *for length* **gram** *for weight* **liter** *for capacity*
deka-	10		
hecto-	100		
kilo-	1000		

23. The primary unit of **length** is the **meter**; it is equal to 39.37 inches, nearly.



Comparison Scale: 10 Centimeters and 4 Inches.

24. **Table of Measures of Length**

10 *millimeters* (mm)	= 1 *centimeter* (cm)	= 0.3937 in.
10 centimeters	= 1 decimeter (dm)	
10 decimeters	= 1 *meter* (m)	= 39.37 in.
10 meters	= 1 dekameter (Dm)	
10 dekameters	= 1 hectometer (Hm)	
10 hectometers	= 1 *kilometer* (Km)	= 200 rd.
10 kilometers	= 1 myriameter (Mm)	

NOTE. Important units are printed in italic type.

25. A quantity expressed in different metric units may be expressed and read without reduction in any denomination desired, or in more than one denomination; just as we now express a sum of money made up of \$7000, \$25, 6 d., 5¢, 2 m., in one denomination, \$7025.652, which is sometimes read *7025.652 dollars*, and sometimes *seven thousand twenty-five dollars, sixty-five cents, two mills.*

Thus, $4^{Km}\ 6^{Dm}\ 5^{m}\ 4^{cm}\ 2^{mm}$ may be expressed 4065.042^{m}, and read *four thousand sixty-five and forty-two thousandths meters*, or *4 kilometers, 65 meters, 42 millimeters.*

26. The metric system is a *decimal system* and therefore reductions from one denomination in it to another may be made by simply moving the decimal point to the right or to the left.

Thus, $1825^{mm} = 182.5^{cm} = 18.25^{dm} = 1.825^{m}$,
and $0.638^{m} = 6.38^{dm} = 63.8^{cm} = 638^{mm}$.

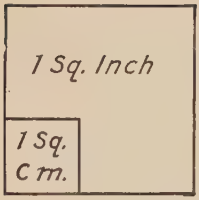
Exercise 6

Read:

1. 7^{Km} in hectometers; in dekameters; in meters.
2. 726^{mm} in centimeters; in decimeters; in meters.
3. 5^{Dm} in meters; in centimeters; in millimeters.
4. $5^{m}\ 6^{dm}$ in decimeters; in centimeters.
5. $4^{Km}\ 65^{m}\ 375^{mm}$ in meters.

Write:

6. 25^{Km} in meters.
7. 375^{mm} in meters.
8. 1425.62^{m} in kilometers, meters, and millimeters.



27. In square measure the units are squares whose sides are the units of length, as in our system. For land measure a square 100 meters long called an **are** (meaning "area") is the unit.

28. **Table of Measures of Surface**

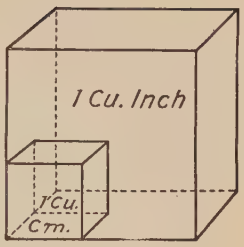
100 *square millimeters* (qmm)	= **1** *square centimeter* (qcm)	= **0.155** *sq. in.*
100 square centimeters	= **1** square decimeter (qdm)	
100 square decimeters	= **1** *square meter* (qm), **1** centare (ca)	= **10.764** *sq. ft.*
100 centares	= **1** *are* (a)	= **119.6** *sq. yd.*
100 ares	= **1** hectare (Ha)	= **2.5** *A.*

29. In square measure in the metric system 100 units of any denomination make one of the next higher denomination; therefore, reductions from one denomination to the next higher or lower may be made by moving the decimal point two places to the left or two places to the right.

Exercise 7

Express:

1. 4^{qm} in square decimeters; in square centimeters.
2. 65^{qdm} in square meters.
3. $4^{qm}\ 6^{qdm}$ in square meters; in square decimeters.
4. 425^{qm} in ares.
5. $4^{qdm}\ 6^{qcm}$ in square meters; in square decimeters.
6. 1.5^{qcm} in square meters; in square millimeters.
7. $14^{a}\ 7^{qm}$ in ares; in hectares.
8. 75832.4^{qm} in hectares, ares, and centares.
9. 45.674^{Ha} in hectares, ares, and centares.
10. 456.78^{a} in hectares, ares, and centares.

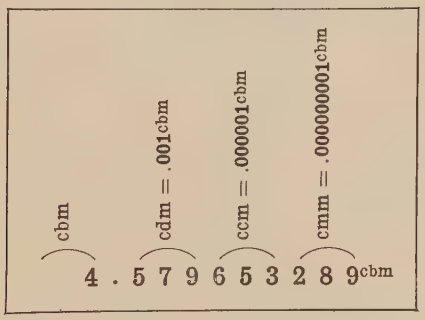


30. In cubic measure the units are cubes whose edges are the linear units. The **cubic meter** (sometimes called the **stere**, meaning "solid") is the unit of volume.

31. Table of Measures of Volume

1000 *cubic millimeters* (cmm)	= **1** *cubic centimeter* (ccm)
1000 cubic centimeters	= **1** *cubic decimeter* (cdm)
1000 cubic decimeters	= **1** *cubic meter* (cbm), or stere (s) = **35.315** cu. ft.

32. Scale of Cubic Measure



Exercise 8

Read and express:

1. 4.579^{cbm} in cubic decimeters.

Thus, 4.579 = 4579 thousandths.
$\therefore 4.579^{cbm} = 4579^{cdm}$.

2. 5.4^{cbm} in cubic decimeters; in cubic centimeters; in cubic millimeters.

3. 7.5^{ccm} in cubic meters; in cubic millimeters.

4. 146.750^{cbm} in cubic meters and cubic decimeters.

5. $1^{ccm}\ 10^{cmm}$ in cubic meters.

SUGGESTION. $1^{ccm}\ 10^{cmm} = .000001^{cbm} + .000000010^{cbm}$.

6. 16^{cdm} in cubic meters; in cubic centimeters; in cubic millimeters.

7. 250^{ccm} in cubic meters; in cubic decimeters; in cubic millimeters.

8. 4462.15^{cdm} in cubic meters, cubic decimeters, and cubic centimeters.

33. The primary unit of capacity is the *liter* which is exactly one cubic decimeter; it is equivalent to 1.0567 liquid quarts or 0.908 dry quarts.

34. **Table of Measures of Capacity**

10 milliliters (ml)	**= 1 centiliter** (cl)	
10 centiliters	**= 1 deciliter** (dl)	
10 deciliters	**= 1 *liter*** (l)	**= { 0.908 qt. (dry) / 1.0567 qt. (liq.)**
10 liters	**= 1 dekaliter** (Dl)	
10 dekaliters	**= 1 hectoliter** (Hl)	**= { 2.8375 bu. / 26.417 gal.**
10 hectoliters	**= 1 kiloliter** (Kl)	

Exercise 9

1. Read 8^{Kl} in hectoliters; in dekaliters; in liters.

SUGGESTION. See Sec. 26.

2. Read 275^{ml} in centiliters; in liters; in deciliters.
3. Express $3^{l}\ 2^{cl}\ 5^{ml}$ in liters.
4. Express $7^{dl}\ 8^{cl}\ 6^{ml}$ in liters; in milliliters.
5. Read 4000^{ml} in centiliters; in deciliters; in liters.
6. Read 380^{ml} in deciliters; in centiliters; in liters.
7. Express $2^{Kl}\ 5^{Hl}\ 3^{l}$ in liters.
8. Express $7^{Hl}\ 4^{Dl}\ 8^{l}$ in liters.

Measures of Weight

35. The primary unit of weight is the **gram**, which is the weight of one cubic centimeter of pure water at a temperature of 39.2 degrees Fahrenheit; it is equivalent to 15.432 grains.

Relative Size of Avoirdupois Ounce, 30-Gram, and Troy Ounce Weights.

36. Table of Measures of Weight

10 *milligrams* (mg)	= 1 *centigram* (cg)	
10 centigrams	= 1 decigram (dg)	
10 decigrams	= 1 *gram* (g)	= 15.432 gr.
10 grams	= 1 dekagram (Dg)	
10 dekagrams	= 1 hectogram (Hg)	
10 hectograms	= 1 *kilogram* (Kg)	= 2.2046 lb. (avoir.)
10 kilograms	= 1 myriagram (Mg)	
10 myriagrams	= 1 quintal (q)	
10 quintals	= 1 *Metric Ton* (MT)	= 2204.6 lb. (avoir.)

NOTE. $1^{mg} = 0.0154$ gr.

NOTE. It is evident that a liter of water weighs 1 kilogram and that 1000 liters weigh 1000 kilograms, or 1 metric ton.

Exercise 10

1. Read 7^{Kg} in hectograms; in grams.

SUGGESTION. See Sec. 26.

2. Read 2^{Kg} in grams; in centigrams; in milligrams.
3. Read 2000^{mg} in centigrams; in decigrams; in grams.
4. Read 2.455^{g} in grams and milligrams.
5. Read 7200.005^{g} in kilograms, grams, and milligrams.
6. Express $2^{Kg}\ 7^{g}\ 3^{mg}$ in grams.

Equivalents

37. Metric Units expressed in Terms of Common Units

Units	Equivalents	Approximate Equivalents
1 kilometer	0.62137 mile	220 rods
1 meter	39.37 inches	1.1 yards
1 centimeter	0.3937 inch	0.4 inch
1 hectare	2.471 acres	2.5 acres
1 square meter	1.1960 square yards	1.2 square yards
1 square centimeter	0.155 square inch	0.16 square inch
1 cubic meter	1.3079 cubic yards	1.3 cubic yards
1 cubic centimeter	0.061 cubic inch	0.06 cubic inch
1 hectoliter	2.83774 bushels	3 bushels
1 liter	0.9081 quart (dry)	0.9 quart (dry)
1 liter	1.05668 quarts (liquid)	1.06 quarts (liquid)
1 metric ton	2204.6 pounds	1 long ton
1 kilogram	2.20462 pounds (avoir.)	2.2 pounds
1 gram	15.4324 grains	15.4 grains

38. Common Units expressed in Terms of Metric Units

Units	Equivalents	Approximate Equivalents
1 inch	25.4001 millimeters	25 millimeters
1 foot	0.304801 meter	0.3 meter
1 yard	0.914402 meter	0.9 meter
1 mile	1.60935 kilometers	1.6 kilometers
1 square inch	6.4516 square centimeters	6.5 square centimeters
1 acre	0.4047 hectare	0.4 hectare
1 square yard	0.8361 square meter	0.8 square meter
1 cubic inch	16387.2 cubic millimeters	16 cubic centimeters
1 cubic yard	0.7646 cubic meter	0.75 cubic meter
1 gallon	3.78543 liters	3.75 liters
1 bushel	0.35239 hectoliter	0.35 hectoliter
1 ounce (troy)	31.10348 grams	31.1 grams
1 ounce (avoir.)	28.3495 grams	28.35 grams
1 pound (troy)	0.37324 kilogram	0.38 kilogram
1 pound (avoir.)	0.45359 kilogram	0.45 kilogram

39. The Metric System of Weights and Measures was first adopted in France in 1799. The system is almost universally used for scientific measurements. The following objections have been seriously urged against its general use, but as the public is becoming more and more familiar with it, these objections are disappearing.

It has been urged that:

1. The unit base, the meter, is too large to be clearly apprehended by the young and uninstructed.
2. The decimal division is too difficult, and is unsuited to practical purposes.
3. The unit of length should be some dimension of the human body.
4. It is inconvenient to think in one system and deal in another.
5. The decimal unit has failed when applied to the circle.
6. The adoption of the system would invalidate land titles.
7. The meter has never been correctly determined and never will be.

Some advantages are as follows:

1. Easiness of apprehension by pupils old enough to understand the decimal system of notation.
2. The facility with which the tables may be learned owing to the uniformity of scale.
3. The facility of reduction from one denomination to another of the same kind of quantity.
4. The simplicity of the relations of the units of surface, solids, capacity, and weight, to the units of length.
5. Its adoption would simplify international communication, since it is so generally used by other nations.

Exercise 11

(In changing from one system to another, use equivalents, page 352.)

1. Find, correct to the nearest hundredth, the number of millimeters in 2 inches.
2. Find the approximate number of kilometers in 5 miles.

3. A cask containing 120 liters contains, approximately, how many liquid quarts?

4. 2000 pounds (avoir.) are equal, approximately, to how many kilograms?

5. 10 miles are equal, approximately, to how many kilometers?

6. A square foot is equal, approximately, to how many square centimeters?

7. A cubic centimeter of water weighs a gram. Show that a cubic decimeter, or a liter, of water weighs a kilogram, and that a cubic meter weighs a metric ton.

8. Find the weight of a liter of milk, if it is 1.03 times as heavy as water.

9. Find the weight of 2^{ccm} of silver, which is 10.47 times as heavy as water.

10. How many cubic centimeters does a flask hold that has a capacity of 5^{cl}?

11. Find the weight of a milliliter of water.

12. A certain flask holds 2.5^{Kg} of water; how many liters does it hold? how many cubic decimeters?

13. Find in milligrams the weight of a centiliter of alcohol, which is .792 times as heavy as water.

14. If a field yielded on an average 15^{Hl} of wheat per hectare and each hectoliter of wheat yielded 55^{Kg} of flour, how many metric tons of flour did the field yield, if it is 50.5^{Dm} long and 24^{Dm} wide?

15. The weight of the olive oil in a certain flask is $36^{g}\ 600^{mg}$. How many cubic centimeters of this oil are there, if it is .915 times as heavy as water?

16. A piece of steel bar 1^{m} long, 8^{cm} wide, and 5^{cm} thick weighs how many kilograms, steel being 7.81 times as heavy as water?

17. How many kiloliters of water will a tank hold that is 4.5^{m} long, 2.4^{m} wide, and 4^{m} deep? Find the weight of the water in metric tons.

COMPOUND PROPORTION

40. A **compound ratio** is an expression denoting the product of two or more simple ratios.

Thus, $\left\{\begin{matrix}1:2\\3:4\\5:6\end{matrix}\right\}$ is equivalent to $\frac{1}{2}\times\frac{3}{4}\times\frac{5}{6}$, and may be read, *1 measured by 2, times 3 measured by 4, times 5 measured by 6.*

41. A **compound proportion** is a proportion one or both of whose ratios are compound.

42. In a proportion whose second ratio is compound, the first extreme equals the product of all the terms in the means divided by the product of all the terms in the last extreme.

$$\text{Thus, let } x:3=\left\{\begin{matrix}4:5\\6:7\\8:9\end{matrix}\right\}$$

$$\text{then, } \frac{x}{3}=\frac{4}{5}\times\frac{6}{7}\times\frac{8}{9}.$$

Multiplying by $3\times5\times7\times9$, we have $\cancel{3}\times5\times7\times\cancel{9}\times\frac{x}{\cancel{3}}=\cancel{3}\times\cancel{5}\times\cancel{7}\times\cancel{9}\times\frac{4}{\cancel{5}}\times\frac{6}{\cancel{7}}\times\frac{8}{\cancel{9}}$;

$$\text{or, } 5\times7\times9\times x=3\times4\times6\times8.$$

Illustrative Examples

1. If 20 horses, each consuming 12 quarts of oats daily, eat $262\frac{1}{2}$ bushels in 5 weeks, how many horses, each consuming 8 quarts daily, will consume 210 bushels in 10 weeks ?

Let x = the number of horses required.

Amount of stock	*Daily allowance*	*Quantity*	*Time*
x horses	8 qt.	210 bu.	10 wk.
20 horses	12 qt.	$262\frac{1}{2}$ bu.	5 wk.

The amount of stock required is inversely proportional to the daily allowance of a horse, and to the length of time the horses are fed, and directly proportional to the quantity consumed.

$$x : 20 = \left\{ \begin{array}{c} 12 : \quad 8 \\ 210 : 262\frac{1}{2} \\ 5 : \quad 10 \end{array} \right\}$$

Hence, $x = \dfrac{20 \times 12 \times 210 \times 5}{8 \times 262\frac{1}{2} \times 10}$, or 12.

Exercise 12

1. If 21 bu. of oats are required to feed 8 horses 7 da., how many bushels would be required to feed 16 horses 3 da.?

2. If a train running at the rate of 30 mi. per hour runs 240 mi. in 8 hr., in how many hours would it run 360 mi., running at the rate of 45 mi. per hour?

3. A lot 121 yd. long and 80 yd. wide contains 2 A.; how wide must a lot 220 yd. long be to contain $3\frac{1}{2}$ A.?

4. A wheel 6 ft. in circumference makes 880 revolutions in going a mile; how many revolutions would a wheel 4 ft. in circumference make in going 3520 ft.?

5. If a man working 10 hr. per day can build a wall 24 ft. long, 10 ft. high, and two bricks thick, in 2 da., in how many days of 8 hr. each can 3 men build a wall 30 ft. long, 8 ft. high, and 3 bricks thick?

6. If a mow of hay 32 ft. long, 16 ft. wide, and 16 ft. high lasts 8 horses 20 wk., how many weeks will a mow of hay 28 ft. long, 20 ft. wide, and 12 ft. high last 5 horses?

CUBE ROOT

43. The **cube root** of a given number is the number which cubed will produce the given number.

Thus, the cube root of 64 is 4, since $4^3 = 64$; the cube root of $\frac{8}{27}$ is $\frac{2}{3}$, since $(\frac{2}{3})^3 = \frac{8}{27}$; the cube root of .125 is .5, since $(.5)^3 = .125$.

44. The symbol $\sqrt[3]{\ }$, written before a number, indicates that the cube root of the number is to be taken.

Thus, $\sqrt[3]{27}$, read *the cube root of 27*, equals 3.

Exercise 13

1. Name the cube root of: 8; 27; 125; 64; $\frac{1}{8}$; $\frac{1}{27}$.
2. Name the cube root of: 1; 1000; 1,000,000; .001; .000001.
3. Find the cube root of: .008; .027; .064; .125.
4. Find the cube root of: $\frac{8}{27}$; $\frac{27}{64}$; $\frac{8}{125}$; $\frac{27}{125}$; $\frac{64}{125}$.
5. Find the cube root of 216.

Thus, $216 = 2 \times 2 \times 2 \times 3 \times 3 \times 3$
$= (2 \times 3) \times (2 \times 3) \times (2 \times 3)$
$= 6 \times 6 \times 6.$
Therefore, $\sqrt[3]{216} = 6.$

Find the cube root of:

6. 1728 7. 2744 8. 3375 9. 13,824

10. Find the length of a cube which contains 64 cu. in.

Suggestion. Since the number of cubic units in a cube is equal to the cube of the number of units in its length, then the number of units in its length is equal to the cube root of the number of cubic units in the cube.

Find the lengths of cubes that contain:

11. 125 cu. in. 12. 343 cu. in. 13. 729 cu. in.

General Rule for Extracting Cube Root

45. The cube root of numbers may be found by the following rule:

1. *Point off the number into periods of three figures each, beginning with the units.*

2. *Take the cube root of the nearest cube number below that expressed by the first period on the left, for the first figure of the root.*

3. *Subtract the cube of the first figure of the root from the first period of the number, and annex the next period to the remainder for a dividend.*

4. *Take three times the square of the root found for a trial divisor, and divide the dividend, exclusive of the two right-hand figures, by it for the next figure of the root.*

5. *Take three times the root previously found multiplied by the last figure found for the first correction, and the square of the last figure found for the second correction.*

6. *Place the right-hand figure of the first correction one order to the right of the trial divisor, and the right-hand figure of the second correction one order to the right of the first correction, then take the sum of the trial divisor and the two corrections for the complete divisor.*

7. *Multiply the complete divisor by the last figure of the root now found, and subtract the product from the dividend.*

8. *If the root is not fully found, annex the next period of the number to the remainder for a dividend and again find the trial divisor, the next figure of the root, the corrections, and the complete divisor as before; and so continue till all the periods of the number have been used.*

9. *Should a cipher occur in the root, annex two ciphers to the trial divisor to form the next trial divisor, and annex another period to the dividend.*

Illustrative Examples

1. Extract the cube root of 45,118,016.

	45'118'016 | 356
	27
$3 \times 3 \times 3 = 27$	18118
$3 \times 3 \times 5 = 45$	
$5 \times 5 = 25$	
3175	15875
$3 \times 35 \times 35 = 3675$	2243016
$3 \times 35 \times 6 = 630$	
$6 \times 6 = 36$	
373836	2243016

Explanation:

Point off the number into periods of three figures each, beginning at units. The greatest number whose cube is contained in 45 is 3, which is the first figure of the root. Cube 3; subtract the result from 45, and annex 118 to the remainder, making the dividend 18,118. Take 3 times the square of 3, or 27, for a trial

divisor. Divide 18,118, exclusive of the two right-hand figures, by 27, making allowance for corrections to be added, for the next figure of the root, which is found to be 5. Take 3 times 3 times 5 for the first correction, and set it one place to the right of the trial divisor. Take the square of 5 for the second correction, and set it one figure to the right of the first correction. The sum of the trial divisor and the two corrections, 3175, is the complete divisor. Multiply 3175 by 5, the last figure of the root found, subtract the product 15,875 from 18,118, and annex 016 to the remainder, making the dividend 2,243,016. Take 3 times the square of 35 for the trial divisor. Find as before the next figure of the root 6, the first correction 630, the second correction 36, the complete divisor 373,836, the product 2,243,016, and the remainder 0. The cube root is 356.

2. Extract the cube root of 25, correct to the nearest hundredth.

$$
\begin{array}{rl|l}
 & 25.'000'000 & 2.924 \\
 & 8 & \\
\hline
3 \times 2 \times 2 = 12 & 17000 & \\
3 \times 2 \times 9 = \ 54 & & \\
9 \times 9 = \ 81 & & \\
\hline
 & 1821 & 16389 \\
\hline
3 \times 29 \times 29 = 2523 & 611000 & \\
3 \times 29 \times 2 \ = \ 174 & & \\
2 \times 2 \ = \ 4 & & \\
\hline
 & 254044 & 508088 \\
\hline
3 \times 292 \times 292 = 255792 & 102912000 & \\
3 \times 292 \times \ 4 = \ 3504 & & \\
4 \times \ 4 = \ 16 & & \\
\hline
 & 25614256 & 102457024
\end{array}
$$

Explanation:

Separate the number into periods of three figures each rightward and leftward from the decimal point. The cube root is found to be nearer to 2.92 than 2.93. Therefore 2.92 is the cube root of 25, correct to the nearest hundredth.

Exercise 14

Extract the cube root of:

1. 3375	2. 195,112	3. 17,576
4. 50,653	5. 421,875	6. 681,472
7. 1,442,897	8. 16,777,216	9. 28,094,464
10. 45,118,016	11. 131,096,512	12. 175,616,000

Extract the cube root, correct to the nearest hundredth, of:

13. 25	14. 126	15. 4.5
16. 3.75	17. 300	18. .005

19. Extract the cube root of $\frac{27}{64}$.

SUGGESTION. Since $\frac{3}{4} \times \frac{3}{4} \times \frac{3}{4} = \frac{27}{64}$, the cube root of $\frac{27}{64} = \frac{3}{4}$.

NOTE. If a fraction in its lowest terms is a perfect cube, its terms are perfect cubes. If the terms of a fraction in its lowest terms are not perfect cubes, it is best to reduce the fraction to a decimal before extracting the cube root.

Extract the cube root of:

20. $\frac{64}{1728}$ 21. $\frac{125}{1728}$ 22. $12\frac{19}{27}$ 23. $\frac{3}{5}$ to hundredths

24. $\frac{729}{1000}$ 25. $\frac{9261}{10648}$ 26. $5\frac{104}{125}$ 27. $\frac{2}{7}$ to hundredths

TABLES OF DENOMINATE AMOUNTS

(For Reference)

United States Money

10 mills = 1 cent
10 cents = 1 dime
10 dimes = 1 dollar
10 dollars = 1 eagle

Although the gold dollar is no longer coined, it is represented by 25.8 grains of standard gold .9 pure. The standard silver dollar consists of 412.5 grains of standard silver. All gold and silver coins are .9 pure metal and .1 copper.

Counting

12 units = 1 dozen
12 dozen = 1 gross
12 gross = 1 great gross
20 units = 1 score

Paper

24 sheets = 1 quire
20 quires = 1 ream
2 reams = 1 bundle
5 bundles = 1 bale

Dry Measure

2 pints = 1 quart
2 quarts = 1 small measure
8 quarts = 1 peck
4 pecks = 1 bushel stricken measure
5 pecks = 1 heaped bushel, nearly

The heaped bushel, which contains about 5 pecks, or $\frac{5}{4}$ of a stricken bushel, is used in measuring corn in the ear, potatoes, apples, lime, turnips, and various other substances.

A stricken bushel equals about $\frac{5}{4}$ of a cubic foot, and a cubic foot equals about $\frac{4}{5}$ of a stricken bushel.

A heaped bushel is about $\frac{25}{18}$ of a cubic foot, and a cubic foot is about $\frac{18}{25}$ of a heaped bushel.

Two bushels of corn in the ear are considered the equivalent of one bushel of shelled corn.

The standard bushel is a cylindrical vessel whose inner diameter is 18.5 in. and depth 8 in.; it contains 2150.42 cu. in.

Liquid Measure

4 gills = 1 pint
2 pints = 1 quart
4 quarts = 1 gallon
31½ gallons = 1 barrel
2 barrels = 1 hogshead

Nearly all kinds of liquids are sold in the United States on the basis of 231 cu. in. to the gallon; but ale and beer are sold on the basis of 282 cu. in. to the gallon.

The barrel (31½ gal.) and the hogshead (63 gal.) in the above table represent *measures*. In practice a barrel holds from 28 gal. to 43 gal., and a hogshead from 54 gal. to 63 gal.

In mixing liquid medicines druggists use **Apothecaries' Fluid Measure**, which is as follows:

60 minims (♏) = 1 fluid dram (ƒ ℨ)
8 fluid drams = 1 fluid ounce (ƒ ℨ)
16 fluid ounces = 1 pint (O.)
8 pints = 1 gallon (Cong.)

A minim is about a drop.

An ounce bottle holds a fluid ounce, or $\frac{1}{16}$ of a pint; a 2-ounce bottle, $\frac{1}{8}$ of a pint; a 3-ounce bottle, $\frac{3}{16}$ of a pint; and so on.

Troy Weight

24 grains = 1 pennyweight
20 pennyweights = 1 ounce
12 ounces = 1 pound

Note. The unit of weight for gems and precious stones is the **carat**, which equals 200 milligrams, or about 3.08 troy grains. The term *carat* is also used to denote the purity, or fineness, of gold, 24 carats denoting pure gold.

The troy pound contains 5760 grains, which is also the unit of apothecaries' weight.

Medicines are sold in prescriptions by apothecaries' weight, which is as follows:

1 pound (℔) = 12 ounces (ℨ) = 96 drachms (ℨ) = 288 scruples (℈) = 5760 (gr.)

Avoirdupois Weight

16 ounces (oz.) = 1 pound (lb.)
25 pounds, 28 pounds = 1 quarter (qr.)
4 quarters = 1 hundredweight (cwt.)
20 hundredweight = 1 Ton (T.)

Note 1. 1 lb. avoir. = 7000 troy grains.

Note 2. The long hundredweight, 112 pounds, and the long ton, 2240 pounds, are used in custom houses and in some wholesale and retail transactions.

Weights of a Bushel of Grain, Seed, and Other Produce

Commodities	Lb.	Commodities	Lb.
Barley	48	Clover seed	60
Beans	60	Oats	32
Buckwheat	48	Peas	60
Bran	20	Rye	56
Corn (shelled)	56	Timothy seed	45
Corn (on cob)	70	Wheat	60

In Maryland and Pennsylvania the legal weight of a bushel of potatoes is 56 lb.; in the other states it is 60 lb.

Linear Measure

12 inches = 1 foot
3 feet = 1 yard
5½ yards = 1 rod
320 rods = 1 statute mile

The **furlong** (40 rd.) is now seldom used.

Other Linear Units

(1) A **fathom** = 6 feet, used at sea to measure cables, cordage, and the depth of the sea.

(2) A **hand** = 4 inches, used in measuring a horse's height.

(3) A **geographic mile**, or **knot** = 6086 ft. = the length of one minute of the equatorial circumference of the earth.

(4) A **league** (England and U. S.) = 3 geographic miles, used to measure distances at sea.

(5) A **pace** = 2½ feet in ordinary walking, but 3.3 feet, or ⅕ of a rod, in stepping distances.

(6) 7.92 inches = 1 link.

(7) 100 links = 1 chain.

Land Measure

10,000 square links (sq. li.) = 1 square chain
16 square rods (sq. rd.) = 1 square chain
10 square chains = 1 acre
160 square rods = 1 acre
640 acres = 1 square mile

A square rod is sometimes called a **perch**.

Square Measure

144 square inches (sq. in.) = 1 square foot
9 square feet = 1 square yard
30¼ square yards = 1 square rod

Cubic Measure

1728 cubic inches = 1 cubic foot
27 cubic feet = 1 cubic yard

1 cubic yard of earth is called a **load**.

A pile of wood 8 ft. long, 4 ft. wide, and 4 ft. high is called a **cord**.

24¾ cu. ft. of masonry is called a **perch**.

Angles and Arcs

60 seconds = 1 minute
60 minutes = 1 degree
90 degrees = 1 right angle
90 degrees = 1 quadrant
360 degrees = 1 circumference

Weights of Everyday Things

A barrel of flour weighs	196 lb.
A barrel of salt weighs	280 lb.
A barrel of pork, beef, or fish weighs	200 lb.
A bushel of coarse salt weighs	56 lb.
A keg of nails weighs	100 lb.
A firkin of butter weighs	56 lb.
A keg of powder weighs	25 lb.
A bushel of Portland cement weighs	96 lb.
A bushel of loose lime weighs	70 lb.
A cubic foot of water weighs	62½ lb.
400 cu. ft. of hay in the mow weighs	1 ton

Time

60 seconds	= 1 minute
60 minutes	= 1 hour
24 hours	= 1 day
7 days	= 1 week
52 weeks	= 1 year (nearly)
365 days	= 1 common year
366 days	= 1 leap year
100 years	= 1 century

The exact length of the year is 365 days 5 hours 48 minutes 49.7 seconds. But, as it is not convenient to have other than a whole number of days in a year, it has been decreed that:

Every year whose number is not divisible by 4 shall consist of 365 days, called a **common year**.

Every year whose number is divisible by 4 shall consist of 366 days, called a **leap year**, except when it is a centennial year.

A centennial year whose number is divisible by 400 shall be a leap year, and when not so divisible, a common year.

Foreign Money

4 farthings	= 1 penny
12 pence	= 1 shilling
20 shillings	= 1 pound

1 guinea = 21*s*. 1 crown = 5*s*.
1 florin = 2*s*. £1 = $4.8665

NOTE. In Canada the units of money are the same as in the United States.

The standard unit of value in France is the **franc**; its value in U. S. money is $0.193, or $1 = about 5.18 francs.

The standard unit of value in Germany is the **reichsmark**, or the **mark**; its value in U. S. money is $0.2385, or 4 marks equals $0.945.

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ANSWERS

Book Three

Exercise 6, page 16

1. 64,127,036. **2.** 67,869,986. **3.** 9,797,161. **4.** 5,860,854. **5.** 48,194,000.

Exercise 7, page 18

1. 4131. **2.** 20,546. **3.** 27,496. **4.** $2193.10. **5.** 2151.
6. 16,663. **7.** 22,582. **8.** $2239.50. **9.** 21,183. **10.** 359,627.

Exercise 9, page 20

1. 21,745,584. **2.** 271,777,406. **3.** 139,854,276. **4.** 152,843,769.
5. 157,326,849. **6.** 215,384,976. **7.** 245,893,761. **8.** 254,817,369.

Exercise 11, page 24

1. 401. **2.** 235. **3.** 260. **4.** 2183.

Exercise 12, page 25

1. 208,953,942. **2.** $446,800,000,000. **3.** 195 mi. **4.** 5,103,000,000.
5. $1075. **6.** 1047. **7.** 70. **8.** 33; 1500.
9. 25. **10.** 40. **11.** 12. **12.** 88,200 lb.
13. 119,988. **14.** 572. **15.** 245 mi. **16.** $1,532,876,200.
17. 3800 lb. **18.** 109 gal. **19.** 1152 bu. **20.** 96,000,000.
21. $650. **22.** 28; 1515. **23.** 10,368. **24.** 125,671 sq. mi.
25. 1092. **26.** $150. **27.** $1, $2, $5, $10. **28.** 78.
29. 33,350. **30.** 248,832. **31.** 899. **32.** $78.
33. 75. **34.** 30 da. **35.** 794.

Exercise 13, page 31

1. 4. **2.** 24. **3.** 4. **4.** 20. **5.** 19. **6.** 760. **7.** 700. **8.** 304

Exercise 16, page 35

1. 7. 2. 8. 3. 10. 4. 9. 5. 4.
6. 7. 7. 138. 8. 48. 9. 165. 10. 108.
11. 112. 12. 78. 13. 625. 14. 56. 15. 2, 5, 10.
16. 2, 3, 5, 6, 10, 15, 30. 17. 2, 3, 5, 6, 10, 15, 25, 30, 50, 75, 150.
18. 2, 3, 5, 6, 7, 10, 14, 15, 21, 30, 35, 42, 70, 105, 210.

Exercise 17, page 36

1. 420. 2. 2772. 3. 720. 4. 750. 5. 7920.
6. 6300. 7. 22,000. 8. 3600. 9. 1500. 10. 10,800.

Exercise 18, page 38

1. 1500. 2. 17,500. 3. 1400. 4. 4000. 5. 6.
6. 4. 7. 5. 8. 4. 9. 12. 10. 35.
11. 100. 12. 108. 13. 27. 14. 100. 15. 12.
16. 54. 17. 360. 18. 12. 19. 9. 20. 28.
21. 588. 22. 1.

Exercise 21, page 46

1. $\frac{99}{8}$. 2. $\frac{75}{2}$. 3. $\frac{100}{7}$. 4. $\frac{200}{3}$.
5. $\frac{127}{8}$. 6. $\frac{232}{6}$. 7. $\frac{342}{4}$. 8. $\frac{987}{8}$.
9. $\frac{1412}{9}$. 10. $\frac{1562}{5}$. 11. $\frac{7338}{15}$. 12. $\frac{15773}{10}$.
13. $\frac{86332}{25}$. 14. $\frac{123401}{50}$. 15. $\frac{101123}{20}$. 16. $\frac{7502}{25}$.
17. 100. 18. $33\frac{1}{3}$. 19. $14\frac{2}{7}$. 20. $21\frac{7}{8}$.
21. $11\frac{1}{2}$. 22. 28. 23. $63\frac{1}{3}$. 24. $16\frac{1}{12}$.
25. $13\frac{1}{5}$. 26. 80. 27. 144. 28. $87\frac{3}{5}$.
29. 50. 30. $133\frac{1}{3}$. 31. 250. 32. 125.
33. $\frac{3}{8}$. 34. $\frac{4}{5}$. 35. $\frac{3}{4}$. 36. $\frac{11}{14}$.
37. $\frac{33}{50}$. 38. $\frac{103}{180}$. 39. $\frac{4}{5}$. 40. $\frac{15}{16}$.
41. $\frac{27}{28}$. 42. $\frac{31}{32}$. 43. $\frac{20}{31}$. 44. $\frac{41}{41}$.
45. $\frac{0}{10}$. 46. $\frac{1}{3}$. 47. $\frac{5}{8}$. 48. $\frac{7}{8}$.
49. $\frac{40}{60}$, $\frac{45}{60}$, $\frac{48}{60}$. 50. $\frac{20}{30}$, $\frac{24}{30}$, $\frac{25}{30}$. 51. $\frac{16}{24}$, $\frac{14}{24}$, $\frac{9}{24}$.
52. $\frac{3}{5}$, $\frac{2}{5}$, $\frac{3}{5}$. 53. $\frac{24}{30}$, $\frac{21}{30}$, $\frac{4}{30}$. 54. $\frac{18}{48}$, $\frac{12}{48}$, $\frac{9}{48}$.
55. $\frac{35}{60}$, $\frac{9}{60}$, $\frac{2}{60}$. 56. $\frac{8}{36}$, $\frac{2}{36}$, $\frac{20}{36}$. 57. $\frac{20}{48}$, $\frac{21}{48}$, $\frac{2}{48}$.
58. $\frac{320}{600}$, $\frac{48}{600}$, $\frac{45}{600}$. 59. $\frac{72}{600}$, $\frac{140}{600}$, $\frac{135}{600}$. 60. $\frac{45}{144}$, $\frac{42}{144}$, $\frac{88}{144}$.
61. $\frac{504}{1800}$, $\frac{135}{1800}$, $\frac{280}{1800}$. 62. $\frac{180}{200}$, $\frac{88}{200}$, $\frac{15}{200}$. 63. $\frac{30}{84}$, $\frac{20}{84}$, $\frac{27}{84}$.
64. $\frac{60}{72}$, $\frac{14}{72}$, $\frac{5}{72}$. 65. $\frac{28}{60}$, $\frac{9}{60}$, $\frac{16}{60}$. 66. $\frac{105}{180}$, $\frac{132}{180}$, $\frac{50}{180}$.
67. $\frac{45}{90}$, $\frac{20}{90}$, $\frac{27}{90}$. 68. $\frac{35}{120}$, $\frac{66}{120}$, $\frac{68}{120}$. 69. $\frac{8}{100}$, $\frac{6}{100}$, $\frac{7}{100}$.

Exercise 22, page 49

1. $\frac{23}{30}$. 2. $\frac{29}{36}$. 3. $\frac{3}{10}$. 4. $\frac{7}{64}$. 5. $\frac{1}{12}$.
6. $\frac{19}{72}$. 7. $1\frac{13}{60}$. 8. $1\frac{17}{24}$. 9. $1\frac{23}{36}$. 10. $\frac{41}{150}$.
11. $\frac{21}{32}$. 12. $1\frac{47}{60}$. 13. $\frac{29}{72}$. 14. $\frac{1}{50}$. 15. $\frac{21}{100}$.
16. $\frac{81}{100}$. 17. $\frac{21}{64}$. 18. $1\frac{33}{150}$. 19. $2\frac{29}{120}$. 20. $1\frac{13}{32}$.
21. $1\frac{1}{15}$. 22. $\frac{3}{5}$. 23. $\frac{23}{28}$. 24. $\frac{203}{800}$. 25. $3\frac{29}{80}$.
26. $5\frac{89}{120}$. 27. $14\frac{3}{20}$. 28. $\frac{29}{30}$. 29. $2\frac{1}{24}$. 30. $\frac{29}{48}$.
31. $1\frac{7}{20}$. 32. $24\frac{29}{32}$. 33. $1\frac{1}{2}$. 34. $1\frac{4}{7}$. 35. $20\frac{9}{10}$.
36. $20\frac{5}{12}$. 37. $3\frac{111}{120}$. 38. $1\frac{3}{4}$. 39. $2\frac{49}{150}$. 40. $1\frac{5}{48}$.
41. $1\frac{59}{144}$. 42. $1\frac{1}{20}$. 43. $129\frac{37}{42}$. 44. 66. 45. $63\frac{2}{3}$.
46. $15\frac{5}{8}$.

Exercise 24, page 53

1. $1\frac{1}{3}$. 2. $1\frac{3}{4}$. 3. $18\frac{3}{4}$. 4. $1\frac{11}{25}$. 5. $1\frac{2}{5}$.
6. $2\frac{4}{5}$. 7. $1\frac{11}{16}$. 8. 7. 9. $5\frac{2}{3}$. 10. $2\frac{1}{3}$.
11. $1\frac{1}{8}$. 12. $1\frac{1}{4}$. 13. $\frac{21}{40}$. 14. $\frac{85}{64}$. 15. $\frac{9}{10}$.
16. $1\frac{1}{14}$. 17. $\frac{3}{64}$. 18. $\frac{4}{9}$. 19. $33\frac{1}{3}$. 20. $22\frac{2}{3}$.
21. $28\frac{1}{8}$. 22. $41\frac{1}{4}$. 23. $28\frac{1}{8}$. 24. 25. 25. $\frac{1}{36}$.
26. $\frac{1}{15}$. 27. $\frac{2}{5}$. 28. $\frac{7}{18}$. 29. 1878. 30. 7576.
31. 13,420$\frac{4}{5}$. 32. 34,951$\frac{1}{9}$. 33. 3,437$\frac{1}{2}$. 34. 14,208. 35. 30,043$\frac{3}{4}$.
36. 31,521$\frac{3}{5}$. 37. 11,260$\frac{1}{12}$. 38. 22,765$\frac{1}{6}$. 39. 39,823$\frac{1}{2}$. 40. 32,687$\frac{2}{5}$.

Exercise 25, page 56

1. $\frac{4}{25}$. 2. $\frac{8}{125}$. 3. $\frac{3}{50}$. 4. $\frac{1}{49}$. 5. $\frac{3}{5}$.
6. $\frac{2}{45}$. 7. $\frac{1}{15}$. 8. $\frac{24}{35}$. 9. $1\frac{1}{4}$. 10. $3\frac{1}{7}$.
11. $6\frac{2}{3}$. 12. $6\frac{2}{3}$. 13. $2\frac{1}{2}$. 14. $2\frac{7}{9}$. 15. $12\frac{1}{2}$.
16. $12\frac{1}{2}$. 17. 63. 18. 108. 19. $10\frac{1}{2}$. 20. 20.
21. 60. 22. 75. 23. 27. 24. $23\frac{1}{7}$. 25. 6.
26. 9. 27. 6. 28. $6\frac{6}{7}$. 29. $\frac{5}{12}$. 30. $\frac{15}{16}$.
31. $\frac{32}{125}$. 32. $3\frac{1}{3}$. 33. $\frac{1}{2}$. 34. 33. 35. $\frac{1}{6}$.
36. $\frac{2}{3}$. 37. $3\frac{3}{4}$. 38. $\frac{7}{15}$. 39. $44\frac{3}{50}$. 40. $1\frac{1}{4}$.
41. $13\frac{7}{11}$. 42. $21\frac{23}{37}$. 43. $9\frac{8}{100}$. 44. $71\frac{7}{69}$. 45. $2\frac{271}{1182}$.
46. $6\frac{231}{580}$.

Exercise 26, page 57

1. $\frac{5}{24}$. 2. $73\frac{1}{8}$. 3. $\frac{23}{96}$. 4. 15. 5. $3\frac{9}{25}$.
6. $\frac{16}{185}$. 7. $47\frac{17}{42}$. 8. 6. 9. $4\frac{1}{2}$. 10. 17.
11. $\frac{9}{16}$. 12. $\frac{1}{21}$. 13. $2\frac{2}{11}$. 14. $\frac{3}{29}$. 15. $\frac{19}{380}$.
16. $\frac{3}{160}$. 17. $3\frac{7}{80}$. 18. $1\frac{5}{12}$.

Exercise 27, page 57

1. 26. **2.** $6\frac{1}{2}$. **3.** 18. **4.** $2\frac{1}{2}$. **5.** $6\frac{3}{4}$.
6. $\frac{1}{12}$ greater. **7.** $\frac{2}{15}$ less. **8.** \$1.55. **9.** 6. **10.** $1\frac{7}{8}$.
11. $\frac{8}{9}$. **12.** 55 mi. **13.** $7619\frac{1}{2}$. **14.** $35\frac{1}{5}$. **15.** $90\frac{5}{12}$.
16. $\frac{1}{4}$; $\frac{3}{4}$. **17.** $\frac{1}{4}$; $\frac{1}{3}$; $\frac{1}{2}$; $\frac{2}{3}$; $\frac{7}{6}$. **18.** $1\frac{5}{7}$ da. **19.** $\frac{25}{28}$. **20.** $2\frac{1}{2}$ da.
21. The latter by \$10.
22. 12 cups; 18 cups; 2 doz.; 6 teaspoonfuls; 18 teaspoonfuls.
23. 208 bu.; 242 bu.; 640 bu.; 61 bu. **24.** 6300 lb.
25. 20¢. **26.** 60; 100.
27. 18 teaspoonfuls; 6 cups; 6 cups; 12 teaspoonfuls; 6 lemons; 1 doz.; 6 teaspoonfuls.
28. $\$1\frac{1}{4}$. **29.** $2\frac{11}{12}$ hr. **30.** 50; 48.
31. 64. **32.** \$432. **33.** $\frac{2}{7}$. **34.** $\frac{2}{3}$.
35. 4 cups; 1 teaspoonful; 6 teaspoonfuls; 6 tablespoonfuls; 4 eggs; 3 cups; 8 tablespoonfuls; 8 apples; 1 teaspoonful. **36.** 31 ft. **37.** $\$1\frac{1}{4}$.
38. \$350. **39.** \$3.06. **40.** $\frac{8}{11}$. **41.** $90\frac{1}{4}$; $59\frac{3}{4}$. **42.** $\$15\frac{1}{2}$; \$10.
43. 3 mi. **44.** $40\frac{3}{4}$; $34\frac{3}{4}$. **45.** $52\frac{1}{2}$ lb.

Exercise 28, page 65

28. .7. **29.** .45. **30.** .03. **31.** .009.
32. .025. **33.** 2.7. **34.** .135. **35.** 1.8.
36. 3.75. **37.** .0008. **38.** $.37\frac{1}{2}$. **39.** $.87\frac{1}{2}$.
40. .707. **41.** 700.007. **42.** 3.03. **43.** .4325
44. $.0062\frac{1}{2}$. **45.** 300.03.

Exercise 29, page 67

1. $\frac{2}{25}$. **2.** $\frac{1}{40}$. **3.** $\frac{3}{8}$. **4.** $\frac{5}{8}$.
5. $4\frac{1}{4}$. **6.** $5\frac{1}{20}$. **7.** $3\frac{3}{4}$. **8.** $\frac{9}{20}$.
9. $\frac{1}{250}$. **10.** $\frac{3}{80}$. **11.** $6\frac{1}{200}$. **12.** $\frac{7}{8}$.
13. $\frac{5}{6}$. **14.** $\frac{5}{12}$. **15.** $\frac{5}{8}$. **16.** $\frac{5}{8}$.
17. $\frac{5}{16}$. **18.** $\frac{3}{8}$. **19.** $\frac{2}{3}$. **20.** $\frac{7}{8}$.
21. $\frac{5}{6}$. **22.** $\frac{2}{9}$. **23.** $\frac{3}{16}$. **24.** $\frac{1}{7}$.
25. $\frac{4}{9}$. **26.** $\frac{2}{7}$. **27.** $1\frac{1}{8}$. **28.** $1\frac{3}{8}$.
29. .75. **30.** .375. **31.** .875. **32.** .1875.
33. .1. **34.** .225. **35.** .21875. **36.** .140625.
37. $.6\frac{2}{3}$. **38.** $.5\frac{5}{9}$. **39.** $.0\frac{5}{8}$. **40.** $.4\frac{2}{7}$.
41. $.83\frac{1}{3}$. **42.** $.88\frac{8}{9}$. **43.** $.41\frac{2}{3}$. **44.** $.46\frac{2}{3}$.
45. $.428\frac{4}{7}$. **46.** $.444\frac{4}{9}$. **47.** $.583\frac{1}{3}$. **48.** $.733\frac{1}{3}$.
49. $.41\frac{2}{3}$. **50.** $.83\frac{1}{3}$. **51.** $.166\frac{2}{3}$. **52.** $.142\frac{6}{7}$.
53. $.583\frac{1}{3}$. **54.** $.2857\frac{1}{7}$. **55.** $.12\frac{1}{2}$. **56.** $.63\frac{3}{4}$.
57. $.5\frac{5}{8}$. **58.** $.33\frac{1}{3}$.

Exercise 30, page 69

1. 38.291. 2. 141.0412. 3. 134.5185. 4. 26.6365.
5. 354.734$\frac{1}{9}$. 6. 2.041$\frac{2}{3}$. 7. .302. 8. .000192.
9. 1.0819. 10. 3.333$\frac{1}{3}$. 11. 2.5666$\frac{2}{3}$. 12. 11.86$\frac{1}{3}$.
13. 142.8312. 14. 5.2212.

Exercise 32, page 71

1. 61.62. 2. 231. 3. .5792. 4. 116.92.
5. 203. 6. .2618. 7. 2.1875. 8. 5.056.
9. .08208. 10. 48.006. 11. 58.272. 12. 1.8672.
13. 1.4592. 14. .000384. 15. .07182.

Exercise 33, page 72

1. $19.61. 2. $540. 3. $70. 4. $720.
5. $2.88. 6. Wheat, 16,016,000; Oats, 10,500,000; Barley, 2,490,000; Rye, 198,000.
7. $326.50. 9. $235.79. 10. $3.39.

Exercise 34, page 75

1. .1. 2. .013. 3. .041. 4. .00006.
5. 3.25. 6. .0327. 7. .0456. 8. 3.6.
9. .000003. 10. .0024. 11. 1000. 12. .01.
13. 3.6. 14. 1.26. 15. 12. 16. 30,250.
17. 2.5. 18. 24. 19. 2300. 20. .14.

Exercise 35, page 76

1. .097. 2. .057. 3. 150. 4. 2.1.
5. .068. 6. 99. 7. .087. 8. 26.
9. 10.6. 10. 108. 11. 30.9. 12. .00406.
13. .404. 14. .708. 15. 809. 16. 8060.
17. 7600. 18. .605. 19. 609. 20. 909.

Exercise 36, page 77

1. .4667. 2. .3214. 3. .3409. 4. .1985.
5. .0016. 6. .0016. 7. .0314. 8. .0047.
9. .0005. 10. .3947. 11. .1067. 12. .0033.
13. .0148. 14. .0076. 15. .0615. 16. 7.4146.
17. .3529. 18. .9032. 19. .4043. 20. .0571.
21. .0653. 22. .0821. 23. .9913. 24. .6667.

25. .3333.	**26.** .3003.	**27.** .6667.	**28.** .0615.
29. 7.4146.	**30.** .3529.	**31.** .9032.	**32.** .4043.
33. .0571.	**34.** .0653.	**35.** .0821.	**36.** 1.1217.
37. .7778.	**38.** .3745.	**39.** .6006.	**40.** .7619.
41. .3002.	**42.** .1286.	**43.** .7147.	

Exercise 37, page 78

3. $7.60.	**4.** $66.95.	**5.** $55.54.	**6.** $165.
7. $48.55.	**8.** $381.30.		

Exercise 38, page 83

1. 6¢.	**2.** 8¢.	**3.** 35¢.	**4.** 62¢.
5. 24¢.	**6.** 2¢.	**7.** 10¢.	**8.** 35¢.
9. 7¢.	**10.** 10¢.	**11.** 8¢.	**12.** 10¢.

Exercise 39, page 88

1. $0.88; $0.71; $0.60; $0.90.
2. $1.84; $1.84; $1.00; $1.50.
3. $2.05; $2.05; $1.00; $1.50.
4. $1.15; $0.92; $0.75; $1.13.
5. $1.30; $0.75; $0.36; $0.54.
6. $0.47; $0.36; $0.25; $0.38.
7. $1.80; $1.40; $0.60; $0.90.
8. $0.80; $0.65; $0.60.
9. $1.98; $1.98; $1.00; $1.50.
10. $1.25; $1.15; $0.50; $0.75.
11. $0.77; $0.67; $0.50; $0.75.
12. $0.60; $0.50.
13. $1.80; $1.40; $0.60; $0.90.
14. $0.98; $0.88; $0.50; $0.75.
15. $1.70; $1.60; $0.50; $0.75.
16. $0.37; $0.31; $0.25.
17. $0.79; $0.47; $0.35; $0.53.
18. $1.00; $0.60; $0.30; $0.45.
19. $0.59; $0.42; $0.25; $0.38.

Exercise 40, page 89

1. 18 lb. **2.** $8.64. **3.** 6¢; 6¢; 8¢; 10¢. **4.** 3¢; 2¢; 4¢; 4¢.
5. 3¢; 4¢; 4¢; 3¢. **6.** 3¢; 4¢; 2¢; 2¢. **7.** 18¢. **8.** 14¢.
9. 14¢. **10.** $7.17. **11.** $11.15. **12.** 7¢.
13. 7.45. **14.** $38.50. **15.** 230.4. **18.** 709.4 lb.
19. $67.50. **20.** $1.93. **21.** 7.48. **22.** 643.75 lb.
23. $31.24. **24.** 8.61 lb. **25.** $4\frac{4}{5}$. **26.** 16 bu.
27. $4.68. **28.** 9.31. **29.** .3. **30.** .045.
31. $27. **32.** 18.764 yr. **33.** 2. **34.** 168.
35. $383.33. **36.** 444. **37.** $134.40; $3897.60; $84.40; $2447.60.

Exercise 41, page 95

1. 96 qt.	**2.** 36 pt.	**3.** 126 in.	**4.** 4480 lb.
5. 45.375 ft.	**6.** 420 hr.	**7.** 5760 min.	**8.** 4212 sq. in.
9. $\frac{2}{15}$ ft.	**10.** 283.5 gal.	**11.** 1440 sheets.	**12.** $3\frac{1}{8}$ bu.
13. $9\frac{3}{8}$ gal.	**14.** 8 yd.	**15.** $3\frac{1}{4}$ A.	**16.** $4\frac{1}{6}$ rd.

Exercise 42, page 96

1. 80.	**2.** 1.12.	**3.** 180.	**4.** $5\frac{1}{8}$.
5. $83\frac{1}{3}$.	**6.** 32 rd.	**9.** 1296.	**10.** 80¢.
11. 5.	**12.** 28¢.	**13.** 20¢.	**14.** \$9.31.
15. \$16.74.	**16.** \$29.61.	**17.** \$10.40.	**18.** \$26.79.
19. 67.20.	**20.** 40¢.	**21.** 24¢.	**22.** 32.
23. $12\frac{1}{2}$.	**25.** \$1.25.	**26.** \$21.88.	**27.** June 30.
28. Aug. 29.	**29.** Apr. 28.	**30.** $\frac{5}{8}$.	**31.** $\frac{1}{4}$.
32. 58 yr.	**33.** 350 yr.	**34.** Dec. 30.	

Exercise 43, page 101

3. 36.	**4.** $\frac{1}{4}$.	**6.** 96.	**7.** 720.	**8.** $6\frac{1}{4}$; 165.
9. 12 sq. ft.	**10.** 48.	**11.** 54.	**12.** 160.	

Exercise 44, page 103

1. $7\frac{1}{2}$.	**2.** 6.	**3.** \$141.66.	**4.** 80 yd.
5. 60; 120; \$600.	**6.** 1920.	**7.** \$7200.	

Exercise 45, page 105

1. 6.	**2.** 6.	**3.** $68\frac{1}{2}$.	**4.** 51.	**5.** \$65.33.
6. 5.	**7.** $30\frac{5}{8}$.	**8.** \$1.	**9.** 12.	**10.** \$11.90.
11. 18 ft. 9 in.; 12 ft. 9 in.			**12.** 16; 20.	

Exercise 46, page 108

1. 120.	**2.** 240.	**3.** 252.	**4.** 480.
5. 36.	**6.** 28.	**7.** $10\frac{2}{3}$.	**8.** 288.
9. 54.	**10.** $31\frac{1}{2}$.	**11.** 9i.	**12.** \$15.46.
13. \$51.76.			

Exercise 47, page 111

1. 3; 4; 4.	**2.** 14; 16.	**3.** 11; 14.	**4.** 5; 12 ft.	**5.** 4; 3 ft.
6. 5.	**7.** 8.	**8.** \$9.17.	**9.** \$8.88.	**10.** \$6.23.

Exercise 48, page 113

1. 3250. **2.** \$0.76. **3.** \$27.30. **4.** \$6.39. **5.** \$13.87.
6. \$40.60. **7.** \$31.62. **8.** 12.384. **9.** 11.088. **10.** \$40.50.

Exercise 49, page 116

1. 120. **2.** $7\frac{1}{2}$ in. **3.** 320. **4.** 480. **5.** $166\frac{2}{3}$. **6.** 7840. **7.** \$190.97.
8. 98. **9.** \$14; 747. **10.** \$99.01. **11.** \$5.56. **12.** \$8.83. **13.** \$153.80.

Exercise 50, page 119

1. 24 ft. **2.** 27. **3.** $15\frac{5}{8}$. **6.** 8. **7.** 13,824. **8.** $\frac{1}{8}$.
9. $\frac{1}{27}$. **10.** 231. **11.** 128. **12.** 288. **13.** 10.

Exercise 51, page 122

1. 32. **2.** \$110.55. **3.** \$1493.33. **4.** 16; 8.
5. 23,156. **6.** \$537.60. **7.** \$93.60; \$102.08.
8. \$312.50. **9.** \$255. **10.** 20; 30; 60.

Exercise 52, page 126

1. 1500′. **2.** 1920′. **3.** 2700′. **4.** 5400′.
5. 7200′. **6.** 720″. **7.** 1440″. **8.** 1800″.
9. 3000″. **10.** 3480″. **11.** 5′. **12.** 7′.
13. 9′. **14.** 12′. **15.** 4°. **16.** 6°.
17. 14°. **18.** $19\frac{1}{2}$°. **19.** $\frac{1}{4}$. **20.** $\frac{1}{60}$.
21. $\frac{1}{8}$. **22.** 6 ft. **23.** 150. **24.** 21,600.
25. $\frac{5}{12}$. **26.** 47.124 in. **27.** 125.664 ft. **28.** 204.204 ft.
29. 22.61952 ft. **30.** 5.4978 rd. **31.** 3.927 yd. **32.** 14.45136 ch.
33. 23.436336 ch. **34.** 56.5488 in. **35.** 150.7968 ft. **36.** 204.204 ft.
37. 102.102 yd. **38.** 75.3984 rd. **39.** 100.5312 ch. **40.** 79.16832 rd.
41. 89.5356 ch. **42.** 5 in. **43.** 7 ft. **44.** 10 yd.
45. 2 in. **46.** 4 ft. **47.** 10 yd. **48.** 12.5664 sq. in.
49. 113.0976 sq. ft. **50.** 201.0624 sq. yd. **51.** \$25.13. **52.** 17.67.
53. 7.854 A.

Exercise 53, page 128

1. 251.328 sq. in. **2.** 251.328 sq. ft. **3.** 72 sq. ft. **4.** 188.496 sq. ft.
5. 376.992. **6.** 226.1952 sq. in. **7.** 143.66. **8.** 565.488.
9. 1570.8. **10.** 2150.4252. **11.** 4512 gal. **12.** 4230 gal.

Exercise 54, page 132

1. 29 pt. **2.** 8805 lb. **3.** 154 qt. **4.** 15,912″.
5. 50 ft. **6.** 771½ sq. ft. **7.** 10⅛ gal. **8.** $4\frac{11}{18}$ bu.
9. 2.2825 T. **10.** $16\frac{2}{3}$ rd. **11.** $1\frac{7}{18}°$. **12.** $1\frac{125}{242}$ A.
13. 776½ gal. **14.** 45½ ft. **15.** 19¼ pk. **16.** $340\frac{4}{11}$ sq. rd.
17. 64.3 cwt. **18.** 4 yd. 4.5 in. **19.** 14 hr. 24 min.
20. 22 sq. yd. 6 sq. ft. 27 sq. in. **21.** 58 rd. 1 yd. **22.** 1 A. 64 sq. rd.
23. 18 sq. yd. 1 sq. ft. 50.4 sq. in. **24.** 2° 4′ 12″. **25.** 4 yd. 4.5 in.

Exercise 55, page 134

1. 47 yr. 7 mo. 7 da. **2.** 9 bu. 3 pk. 5 qt. **3.** 222 rd. 4 yd. 10 in.
4. 25 T. 2 cwt. 26 lb. **5.** 24 A. 147 sq. rd. 5 sq. yd. 6 sq. ft. 108 sq. in.
6. 180° 9′ 43″. **7.** 25 yr. 8 mo. 15 da. **8.** 11 rd. 4 yd. 6 in.
9. 104° 29′ 15″. **10.** 7 A. 119 sq. rd. 29 sq. yd. 2 sq. ft. 36 sq. in.
11. 8 T. 8 cwt. 25 lb. **12.** 1828 yr. 7 mo. 16 da.

Exercise 56, page 135

1. 51 bu. 2 pk. 4 qt. **2.** 195 yr. 1 mo. 15 da. **3.** 407° 2′ 15″.
4. 3 mi. 129 rd. 1 yd. 1 ft. 6 in. **5.** 124 T. 14 cwt. 50 lb.
6. 30 A. 150 sq. rd. 15 sq. yd. **7.** 30 yr. 11 mo. 26 da. 12 hr.
8. 21 bu. 3 pk. $5\frac{2}{3}$ qt. **9.** 19° 30′ $15\frac{2}{3}$″. **10.** 13 T. 19 cwt. $12\frac{3}{7}$ lb.
11. 1 A. 77 sq. rd. 27 sq. yd. 121½ sq. in. **12.** 3 rd. 3 yd. 7 in.

Exercise 57, page 135

1. Dec. 14, 1799. **2.** 11 hr. 34 min. **3.** $5.31. **4.** 20 cwt. 11 lb.
5. 85. **6.** 22 yd. 6 in. **7.** 7 T. 15 cwt. 40 lb.
8. 2 da. 10 hr. 55 min. **9.** 2 hr. 34 min. 30 sec. **10.** 38 yd. 2 ft.
11. 4° 8′ 10″. **12.** $42.90. **13.** 64 ch. 95 li.
14. 17° 45′ 20″.

Exercise 58, page 139

1. 720*d*. **2.** 528*d*. **3.** £12. **4.** £11 9*s*. 10*d*
5. $19.30. **6.** $238. **7.** $38.60.
8. 4 marks; 5 francs; 5 crowns; 5 lire; 5 pesetas. **9.** $103.
10. Half dollar. **11.** Austria-Hungary, 20¢; Great Britain, $1.20.
12. £205.49. **13.** 518 francs. **14.** 420 marks. **15.** 5760.
16. 12.9 gr. **17.** 2 oz. 8 pwt. 9 gr. **18.** 15; 191.25. **19.** 36.96 gr.
20. 240. **21.** 3 pwt. 12 gr. **22.** 8. **23.** $4\frac{23}{48}$.
24. 68. **25.** 29.26.

Exercise 59, page 141

1. $2,150,625.	**2.** 864.	**3.** 112.	**4.** $119.
5. 25.	**6.** 100.	**7.** $2.80.	**8.** March 1, 1914.
9. $15.	**10.** $5500.	**11.** $5.30.	**12.** 10,800.
13. Feb. 28, 1914.	**14.** 40.	**15.** 432.	**16.** 3380 lb.
17. $2048.	**18.** 60¢.	**19.** 160.	**20.** 36.
21. $33.15.	**22.** 11.52.	**23.** 216.	**24.** 1838.
25. 437.5.	**26.** 528.	**27.** 269.	**28.** $192.60.
29. $14.80.	**30.** $22.50.	**31.** 67.5.	**32.** 1344.
33. 360°; 360°.	**34.** 129.6.	**35.** 1440.	**36.** 1440.
37. 2 lb.	**38.** 30.	**39.** $12.71.	**40.** $40.
41. 8.	**42.** 68.	**43.** $4.	**44.** 88.
45. 186.49 lb.	**46.** $4083.79.	**47.** $756.	**48.** $7.39.
49. $1129.10.	**50.** 20.	**51.** 13.2.	

Exercise 60, page 146

1. 2,400,000,000,000.	**2.** 6.	**3.** 937.5 lb.	**4.** $26,586.01.
5. 6.25.	**6.** $1.25.	**7.** $4.00.	**8.** 3¢.
9. $1\frac{1}{8}$.	**10.** 1.15.	**11.** $18.75.	**12.** $540.
13. $5,856,000,000.	**14.** 8 ft.	**16.** $17.20.	
17. 312.5 lb.; 125 lb.; 78.125 lb.		**18.** 4.	**19.** $24.43.
20. .2.	**21.** 640.	**22.** 60.	**23.** $4014.18.
24. $487.50.	**26.** 3.3.	**27.** $25.3058.	**28.** 38.
29. 10 hr.	**30.** $1.27025.	**31.** 12.	**32.** $18,150.
33. 79 lb.	**34.** $22.50.	**35.** 9300.	**36.** $349.45.

Exercise 61, page 152

2. 2.	**3.** 5.	**4.** 1.	**5.** 10.	**6.** 5.
7. 7.	**8.** 1.	**9.** 4.	**10.** 2.	**11.** 5.
12. 4.	**13.** 2.	**17.** 8.	**18.** $2\frac{1}{3}$.	**19.** $3\frac{3}{4}$.
20. $4\frac{1}{5}$.	**21.** $\frac{1}{4}$.	**22.** $\frac{1}{2}$.	**23.** 8.	**24.** 15.
25. 2.	**26.** $1\frac{1}{3}$.	**27.** $12\frac{1}{2}$.	**28.** 100.	**29.** 9.
30. 12.	**31.** 15.	**32.** $10\frac{1}{2}$.	**33.** $13\frac{1}{3}$.	**34.** $18\frac{3}{4}$.
35. $\frac{2}{3}$.	**36.** $1\frac{1}{8}$.	**37.** $3\frac{1}{3}$.	**38.** $4\frac{1}{6}$.	**39.** 9.
40. 6.	**42.** 3.	**43.** 2.	**44.** 4.	**45.** 1.
46. $\frac{1}{2}$.	**47.** $\frac{3}{4}$.	**48.** 2.	**49.** 16.	**50.** 16.

Exercise 62, page 155

1. 24; 6. **2.** 5; 10. **3.** 40¢; 10¢. **4.** 20; 10. **5.** 280 A.; 70 A.
6. $48. **7.** $12.50. **8.** $200. **9.** $100. **10.** 35; 30. **11.** 400 A.

Exercise 65, page 162

1. 16.5.	**2.** 31.2.	**3.** 35.52.	**4.** 81.25.
5. $117.75.	**6.** $57.50.	**7.** 171 bu.	**8.** 574 ft.
9. $2400.	**10.** $69.375.	**11.** 540 pupils.	**12.** $100.
13. 50 A.	**14.** $350.	**15.** $25.	**16.** $94.
17. 450.	**18.** 750.	**19.** $875.	**20.** $2800.
21. $720.	**22.** $600.	**23.** $360.	**24.** 432 bu.
25. 880 A.	**26.** $240.	**27.** 5000	**28.** 6000.
29. 25%.	**30.** 45%.	**31.** 18%.	**32.** 20%.
33. $33\frac{1}{3}$%.	**34.** 16%.	**35.** $66\frac{2}{3}$%.	**36.** 6%.
37. 8%.	**38.** 4%.	**39.** 2%.	**40.** 25%.
41. 26.67%.	**42.** 9.33%.	**43.** 12.17%.	**44.** 14.77%.
45. 15.11%.	**46.** 9.99%.	**47.** 3%.	**48.** 9.48%.
49. 0.48%.	**50.** 0.40%.		

Exercise 66, page 165

1. 32.	**2.** 24.	**3.** 5.	**4.** 40.
5. 80%.	**6.** 10%.	**7.** $13\frac{1}{2}$ lb.	**8.** 200%.
9. 90%.	**10.** 6%.	**11.** 21 ft.	**12.** 1500%.
13. $2000.	**14.** 700 bu.	**15.** $2\frac{1}{2}$¢.	**16.** 20.
17. 30.	**18.** $133\frac{1}{3}$ lb.	**19.** 40%; 60%.	**20.** $12\frac{1}{2}$%.
21. 240 sq. ft.	**22.** 64.	**23.** 10%; 90%.	**24.** 75%.
25. $330.	**26.** 41.25 gr.; 371.25 gr.	**27.** $68.75.	**28.** 110.
29. 100 lb.; 30 lb.; 650 lb.		**30.** 1495.2.	**31.** 32.4.
32. 40.	**33.** $45.	**34.** $1530.75.	**35.** 60.48.
36. $200.	**37.** $7\frac{1}{2}$%.	**38.** $8\frac{1}{3}$%.	
39. $14\frac{2}{7}$%; $28\frac{4}{7}$%; $57\frac{1}{7}$%.		**40.** 14%.	**41.** 20%.
42. $42.	**43.** 95.39%; 4.61%.	**44.** 5.56%.	**45.** 311%.
46. 38.1%.	**47.** 91%.	**48.** 2.4%.	**49.** 12%.
50. $3000.			

Exercise 67, page 171

11. $389.69.	**12.** $480.	**13.** 15¢.	**14.** $3.25.
15. $198.	**16.** $190.	**17.** $106.40.	
18. $350; $339.50; $346.50.		**19.** $15.75.	**20.** $316.
21. 60%.	**22.** 44%.	**23.** 46%.	**24.** 36%.
25. 19%.	**26.** 32%.	**27.** 56.8%.	**28.** 32%.
29. 75%.	**30.** 25%.	**31.** 54%.	**32.** 20%.
33. 40%.		**34.** R^{yy}; A^{ar}; K^{uc}; T^{un}.	

Exercise 68, page 176

4. \$3.50; \$7. **5.** \$0.30; \$2.10. **6.** 3¢; 72¢.
7. \$250; \$1750. **8.** \$2.40; \$6. **9.** \$1.125; \$46.125.
10. \$112.50; \$262.50. **11.** \$150; \$2150. **12.** \$48; \$12.
13. \$64; \$8. **14.** \$2.10; \$4.20. **15.** \$4; \$0.80.
16. \$5; \$2.50. **17.** \$23.10. **18.** \$15.25.
19. \$6. **20.** \$7.20. **21.** 40% gain.
22. $66\frac{2}{3}$% loss. **23.** $2\frac{1}{2}$% gain. **24.** 2% loss.
25. 25% loss. **26.** $12\frac{1}{2}$% gain. **27.** 20%.
28. $16\frac{2}{3}$%. **29.** $33\frac{1}{3}$%. **30.** $6\frac{2}{3}$%. **31.** \$1.08.
32. \$3.36. **33.** \$7.20. **34.** \$8.75. **35.** 200%.
36. 50%. **37.** 20%. **38.** \$20. **39.** \$2.50.
40. \$2. **41.** \$0.90. **42.** Neither. **43.** 20% loss.
44. $17\frac{1}{4}$% gain. **45.** 50% gain. **46.** 55% gain. **47.** 24¢.
48. 125%. **49.** 25%. **50.** $6\frac{2}{3}$ oz. **51.** $7\frac{1}{2}$¢.
52. \$48. **53.** 25¢. **54.** 5% gain. **55.** 20%.
56. \$2.50. **57.** 25%. **58.** $66\frac{2}{3}$%. **59.** $42\frac{2}{7}$. **60.** 8%.

Exercise 69, page 183

19. \$12.50; \$117.50. **20.** \$400; \$430. **21.** 8%; \$16.
22. \$500; \$20. **23.** \$210. **24.** \$108.65. **25.** \$1005.67.
26. \$197.93. **27.** \$1127.50. **28.** \$565.50. **29.** \$8431.70.
30. 1000. **31.** \$106.75. **32.** \$20. **33.** 423; \$1.325.
34. \$390. **35.** 15%. **36.** \$12.50. **37.** $12\frac{1}{2}$%.
38. \$65. **39.** 5%. **40.** \$9.30.

Exercise 71, page 194

1. \$10.25. **2.** \$28.63. **3.** \$56.375. **4.** \$4.92.
5. \$9.60. **6.** \$20.85. **7.** \$27.50. **8.** \$7.70.
9. \$7.59. **10.** \$64.72. **11.** \$21.825. **12.** \$98.17.
13. \$28.59. **14.** \$191.25. **15.** \$0.71. **16.** \$45.25.
17. \$15.50. **18.** \$19.26. **19.** \$3.50; \$353.50.
20. \$9.60; \$489.60. **21.** \$128; \$86.88. **22.** \$1; \$1001.
23. \$0.50; \$250.50. **24.** \$3.40; \$683.40. **25.** \$7.17; \$485.17.
26. \$3.74; \$343.74. **27.** \$17.50; \$367.50. **28.** \$7.41; \$285.41.
29. \$1.49; \$41.09. **30.** \$0.87; \$83.81. **31.** \$2.22; \$102.22.
32. \$3.84; \$101.84. **33.** \$5.78; \$213.63. **34.** \$3.60.
35. \$7.40. **36.** \$10.38. **37.** \$25.57. **38.** \$4.29.

Exercise 72, page 197

1. $200. 2. $300. 3. $500. 4. $300. 5. $120.
6. $450. 7. $225. 8. $273.85. 9. $965.86.

Exercise 73, page 199

1. $4\frac{1}{2}\%$. 2. $4\frac{4}{5}\%$. 3. 5%. 4. 4%. 5. 4%.
6. 4.8%. 7. 4%. 8. 5%. 9. $6\frac{2}{3}\%$.

Exercise 74, page 200

1. 3 yr. 72 da. 2. 2 yr. 315 da. 3. 1 yr. 4. 1 yr. 90 da.
5. 3 yr. 120 da. 6. 11 yr. 40 da. 7. Sept. 1. 8. July 19.
9. 25 yr. 10. 20 yr. 11. 11 yr. 40 da.

Exercise 75, page 202

1. $675.31; $75.31. 2. $579.64; $79.64. 3. $552.24; $52.24.
4. $476.41; $76.41. 5. $350.96; $50.96. 6. $296.92; $46.92.
7. $3479.08; $479.08. 8. $2377.37; $377.37. 9. $848.56; $98.56.
10. $2390.19; $390.19.

Exercise 76, page 205

1. $2. 2. $15. 3. $13.50. 4. $52.50.
5. $18. 6. $39.375. 7. $16.80. 8. $59.50.
9. $106.25. 10. $15. 11. 1 mill. 12. 6 mills.
13. 4 mills. 14. 16 mills. 15. 5 mills. 16. 9 mills.
17. 6 mills. 18. 8 mills. 19. 1.6 mills. 20. $1\frac{1}{2}$ mills.
21. $360. 22. $195. 23. $50. 24. $358.40.
25. $180. 26. $750. 27. $55.65. 28. $485.10.
29. $22.50. 30. $4\frac{1}{2}$ mills. 31. $7\frac{1}{2}$ mills; $56.25. 32. $3000.
33. $4\frac{1}{2}$ mills. 34. $7240; $144.80. 35. $214.20. 36. $2454.65.
37. $306.60.

Exercise 77, page 208

1. $11.20. 2. $4.80. 3. $6.875. 4. $16.64. 5. $15.
6. $8. 7. $5.95. 8. .16%; $1.60. 9. $0.40. 10. $400; $40.

Exercise 78, page 211

1. $46.98. 2. $49.26. 3. $23.31. 4. $49.03.
5. $38.96. 6. $20.55. 7. $33.44. 8. $30.63.
9. $31.48. 10. $46.62. 11. $247.05. 12. $247.85.

Exercise 79, page 212

1. 100%. **2.** $33\frac{1}{3}\%$. **3.** $292.80. **4.** 30¢.
5. $12\frac{1}{2}\%$. **6.** 25%. **7.** $33\frac{1}{3}\%$. **8.** $12\frac{1}{2}\%$ gain.
9. $175. **10.** 25%. **11.** 4.8%. **12.** $12.
13. The latter by 44¢. **14.** 1.4%. **15.** $6\frac{1}{4}$. **16.** $1737.
17. $1. **18.** 75%. **19.** $53\frac{1}{3}\%$. **20.** $6.07.
21. $4926. **22.** Red, $9.98; Spot, $10.63; Bess, $10.58; Beauty, $9.72.
23. 12¢. **24.** $12.40. **25.** 100%. **26.** 40%.
27. $3.725. **28.** $27\frac{7}{9}\%$. **29.** $12\frac{1}{2}\%$ gain. **30.** .06%.
31. $6.50. **32.** $4,651,815; $265,818,000. **33.** $20.
34. 60%. **35.** $34.20. **36.** $69.60. **37.** $40; $50; $62.50.

Exercise 80, page 223

1. $1632.50. **2.** $3184.375. **3.** $701. **4.** $13,625.
5. $10,507.50. **6.** $45,275. **7.** $883.75. **8.** $1273.50.
9. $1312.34. **10.** $3453.75. **11.** $508.125. **12.** $206.75.
13. $10,262.50. **14.** $5902.50. **15.** 10. **16.** 54; $40.125.
17. 21; $35.875. **18.** 19; $27.94. **19.** 87; $92.875. **20.** 12.
21. 16. **22.** 38; $52.25. **23.** $120. **24.** $150.
25. $160. **26.** $75. **27.** $700. **28.** $400.
29. $64. **30.** $800. **31.** $1137.50. **32.** $275.
33. $140. **34.** $144. **35.** $24. **36.** $320.
37. $120. **38.** $160. **39.** $400. **40.** 44.
41. $2688.75. **42.** $5377.50. **43.** 4.82%. **44.** 4.87%.
45. 4.17%. **46.** 3.55%. **47.** $119.875. **48.** $124.875.

Exercise 81, page 231

1. $557.15. **2.** $667.75. **3.** $523.23. **4.** $301.20.
10. $852.45. **11.** $362.90.

Exercise 82, page 234

1. $254.90. **2.** $384.08. **3.** $441.22.

Exercise 83, page 238

1. $1, $2. **2.** $1-certificate and savings card with 4 stamps. **3.** $100.
4. Dec. 1. **5.** Aug. 1; 2%; Aug 1, next year. **6.** Mar. 1; Mar 1, next year; $1. **7.** Dec. 1; $1.60. **8.** July 1; $5. **9.** Jan. 1, next year; $3.75.
10. $1. See § 311.

Exercise 84, page 244

3. Mar. 31, 1915. 4. Feb. 28, 1914. 7. $155.625.
9. July 2, 1912; $222.475. 10. $15. 11. Feb. 29, 1916.

Exercise 86, page 253

1. $357.56. 2. $555.90. 3. $65.76. 4. $119.87.

Exercise 87, page 256

1. $588.10. 2. $197. 3. $247.50. 4. $220.54.
5. $148.12. 6. $117.29. 7. $491.25. 8. $196.97.
9. $311.48. 10. $147.92. 11. $98.72. 12. $152.70.
13. $454.99. 14. $264.02. 15. $202.92. $202.88.* 16. $355.07. $355.01.*
17. $502.32. $502.23.* 18. $280.37. $280.32.* 19. $254.97. $254.93.* 20. $219.49. $219.45.*
21. $594. 22. $267.18. 23. $242.84. 24. $353.54.
25. $101.03. 26. $304.57 in common years. $304.62 in leap years.

Exercise 88, page 259

1. $251.25. 2. $316.41. 3. $147.38.
4. $274.66. 5. $218.075. 6. $198.27.

Exercise 89, page 261

3. $\frac{2}{5}$. 4. $\frac{1}{2}$. 5. $\frac{16}{15}$. 6. $\frac{7}{30}$.
7. $\frac{8}{15}$. 8. $\frac{6}{7}$. 9. $2\frac{1}{2}$. 10. 48.
11. 3. 12. $\frac{3}{16}$. 13. 1. 14. 12.
15. $\frac{1}{8}$. 16. 1.25. 18. $\frac{25}{28}$. 19. 21.53.
20. 200.

Exercise 90, page 264

1. 2. 2. 6. 3. 4. 4. 125.
5. $\frac{1}{4}$. 6. $\frac{1}{4}$. 7. $2\frac{1}{4}$. 8. $8\frac{1}{3}$.
9. $3\frac{2}{3}$. 10. 6. 11. $75. 12. $4.

* In Pennsylvania, Delaware, Maryland, Missouri, and the District of Columbia.

Exercise 91, page 267

2. 29.82 lb. **3.** 360 bu. **4.** $7.60. **5.** $21.
6. 42 lb. **7.** 600 mi. **8.** 1160 lb. **9.** 20 in.
10. $2.40. **11.** 8 hr. **12.** 80 lb. **13.** $266\frac{2}{3}$.
14. 50 in. **15.** $5\frac{3}{5}$ da. **16.** 225.

Exercise 92, page 270

1. $750; $500. **2.** 100; 150; 250. **3.** 270; 210.
4. 300; 250. **5.** $157.50; $67.50. **6.** 12; 8.
7. 2 lb. tin; 3 lb. lead. **8.** $7.20; $10.80; $18. **9.** $15; $22.50; $25.
10. 120 lb.; 40 lb.; 40 lb.; 60 lb.; 100 lb.

Exercise 93, page 270

1. $400; $500; $600. **2.** A, $3424; B, $2996.
3. A, $1600; B, $888.89; C, $711.11.
4. C's capital, $9900; A's gain, $100; B's gain, $80.
5. A's, $270; B's, $240. **6.** A's, $282.20; B's, $188.14; C's, $84.66.
7. A, $1800; B, $2000; C, $200. **8.** A's, $3240; B's, $3000; C's, $4410.
9. A's, $600; B's, $900; C's, $1500.

Exercise 95, page 275

5. 14. **6.** 15. **7.** 22. **8.** 33.
9. 35. **10.** 36. **11.** 51. **12.** 55.
13. 24. **14.** 45. **15.** 39. **16.** 42.
17. 105. **18.** 8 ft. **19.** 10 rd. **20.** 25 yd.
21. .05 mi.

Exercise 96, page 278

1. 32. **2.** 86. **3.** 724. **4.** 706.
5. 46. **6.** 95. **7.** 508. **8.** 760.
9. 59. **10.** 119. **11.** 564. **12.** 802.
13. 7.3. **14.** 2.78. **15.** 76.5. **16.** 8.08.
17. 5.66. **18.** 17.92. **19.** 1.85. **20.** .15.
22. $\frac{11}{12}$. **23.** $\frac{16}{27}$. **24.** $2\frac{7}{8}$. **25.** .94.

Exercise 97, page 283

4. 5. **5.** 50°. **6.** 50; 80. **7.** 60.
8. 70; 90. **9.** Acute. **10.** Obtuse. **11.** Right.
14. 10 ft. **15.** 6 ft. **16.** 6 ft. **17.** 10 ft.
18. 20 ft. **19.** 9 ft. **20.** 16 ft. **21.** 65 ft.
22. 59.90 ft. **23.** 30 ft.

Exercise 98, page 290

1. 80 sq. ft.	**2.** 45 sq. ft.	**3.** 168 sq. ft.	**4.** 66 sq. yd.
5. 57 sq. ft.	**6.** 98 sq. ft.	**7.** 42 sq. ft.	**8.** 105 sq. ft.
9. 28 sq. yd.	**10.** 408 sq. ft.	**11.** 96 sq. rd.	**12.** 15 sq. ft. 63 sq. in
13. 50 sq. ft.	**14.** 105 sq. ft.	**15.** 121.5 sq. ft.	**16.** 10.5 sq. ft.
17. 33 sq. ch.	**18.** 34.5 sq. ft.	**19.** 144.	**20.** 6.
21. 8.	**22.** 160.	**23.** 15.	**24.** 10.
25. 4.	**26.** 20.	**27.** 20.	**28.** 166.32 sq. ft.
29. 482.8 sq. ft.	**30.** 27.5 sq. ft.	**31.** 84 sq. ft.	**32.** 60 ft.
33. 48 rd.	**34.** 768 sq. ft.	**35.** 208.7.	**36.** 289 sq. ft.
37. 20.	**38.** 10.	**39.** 10 ch.; 8 ch.	**40.** 27.216. **41.** 3 A.

Exercise 99, page 294

1. 113.0976 sq. in.	**2.** 50.2656 sq. yd.	**3.** 452.3904 sq. rd.
4. 12.5664 sq. ch.	**5.** 19.635 sq. ft.	**6.** 78.54 sq. yd.
7. 19.635 sq. ch.	**8.** 32.169984 sq. ft.	**9.** 5.9395875 sq. rd.
10. 27.339774 sq. ch.	**11.** 28.2744 sq. ft.	**12.** 78.54 sq. rd.
13. .7854 sq. ch.	**14.** 12.5664 sq. in.	**15.** 31,416 sq. yd.
16. 1963.5 sq. ft.	**17.** 32 ft.; 64 sq. ft.	**18.** 10.3008 ft.
19. 21.46 sq. ft.	**20.** 1243.20 sq. ft.	**21.** 1256.64 sq. yd.
22. 1 sq. ft. 56. sq. in.		

Exercise 100, page 298

1. 72 sq. ft.	**2.** 150.7968 sq. in.	**3.** 519.68 cu. ft.
4. 565.488 cu. in.	**5.** 96 sq. ft.	**6.** 96 sq. ft.
7. 301.5936 sq. in.	**8.** 96 cu. ft.	**9.** 2035.7568 cu. in.
10. .375.	**11.** 1269.04 gal.	**12.** 1570.8.
13. 5 ft.	**14.** 7284.585 lb.	**15.** 24.41 ft.
16. 10 ft.	**17.** 1692.06 gal.	**18.** 10 ft.

Exercise 101, page 303

1. 240 sq. ft.	**2.** 150.7968 sq. in.	**3.** 187.06 cu. yd.
4. 56.5488 cu. in.	**5.** 300 sq. in.	**6.** 678.5856 sq. in.
7. 108 cu. in.	**8.** 84.8232.	**9.** 5 ft.
10. 80 sq. ft.	**11.** 48 cu. ft.	**12.** 8 in.
13. 251.328 sq. in.; 402.1248 cu. in.		**14.** 1357.1712 cu. in.
15. 47.124.	**16.** 960 sq. ft.; 3072 cu. ft.	
17. 1570.8 sq. ft.; 6283.2 cu. ft.		

Exercise 102, page 306

1. 160 sq. ft.	2. 75.3984 sq. ft.	3. 38 cu. in.	4. 360 sq. in.
5. 795.872 cu. in.	6. 179.0712.	7. 105.	8. 15.81.

Exercise 103, page 307

1. 50.2656 sq. in.	2. 113.0976 cu. in.	3. 452.3904 sq. in.
4. 201.0624 sq. ft.	5. 50.2656 sq. rd.	6. 314.16 sq. ch.
7. 128.679936 sq. in.	8. 254.4696 sq. ft.	9. 572.5566 sq. yd.
10. 145.2675 sq. ch.	11. 4.1888 cu. in.	12. 523.6 cu. ft.
13. 1204263.2448 cu. yd.	14. 18816613.2 cu. yd.	15. 14.1372 cu. in.
16. 927589.3396 cu. yd.	17. 1361155.46875 cu. yd.	18. .5236 cu. ft.
19. 50.2656 sq. ft.	20. 113.0976 cu. yd.	21. 3 ft.

22. 201,062,400 sq. mi. 23. 14.1372 cu. in.; 21.2058 cu. in.; 28.2744 sq. in.; 28.2744 sq. in. Volume of sphere $= \frac{2}{3}$ volume of cylinder. Area of sphere = lateral area of cylinder.

Exercise 104, page 310

1. $3\frac{1}{3}$ ft.	2. 96 sq. ft.	3. 15.708 ft.	4. 43.304 sq. ft.
5. $\frac{1}{4}$.	6. 1562.5 cu. ft.	7. 12 rd.	8. 128 lb.
9. 80 lb.	10. 960 gal.	11. 72 hr.	

Exercise 105, page 311

1. 16.	2. 9.6.	3. $952.875.	4. 2310 cu. ft.
5. 120 sq. ft.	6. 20 rd.; 16 rd.	7. 5184 sq. ft.	8. 512 sq. in.
9. 184.	10. 22 bu.	11. 27 cu. in.	12. 93.5 sq. ft.
13. 259.81 sq. ft.	14. $301\frac{5}{7}$.	15. 28.2744.	16. 266.
17. 46.875 lb.	18. $49\frac{5}{27}$.	19. 2981.25 lb.	20. 16 ft.
21. $27.	22. 791.68.	23. 67.0208.	24. 2827.44 sq. ft.
25. 4704.	26. $247\frac{1}{2}$.	27. $170\frac{6}{25}$.	28. $478\frac{5}{7}\frac{3}{7}$.
29. 15.588.	30. 96.	31. 35 sq. yd.	32. $347.25.
33. 24 hr.	34. 115.5 yd.	35. 29.92.	36. $\frac{1}{2}$ in.
37. $49.54.	38. 19.14 ft.	39. 16.2.	

General Review

1. 75.	2. 5000; 20,000.	3. $5000.	4. 55.
7. $743\frac{2}{11}$ mi.	8. 176.	9. $\frac{1}{5}$.	
10. (1) 96; (2) 16; (3) 144; (4) 576.		11. $50.	12. 600 sq. in.
14. $27,390.	15. 150; 45; 30.	16. $155.	17. 270.
18. $9.	19. $1452.	20. $43\frac{3}{4}$; $15\frac{5}{8}$; $9\frac{3}{8}$.	21. $25.

22. \$0.871. **23.** 528; 2816. **24.** $\frac{1}{6}$ bu.; $16\frac{2}{3}$ lb. **25.** 15.2.
26. $2\frac{1}{6}$. **28.** 1.25. **29.** 9¢. **30.** 10.
31. 3.458. **32.** 4.8. **33.** \$463,404.26. **34.** 220; \$33.
35. 200 da. **36.** $\frac{1}{4}$%. **37.** 140%. **38.** 25%.
39. 42.06%; 57.94%. **40.** $41\frac{2}{3}$%. **41.** \$30.
42. 6.4%. **43.** 44%. **44.** 60%. **45.** $233\frac{1}{3}$.
46. 100%. **47.** 12¢. **48.** \$70. **49.** 4.2%.
50. \$1.08. **51.** 20%. **52.** 4.225%. **53.** \$2.80.
54. $4\frac{4}{5}$%. **55.** 4 mills. **56.** \$5. **57.** \$14.97.
58. 25¢. **59.** 50%. **60.** 20%. **61.** 60¢.
62. \$50 loss. **63.** 140%. **64.** \$4480. **65.** 36¢.
66. 400%. **67.** $\frac{2}{3}$. **68.** 200. **69.** \$1760.15.
70. \$60.48. **71.** $48\frac{8}{9}$%. **72.** 5 mills. **73.** \$67.20.
74. \$12 loss. **75.** \$0.32; \$0.56; \$0.80. **76.** \$488.60.
77. Nov. 28. **78.** 2%. **79.** 20¢.
80. 4%; 4.4%; 4.38%; $4\frac{1}{2}$%; $4\frac{1}{2}$%. **81.** \$149.875. **82.** 5.4%.
83. \$61.06. **84.** \$85. **85.** \$4978.125. **86.** \$103.03.
87. \$7632.81. **88.** 4.21%. **89.** \$119.75.
90. 363.64 lb.; 1636.36 lb. **91.** 132. **92.** 50 yd.
93. 10 rd. **94.** \$74.875. **95.** 64 gal. **96.** \$1000.
97. \$12. **98.** \$80; \$11. **99.** 432 sq. ft.; 648 cu. ft.
100. \$2104.15. **102.** \$5.425; Nov. 4, 1913. **103.** 41.5704 sq. rd.
104. \$360. **105.** 72 yd.; 108 yd. **106.** 50 ft. **109.** \$995.
110. \$54.72. **111.** 1500 oz. **112.** 17.321 in. **113.** $4\frac{2}{7}$.
114. $\frac{5}{11}$. **115.** $5\frac{5}{8}$ in. **116.** 10.
117. 270 yd.; 450 yd. **118.** \$750; \$1000; \$1250. **119.** 9408 cu. ft.
120. 7.85 A. **121.** 180. **122.** $26\frac{1}{2}$. **123.** 6376.32 lb.
124. 831 lb. **125.** 23 hr. 15 min. 18.8 sec. **126.** $\frac{1}{2}$; 8; 6; $\frac{1}{2}$.
127. 420. **128.** 1920. **129.** 425. **130.** $\frac{15}{16}$.
131. 439.824 sq. ft. **132.** \$30; \$18. **133.** 3.501. **134.** 12 hr.
135. 24897.27 mi. **136.** \$209.14. **137.** 3 lb. **138.** \$4.68.
139. 10 da. **140.** 18.03 ft. **141.** 489.375 lb. **142.** 272.25.
143. 1963.5 cu. ft. **144.** 98 ft. **145.** 6.9284 sq. rd. **146.** 140 rd.
147. $12\frac{1}{2}$%. **148.** 60¢. **149.** $59\frac{43}{48}$. **150.** 1¢.
151. $80\frac{2}{5}$%. **152.** \$89$\frac{3}{4}$. **153.** \$5234.16.
154. \$3000; \$3600. **155.** 2 pk. **156.** 21. **157.** $64\frac{17}{22}$¢.
158. $566\frac{2}{3}$.

SUPPLEMENT

Exercise 1, page 333

1. 45° N. 2. 65° S. 3. 40° N. 4. 10° S.
5. 85°N. 6. 110° S. 7. 15° E. 8. 65° W.
9. 50° W. 10. 40° E. 11. 80° W. 12. 70° W.

Exercise 2, page 339

1. 105° E. 2. 105° W. 3. 60° W. 4. 150° E.
5. 90° W. 6. 60° W. 7. 90° E. 8. 120° E.
9. 120° E. 10. 15° E. 11. 90° W. 12. 15° W.
13. 90° E. 14. 150° E. 15. 75° W. 16. 150° W
17. 10 P.M. 18. 10 A.M. 19. 1 P.M. 20. Noon.
21. 9 A.M. 22. 6 P.M. 23. 105° W. 24. 60° W. 25. 90° E.

Exercise 3, page 341

1. 4 hr. 50 min. 41.2 sec. P.M.
2. 3 hr. 9 min. $8\frac{3}{8}$ sec. P.M.
3. 4 hr. 45 min. $49\frac{2}{15}$ sec. A.M. Wednesday.
4. 92° 17′ 28.5″ W.

Exercise 4, page 343

1. Central, 11 A.M.; Mountain, 10 A.M.; Pacific, 9 A.M.
2. Pacific, 11 A.M.; Central, 1 P.M.; Eastern, 2 P.M.
3. 1 hr. 4. Too fast by 3 hr. 5. 10 min.
6. Later. 7. 117 hr.

Exercise 5, page 345

1. 36. 2. 640; 320; 160; 40. 12. 80.
13. 80. 14. 40. 15. 40.

Exercise 6, page 347

6. 25,000^{m}. 7. .375^{m}. 8. 1^{Km} 425^{m} 620^{mm}.

Exercise 7, page 348

1. 400^{qdm}; 40,000^{qcm}. 2. .65^{qm}. 3. 4.06^{qm}; 406^{qdm}.
4. 4.25^{a}. 5. .0406^{qm}; 4.06^{qdm}. 6. .00015^{qm}; 150^{qmm}.
7. 14.07^{a}; .1407^{Ha}. 8. 7^{Ha} 58^{a} 32.4^{ca}. 9. 45^{Ha} 67^{a} 40^{ca}.
10. 4^{Ha} 56^{a} 78^{ca}.

Exercise 8, page 349

2. 5400^{cdm}; $5{,}400{,}000^{ccm}$; $5{,}400{,}000{,}000^{cmm}$.
3. $.0000075^{cbm}$; 7500^{cmm}. **4.** $146^{cbm}\ 750^{cdm}$.
6. $.016^{cbm}$; $16{,}000^{ccm}$; $16{,}000{,}000^{cmm}$.
7. $.00025^{cbm}$; $.25^{cdm}$; $250{,}000^{cmm}$.
8. $4^{cbm}\ 462^{cdm}\ 150^{ccm}$.

Exercise 9, page 350

3. 3.025^{l}. **4.** $.786^{l}$; 786^{ml}. **5.** 400^{cl}; 40^{dl}; 4^{l}.
6. 3.8^{dl}; 38^{cl}; $.38^{l}$. **7.** 2503^{l}. **8.** 748^{l}.

Exercise 10, page 351

6. 2007.003^{g}.

Exercise 11, page 353

1. 50.80. **2.** 8. **3.** 127.2. **4.** 900.
5. 16. **6.** 936. **8.** 1.03^{Kg}. **9.** 20.94^{g}.
10. 50. **11.** 1^{g}. **12.** 2.5; 2.5. **13.** 7920^{mg}.
14. 9.999. **15.** 40. **16.** 31.24. **17.** 43.2; 43.2^{MT}.

Exercise 12, page 356

1. 18. **2.** 8. **3.** 77 yd. **4.** 880. **5.** $1\frac{1}{4}$. **6.** $26\frac{1}{4}$.

Exercise 13, page 357

3. .2; .3; .4; .5. **4.** $\frac{2}{3}$; $\frac{3}{4}$; $\frac{2}{5}$; $\frac{3}{5}$; $\frac{4}{5}$. **6.** 12. **7.** 14. **8.** 15.
9. 24. **10.** 4 in. **11.** 5 in. **12.** 7 in. **13.** 9 in.

Exercise 14, page 359

1. 15. **2.** 58. **3.** 26. **4.** 37. **5.** 75.
6. 88. **7.** 113. **8.** 256. **9.** 304. **10.** 356.
11. 508. **12.** 560. **13.** 2.92. **14.** 5.01. **15.** 1.65.
16. 1.55. **17.** 6.69. **18.** .17. **20.** $\frac{1}{3}$. **21.** $\frac{5}{12}$.
22. $2\frac{1}{3}$. **23.** .84. **24.** $\frac{9}{10}$. **25.** $\frac{21}{22}$. **26.** $1\frac{4}{5}$. **27.** .66

40
36
76
9
684

8 X 20 = 360

9 | 10 4 4 | 116 sq. yds
9
14
9
54
54
0

.35 — 8
580
348
40.60

13.87

18
20
360

9)3240 (360
27 .35
54
1800
1080
12600

132
.35
660
396
46.20

18
20
360

9)136 (15-2
9 9
46

32
20
52
20
1040
1040
400
12
164

1.04
12
208
104
12.48

40
36
76
9 .76
9)684
63
54
152